The submitted abstracts are not refereed at this stage and the authors are solely responsible for the claims made in their abstracts.

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Invited Talks

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**On the Hamilton-Waterloo Problem**

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The Hamilton-Waterloo problem is a generalization of the well-known Oberwolfach problem. The problem is to determine the existence of a 2-factorization of $K_{2n+1}$ in which $r$ of the 2-factors are isomorphic to a given 2-factor $R$ and $s$ of the 2-factors are isomorphic to a given 2-factor $S$, with $r + s = n$. In this talk we consider the case when $R$ is a triangle-factor and $S$ is a Hamilton cycle. We will give some background results on this problem as well as discuss some new results.

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**Graph Transformations, Interpolation and Extremal Theorems for Graph Parameters**

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Let $J$ be a class of simple graphs. A graph transformation $\sigma$ on $J$ is a subset of $J \times J$. We will discuss several kinds of graph transformations depending on $J$. We will also discuss the interpolation and extremal property of several graph parameters in special classes of graphs.
Colouring Combinatorial Design: Recent Advances and Trends

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The notion of mixed hypergraph colourings motivated the consideration of colourings of designs with prescribed colour patterns. These are roughly of two kinds: point colourings such that the points of each block have one from a given set of colour patterns (partitions of the block size), and block colourings such that for each point the blocks incident with it have one from a given set of colour patterns (partitions of the replication number).

We discuss the existence of such colourings, and the possible numbers of colours for several types of Steiner system. We also discuss some recent advances on “classical type” colourings, in particular those concerning uniquely colourable Steiner triple systems.

Embeddings of Resolvable Group Divisible Designs and Related Structures: A Survey

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For given positive integers $v, k$ and $m$, a group divisible design, denoted by $GD(k, m; v)$, is a triple $(X, G, A)$ where $X$ is a $v$-set, $G$ is a set of $m$-subsets (called groups) of $X$, $G$ forms a partition of $X$, and $A$ is a set of $k$-subsets (called blocks) of $X$, such that each block intersects each group in at most one point, and each pair of distinct points from distinct groups is contained in a unique block. Let $P$ be a set of blocks, $P$ is called a parallel class if it forms a partition of $X$. A $GD(k, m; v)$ is called resolvable and denoted by $RGD(k, m; v)$ if the block set can be partitioned into parallel classes.

Let $(X, G, A)$ be an $RGD(k, m; v)$ and $(Y, H, B)$ be an $RGD(k, m; u)$, if $X$ is a subset of $Y$, $G$ is a subset of $H$, and each parallel class of $A$ is a part of some parallel class of $B$, then, $(X, G, A)$ is called a subdesign of $(Y, H, B)$, or $(X, G, A)$ is said to be embedded in $(Y, H, B)$. In this talk, we survey the progresses in the study of the embedding problem for resolvable group divisible designs and related structures, including our recent results on the complete solution of the embedding problem for resolvable group divisible designs with block size three.
The Connectivity of a Family of Expanders

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Expander graphs have proven useful in innumerable mathematical contexts, yet few simple explicit constructions for such graphs are known. Here we investigate the connectivity properties of such an explicit family, a collection of 3-regular graphs defined on the vertex sets $\mathbb{Z}_p$, $p$ any prime. In particular, we show that, as expected, the average connectivity $\kappa$ (as defined by Beineke, Oellermann, and Pippert) of such a graph tends to 3 as $p \to \infty$, further justifying the claim that such graphs are “well-connected.” This limiting value is contrasted with the expected value of $\kappa$ for a more general class of related graphs. The proofs will employ elementary number theory and probabilistic methods.

On the Friendly Index Sets of grids

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For a graph $G = (V,E)$ and a binary labeling (coloring) $f : V(G) \to \mathbb{Z}_2$ let $v_f(i) = |f^{-1}(i)|$. The labeling $f$ is said to be friendly if $|v_f(1) - v_f(0)| \leq 1$. The coloring $f : V(G) \to \mathbb{Z}_2$ induces an edge labeling $f^* : E(G) \to \mathbb{Z}_2$ defined by $f^*(xy) = |f(x) - f(y)|$. Let $e_f(i) = |f^*-1(i)|$. The friendly index set of the graph $G$, denoted by $FI(G)$, is defined by

$$FI(G) = \{|e_f(1) - e_f(0)| : f \text{ is a friendly vertex labeling of } G\}.$$ 

The friendly index sets of a number of graphs, including $P_2 \times P_n$ have already been determined. In this talk we will present the friendly index set of $P_n \times P_m$. 
A Result on Chromatic Uniqueness of Edge-Gluing of Graphs

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Let $G$ be a graph and let $P(G; \lambda)$ denote its chromatic polynomial. Then $G$ is said to be chromatically unique if $P(H; \lambda) = P(G; \lambda)$ implies that $H$ is isomorphic to $G$. In this paper, it is shown that the graph obtained by overlapping the cycle $C_m$ ($m \geq 3$) and the complete bipartite graph $K_{3,3}$ at an edge is chromatically unique.

On The Integer Magic Spectra of Halin Graphs

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For $k > 0$, we call a graph $G = (V,E) \mathbb{Z}_k$-magic if there exists an edge labeling $l : E(G) \rightarrow \mathbb{Z}_k^*$ such that the induced vertex labeling $l^+ : V(G) \rightarrow \mathbb{Z}_k$ defined by $l^+(v) = \sum_{(u,v) \in E(G)} l(u,v)$ is a constant map. We denote the set of all $k$ such that $G$ is $k$-magic by $IM(G)$, and call it the integer-magic spectrum of $G$. Halin graphs are planar 3-connected graphs that consist of a tree and a cycle connecting the end vertices of the tree. We determine the integer-magic spectra of some Halin graphs.

On the Balance Index Sets of the Amalgamation of Complete Graphs and Stars

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $A = \{0, 1\}$. A labeling $f : V(G) \rightarrow A$ induces an edge partial labeling $f^* : E(G) \rightarrow A$ defined by $f^*(xy) = f(x)$, if and only if $f(x) = f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f^*(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. If $|e_f(0) - e_f(1)| \leq 1$, then $G$ is said to be balanced. The balance index set of the graph $G$, $BI(G)$, is defined as $\{|e_f(0) - e_f(1)| : \text{the vertex labeling } f \text{ is friendly } \}$. In this paper we investigate the balance index sets of the graphs which are formed by the amalgamation of complete graphs and stars.
Decomposition Into Trails

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Balister has showed that a graph \( G = K_n \) for odd \( n \) and \( G = K_n - I \) for even \( n \), is arbitrarily decomposable into closed trails of prescribed lengths. Similar problems, but in the case of the complete bipartite graphs, were investigated by Horňák and Woźniak. We consider the corresponding question for open and closed trails for some families of graphs.

Neighborhoods of Unbordered Words in the \( n \)-Cube

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A word is simply a string or list. The \( n \)-cube, \( Q_n \), is the graph whose vertices are all words of length \( n \) over the binary alphabet and whose edges join only vertices that differ in exactly one entry. If a word \( w \) has a non-empty prefix which is also a suffix then \( w \) is said to be bordered. Otherwise, it is unbordered. Unbordered words have been studied extensively and arise in applications such as synchronizable coding and pattern matching. The set, \( U_n \), of unbordered words of length \( n \) forms two disjoint connected sets in \( Q_n \) that are dual under reversal. The neighborhood of a word is the set of words at Hamming distance 1 from it. We determine those unbordered words whose neighborhoods are “pure”; i.e., contain only unbordered words.

On a Conjecture Concerning the Friendly Index Sets of Trees

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For a graph \( G = (V, E) \) and a binary labeling \( f : V(G) \to \mathbb{Z}_2 \) let \( v_f(i) = |f^{-1}(i)| \). The coloring \( f \) is said to be friendly if \( |v_f(1) - v_f(0)| \leq 1 \). The coloring \( f : V(G) \to \mathbb{Z}_2 \) induces an edge labeling \( f^* : E(G) \to \mathbb{Z}_2 \) defined by \( f^*(xy) = |f(x) - f(y)| \). Let \( e_f(i) = |f^{*-1}(i)| \). The friendly index set of the graph \( G \), denoted by \( FI(G) \), is defined by

\[
FI(G) = \{ |e_f(1) - e_f(0)| : f \text{ is a friendly vertex labeling of } G \}.
\]

The friendly index sets of a number of graphs, including certain types of trees, have already been determined. Lee-Ng conjectured that the elements of the friendly index set of any tree form an arithmetic progression. In this talk we will present the friendly index set of a specific class of tree and in so doing provide infinitely many counterexamples to disprove the aforementioned conjecture.
Further Contributions to Orthogonal Arrays of Strength Six

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An orthogonal array (O-array) with two elements (say, 0 and 1), \(m\) rows, \(N\) columns, and of strength \(t = 6\) is an \((m \times N)\) matrix \(T\) with symbols 0 and 1 such that in every \((6 \times N)\) submatrix \(T^*\) of \(T\), every \((6 \times 1)\) vector with \(i\) \((0 \leq i \leq 6)\) ones in it appears the same number (say, \(\mu\)) of times. Here, \(\mu\) is called the index of the O-array, and clearly \(N = 2^6 \mu = 64\mu\). In this paper, we present some results on the existence of orthogonal arrays by using the concept of balanced arrays (B-arrays) which have weaker combinatorial constraints as opposed to O-arrays. We briefly discuss our results in relation to those already available in the literature.

The Admissibility of Sporadic Simple Groups

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The Cayley table of a group is a latin square, and it is known that this square has an orthogonal mate if and only if the group admits complete mappings. In 1955 Hall and Paige conjectured that a finite group is admissible, i.e. admits complete mappings, if its Sylow 2-subgroup is trivial or non-cyclic. In a recent paper Wilcox proved that a minimal counterexample must be simple, and further, must be either the Tits group or a sporadic simple group. In this paper we improve on this result by proving that a minimal counterexample must be one of 4 sporadic simple groups.

Balanced Ternary Designs With a Constant Number of Doubletons Per Block

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We give some results about BTD’s in which each block has the same number of doubletons, either exactly one doubleton or, for odd block size \(k\), \((k - 1)/2\) doubletons per block We can show that the necessary conditions are sufficient for the existence of BTD’s with \(k = 4\) and one doubleton per block (completing results of D. Donovan), and likewise for \(k = 5\) with 2 doubletons per block. We illustrate the use of a theorem of H. Agrawal for two important constructions, discuss nesting and signing, and close with a discussion of open problems.
Bipartite Cheesecake Factory Problem

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We will present some results related to the following problem: Find a seating arrangement for two groups of \( n = mk \) persons each over \( d \) nights such that every night they sit around \( m \) rectangular tables with \( 2k \) people each such that there are \( k \) people from the same group along each of the long sides of the table and every person has a conversation with every person in the other group exactly once provided that a person can talk with

1. the person sitting directly across the table, and
2. immediate neighbors of the person sitting directly across the table.

In terms of graph theory we want to factorize \( K_{mk,mk} \) into isomorphic copies of a graph \( mH(k,3) \) where \( mH(k,3) \) stands for a union of \( m \) disjoint copies of \( H(k,3) \) and \( H(k,3) \) is the graph corresponding to the situation described above. (For \( k = 3 \) the graph \( H \) is isomorphic to \( K_3^3 - 2K_2 \).)

The problem was completely solved by the author for \( H(k,3) \) with \( k \) odd. Some new results for \( k \) even will be presented.

Open Dominator Colorings in Graphs

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and

Manuel Cardona, Eduardo Cepeda, Kaitlin McClymont,
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An open dominator coloring in a graph is a proper coloring with the additional property that each vertex in the graph openly dominates an entire color class (that is each vertex must dominate a color class which is not its own color class). The open dominator chromatic number \( \chi_{od}(G) \) is the minimum number of color classes in an open dominator coloring of a graph \( G \). We present several bounds, realization results, and its relationship to the open domination and chromatic numbers.

This project was started during the Mathworks honors math camp in San Marcos, TX.

Vertex and Edge Deletion and the Number of Sizes of Maximal Independent Sets in a Graph

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We say that a graph \( G \) is in the collection \( M_t \) if there are precisely \( t \) different sizes of maximal independent sets of vertices in \( G \). Thus the \( M_1 \) graphs are the well-covered ones (introduced by M. Plummer) where all the maximal independent sets are of one size. Some preliminary observations of the effect of removing a single vertex or a single edge from a graph in \( M_t \) will be outlined.
Three Constructions Using BIBD(v,k,u)s to Construct GDDs with Two Groups and Block Size k+2

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We construct GDDs with two groups and block size $k+2$ with three constructions. The first uses resolution classes of a BIBD($v, k, u$) when the number of classes satisfies a certain (easily satisfied) necessary condition. We illustrate the general theorem giving the construction of a GDD(16, 2, 6; 28, 30) and a GDD(9, 2, 5; $x, y$) for all possible indices ($x, y$). The second construction uses alpha-resolvability. We next construct a class of GDDs with two groups and group size $2k$ having block size $k+2$. This construction uses a decomposition into pairs of the complement of each constituent block of size $k$.

Detour Antipodal Graphs

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For two vertices $u$ and $v$ in a connected graph $G$, the detour distance $D(u, v)$ between $u$ and $v$ is the length of a longest $u-v$ path in $G$. The detour diameter $\text{diam}_D(G)$ of $G$ is the greatest detour distance between two vertices of $G$. Two vertices $u$ and $v$ are detour antipodal in $G$ if $D(u, v) = \text{diam}_D(G)$. The detour antipodal graph $\text{DA}(G)$ of a connected graph $G$ has the same vertex set as $G$ and two vertices $u$ and $v$ are adjacent in $\text{DA}(G)$ if $u$ and $v$ are detour antipodal vertices of $G$. For a connected graph $G$ and a nonnegative integer $r$, define $\text{DA}^r(G)$ as $G$ if $r = 0$ and as the detour antipodal graph of $\text{DA}^{r-1}(G)$ if $r > 0$ and $\text{DA}^{r-1}(G)$ is connected. Then $\{\text{DA}^r(G)\}$ is the detour antipodal sequence of $G$. A graph $H$ is the limit of $\{\text{DA}^r(G)\}$ if there exists a positive integer $N$ such that $\text{DA}^r(G) \cong H$ for all $r \geq N$. We present some results and open questions in this area of research.
Finding a bipartite subgraph with the maximum number of edges in a given graph is a classical problem in combinatorial optimization and extremal graph theory. An elementary heuristic approach to finding large bipartite subgraphs is to start with an arbitrary vertex partition and then make local improvements. Given a partition of the vertices of a graph into two sets, a flip is a move of a vertex from its own set to the other, under the condition that it has more incident edges to vertices in its own set than in the other. Every sequence of flips eventually produces a bipartite subgraph capturing more than half of the edges in the graph. Each flip gains at least one edge. The length of a maximal flip sequence is of interest. For an $n$-vertex loopless multigraph, we show that there is always a sequence of at most $n/2$ flips that ends in a largest bipartite subgraph, and we construct a graph having a sequence of $\frac{2}{5} (n^2 + n - 31)$ flips (answering a question of Cowen and West).

Keliher recently presented an algorithm for evaluating the resistance of the block cipher Camellia against linear and differential cryptanalysis. This algorithm computes security bounds as a function of the number of encryption rounds, beginning with a 2-round base case and applying a 1-round iterative step. We modify this algorithm by moving to a 3-round base case. Our improved algorithm produces tightened Camellia security bounds for both differential and linear cryptanalysis.
Bounds for Signed Edge Domination Numbers in Graphs

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The closed neighborhood $N_G[e]$ of an edge $e$ in a graph $G$ is the set consisting of $e$ and of all edges having a common end-vertex with $e$. Let $f$ be a function on $E(G)$, the edge set of $G$, into the set $\{-1, 1\}$. If $\sum_{x \in N_G[e]} f(x) \geq 1$ for each $e \in E(G)$, then $f$ is called a signed edge dominating function of $G$. The minimum of the values of $\sum_{x \in E(G)} f(x)$, taken over every signed edge dominating function $f$ of $G$, is called the signed edge domination number of $G$ and is denoted by $\gamma'_s(G)$. It has been conjectured that $\gamma'_s(G) \leq n - 1$ for every simple graph $G$ of order $n$. In this talk we see that this conjecture is true for Eulerian simple graphs, simple graphs with all vertices of odd degree and regular graphs. As a result we prove that for any simple graph $G$ of order $n$, $\gamma'_s(G) \leq \lceil \frac{3n}{2} \rceil$. This improves the previous upper bound $\lfloor \frac{11n}{6} - 1 \rfloor$.

On Balanced Index Sets of L-Products with Cycles

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Let $G$ be a graph with vertex set $V$ and edge set $E$, and let $A = \{0, 1\}$. Any vertex labeling $f : V \rightarrow A$ induces a partial edge labeling $f^* : E \rightarrow A$ defined by $f^*(xy) = f(x)$ if and only if $f(x) = f(y)$. For each $i \in A$, let $v_f(i) = |\{v \in V : f(v) = i\}|$ and $e_f(i) = |\{e \in E : f^*(e) = i\}|$. We call $G$ a friendly graph if it admits a vertex labeling $f$ with $|v_f(0) - v_f(1)| \leq 1$. The balance index set of the graph $G$ is defined as $\{|v_f(0) - v_f(1)| : the vertex labeling $f$ is friendly\}$. In this paper, we study the balance index sets of graphs which are L-products of a graph with cycles.

On Circular Flows of Graphs

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For an undirected graph $G$, the circular flow index of $G$ is defined by $\phi_c(G) = \min_D \max_{\emptyset \neq X \subseteq V(G)} \frac{|\delta(X)|}{|\partial_D^+(X)|}$, where the minimum is taken over all orientations of $G$. Galluccio and Goddun in [Combinatorica, 22 (2002), 455-459] proved that if $\kappa'(G) \geq 6$, then $\phi_c(G) < 4$, using linear programming. We present a graph theory proof for the same result. Our result implies other family of graphs which may have edge-connectivity less than 6 can also have $\phi_c(G) < 4$ ([Combinatorica, 27 (2007), 245-246]).
Self-Complementary Magic Squares

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A magic square of order $n$ is an $n \times n$ array of integers from $1, 2, \ldots, n^2$ so that the sum of the integers in each row, column and the diagonal is the same number. Two magic squares are said to be equivalent if one can be obtained from the other by rotation or reflection. If every entry $a$ of a magic square $M$ of order $n$ is replaced by $n^2 + 1 - a$, then we obtain the complement of $M$ (which is also a magic square of order $n$). A magic square is said to be self-complementary if it is equivalent to its complement. In this paper, we prove that for every $n \geq 3$, there exists a self-complementary magic square of order $n$ by construction.

On Two Constructions of Fully Magic Graphs

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For any abelian group $A$, written additively, we denote by $A^* = A - \{0\}$. Any mapping $l : E(G) \rightarrow A^*$ is called a labeling. Given a labeling on edge set of $G$ we can induce a vertex set labeling $l^+ : V(G) \rightarrow A$ as follows:

$$l^+(v) = \sum_{(u,v) \in E(G)} l(u, v).$$

A graph $G$ is known as $A$-magic if there is a labeling $l : E(G) \rightarrow A^*$ such that the sum of the labels of the edges incident with any vertex is a constant, that is, for all vertices $v$, $l^+(v) = c$ for some fixed $c \in A$. We call a graph $G$ fully magic if it is $A$-magic for all non-trivial abelian groups $A$. We present here two constructions of fully magic graphs. We also establish that any connected graph is the induced subgraph of a fully magic graph.

On Balance Index Sets of Halin Graphs of Stars and Double Stars

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $A = \{0, 1\}$. A labeling $f : V(G) \rightarrow A$ induces an edge partial labeling $f^* : E(G) \rightarrow A$ defined by $f^*(xy) = f(x)$ if and only if $f(x) = f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = |\{v \in V(G) : f(v) = i\}|$ and $e_f(i) = |\{e \in E(G) : f^*(e) = i\}|$. A labeling $f$ of a graph $G$ is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. The balance index set of the graph $G$, denoted $BI(G)$, is defined as $\{|e_f(0) - e_f(1)| :$ the vertex labeling $f$ is friendly$\}$. We determine the balance index sets of Halin graphs of stars and double stars.
$\mathbb{Z}_k$-magicness of Direct Products of Graphs

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Let $A$ be an abelian group with $A^* = A - \{0_A\}$. A graph is $A$-magic if there exists an edge labeling by elements of $A^*$ which induces a constant vertex labeling of the graph. In this paper, we analyze the set \{\(k : H \text{ is } \mathbb{Z}_k\)-magic and \(k \geq 2\)\} for various direct products $H = G_1 \times G_2$.

Does There Exist an Equireplicate PBD\{40,\{10,16\},6\}?  

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An equireplicate PBD\{40,\{10,16\},6\}, if it exists, has treatment set $V = \{1, 2, \ldots, 40\}$, and it is composed of:

array2 = 24 blocks of size 10, with replication number $m_2 = 6$, and  
array3 = 30 blocks of size 16, with replication number $m_3 = 12$.

Thus the replication number for the PBD is $m = 18$, and $b$, the total number of blocks, is 54.

We will say why we are interested in this PBD: it comes from a certain type of row-column design called a balanced grid (which is our array1), and it is also a Doehlert-Klee design, since $(18)^2 = 6 \times 54$, i.e., $m^2 = \lambda b$.

Most attempts, so far, have proceeded along the following lines:

Randomly generate an array2. It has replication number 6, so we cannot violate $\lambda\{x,y\} \leq 6$ at this stage. Then we tried to find an array3 to fit this array2. We have tried: hill-climbing, simulated annealing, row-by-row, and element-by-element methods to find array3.

But we have been unsuccessful, so any interest or ideas from audience members will be gratefully received.

Algorithm Performance for Chessboard Separation Problems

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Chessboard separation problems are modifications to chessboard problems such as the N-Queens problem which place obstacles on the chessboard. This talk will focus on the separation problem, N+ k Queens, N+k Queens is a variation on the classic N Queens Problem in which k Pawns and N + k mutually non-attacking Queens are to be placed on an N-by-N chessboard. Results will be presented from performance studies examining the efficiency of sequential and parallel programs that count the number of solutions to the N + k Queens Problem using traditional backtracking, Dancing Links, and Logic Programming. Preliminary results for similar problems, such as the N + k Amazons Problem, are also presented.
Group Divisible Designs With Two Groups and Block Size Five With Fixed Block Configuration

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We present constructions and results about GDDs with two groups and block size five in which each block has Configuration \((s, t)\), that is, in which each block has exactly \(s\) points from one of the two groups and \(t\) points from the other. After some results for a general \(k, s\) and \(t\), we consider the \((2, 3)\) case for block size 5. We give new necessary conditions for this family of GDDs and give minimal or near-minimal index examples for all group sizes \(n \geq 4\) except for \(n = 24s + 17\).

Turán Theorems and Convexity Invariants for Directed Graphs

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This paper is motivated by the desire to evaluate certain classical convexity invariants (specifically, the Helly and Radon numbers) in the context of transitive closure of arcs in the complete digraph. To do so, it is necessary to establish several new Turán type results for digraphs and characterize the associated extremal digraphs. In the case of the Radon number, we establish the following analogue for transitive closure in digraphs of Radon’s classical convexity theorem: in a complete digraph on \(n \geq 7\) vertices with > \(\frac{n^2}{4}\) arcs, it is possible to partition the arc set into two subsets whose transitive closures have an arc in common.

Applications of Chromatic Polynomials Involving Stirling Numbers

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The Stirling numbers of the second kind, \(S(n, k)\), are the number of ways to partition the set \([n] = \{1, 2, \ldots, n\}\) into \(k\)-subsets (none empty). We consider more generally, the numbers, \(S^d(n, k)\), which are defined to be the number of ways to partition \([n]\) into \(k\)-subsets, so that for any \(i, j\) in a given subset we require \(|i - j| \geq d\). The case \(d = 1\) is the classical Stirling numbers. By using our formula for \(S^d(n, k)\), in conjunction with the principle of inclusion and exclusion applied to the chromatic polynomial of the path, we obtain some well-known Stirling number identities.
Binomial Patterns and Binary Periodic Autocorrelation in Cyclic Two-Association-Class Schemes

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Cyclic two-class association schemes have interesting algorithmic patterns. One classic definition of association schemes uses a set of \{0, 1\} matrices to represent a scheme. With appropriate linear and binomial combinations of these matrices, the matrices fall away, and we are left with linear combinations of the scheme parameters \(p_{jk}^i\). Using these ideas, we then show that we can use binary periodic autocorrelation sequences to describe and construct partially balanced incomplete block designs with two associate classes.

On Zero-Sum Magic Graphs and Their Null Sets

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For any \(h \in \mathbb{N}\), a graph \(G = (V, E)\) is said to be \(h\)-magic if there exists a labeling \(l : E(G) \rightarrow \mathbb{Z}_h - \{0\}\) such that the induced vertex set labeling \(l^+ : V(G) \rightarrow \mathbb{Z}_h\) defined by

\[
l^+(v) = \sum_{uv \in E(G)} l(uv)
\]

is a constant map. When this constant is 0 we call \(G\) a zero-sum \(h\)-magic graph. The null set of \(G\) is the set of all natural numbers \(h \in \mathbb{N}\) for which \(G\) admits a zero-sum \(h\)-magic labeling. In this paper several classes of zero sum magic graphs will be determined and their null sets will be identified.

Chromatic Polynomials of \(C_4 \times P_n\) and \(C_5 \times P_n\)

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The chromatic polynomial of a graph \(\Gamma\), \(C(\Gamma; \lambda)\), is the polynomial in \(\lambda\) which counts the number of distinct proper vertex \(\lambda\)-colorings of \(\Gamma\), given \(\lambda\) colors. We compute \(C(C_4 \times P_n; \lambda)\) and \(C(C_5 \times P_n; \lambda)\) in matrix form and will find the generating function for each of these sequences.
Hamiltonian Connected Hourglass free Line Graphs

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Thomassen conjectured that every 4-connected line graph is hamiltonian. An hourglass is a graph isomorphic to $K_5 - E(C_4)$, where $C_4$ is a cycle of length 4 in $K_5$. It is shown that every 4-connected line graph without an induced subgraph isomorphic to the hourglass is hamiltonian connected. We prove that every 3-connected, essentially 4-connected hourglass free line graph is hamiltonian connected.

Sum-Product Graphs

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In a threshold tolerance graph, each vertex $v$ has a tolerance $t(v)$ and a weight $r(v)$ (called “rank” here), and vertices $v$ and $w$ are adjacent if $\min(t(v), t(w)) \leq \sum(r(v), r(w))$. Inspired by threshold tolerance graphs and some other similarly defined classes, Golumbic and Jamison defined rank tolerance graphs by replacing $\min$ and $\sum$ in the definition above by arbitrary commutative binary operations $\phi$ and $\rho$. If we specify that $\phi$ is sum and $\rho$ is product we then have the Sum-Product graphs (SP graphs), in which vertices $v$ and $w$ are adjacent iff $\sum(t(v), t(w)) \leq \text{product}(r(v), r(w))$.

We give an alternate definition for SP graphs, based on a conic in the real projective plane. This geometric interpretation allows us to show easily that some split graphs on 9 vertices are not SP graphs. This work is done with Robert Jamison.

Champions in Graphs of Small Degree

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A champion in a graph is a vertex whose closed neighborhood is larger than that of any other vertex. For given degree larger than three, we prove the existence of a graph with a champion for all but finitely many orders.
On the Friendly Index Sets of Some Forests

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $A$ be an abelian group. A labeling $f : V(G) \to A$ induces an edge labeling $f^* : E(G) \to A$ defined by $f^*(xy) = f(x) + f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = |\{v \in V(G) : f(v) = i\}|$ and $e_f(i) = |\{e \in E(G) : f^*(e) = i\}|$. Let $c(f) = (c_{ij})$ be a matrix with $c_{ij} = |e_f(i) - e_f(j)| : (i, j) \in A \times A$. A labeling $f$ of a graph $G$ is said to be $A$-friendly if $|v_f(i) - v_f(j)| \leq 1$ for all $(i, j) \in A \times A$. If $c(f)$ is a $(0, 1)$-matrix for an $A$-friendly labeling $f$, then $f$ is said to be $A$-cordial. When $A = \mathbb{Z}_2$, the friendly index set of the graph $G$, $FI(G)$, is defined as $\{|e_f(0) - e_f(1)| :$ the vertex labeling $f$ is $\mathbb{Z}_2$-friendly\}. In this paper, the friendly index sets of some forests are determined.