Learning Outcomes

☑ Understand how to measure that nodes with similar characteristics tend to cluster,
  • Based on enumerative characteristics (nationality)
  • Based on scalar characteristics (age, grade)
  • Based on degree.

☑ Analyze network using homophily by identifying the assortativity values based on various characteristics.

☑ Evaluate: consider why behind the what is the assortativity values of your network.
Hubs adjacent to hubs?
Are hubs adjacent to hubs?

• Real networks usually show a non-zero degree correlation (defined later compared to random).
  • If it has a positive degree correlation, the network has assortatively mixed degrees (assort. based other attributes can also be considered).
  • If it is negative, it is disassortative.

• According to Newman, social networks tend to be assortatively mixed, while other kinds of networks are generally disassortatively mixed.

• Homophily or assortativity is a common property of social networks (but not necessary):
  • Papers in citation networks tend to cite papers in the same field
  • Websites tend to point to websites in the same language
  • Political views
  • Race
  • Obesity
Sociologists have observed network partitioning based on the following characteristics:

- Friendships, acquaintances, business relationships
- Relationships based on certain characteristics:
  - Age
  - Nationality
  - Language
  - Education
  - Income level

**Homophily** is the tendency of individuals to choose friends with similar characteristic. “Like links with like.”
Example of homophily

Assortativity by race

The Social Structure of “Countryside” School District

Points Colored by Race

- White
- Black
- Mixed/Other

James Moody
Assortativity by political views

Titter data: political retweet network
Red = Republicans
Blue = Democrats

Note that they mostly tweet and re-tweet to each other
Disassortative

• Of course, some networks show disassortative mixing: “like links with dislike”.

• Disassortative networks are the ones in which adjacent nodes tend to be dissimilar:
  • Dating network (females/males)
  • Food web (predator/prey)
  • Economic networks (producers/consumers)
  • High degree nodes adjacent to small degree nodes

https://stepsandleaps.wordpress.com/2013/08/15/the-financial-ecosystem/
Why care?
• Identifying people of interest could be easy if the network presents homophily

• When assortivity (homophily) is low, Poiec, RedLearnRS (machine learning algorithm that depends on the count of POIs neighbors of nodes) outperforms all other strategies.
• When attributes show high homophily, RedLearnRS performs quite similar to the other algorithms.
How to compute it?
Gephi: Install Circular Layout

- The Radial Axis Layout groups nodes and draws the groups in axes:
- Group nodes by degree, in degree, out degree, etc.
- Group nodes by attribute sort (based on data type of attribute).
- Draw axes/spars in ascending or descending order.
- Allows top, middle or bottom "knockdown" of axes/spars, along with ability to specify number of spars resulting after knockdown.
Homophily in Gephi

Run Radial Axis Layout [here](https://gephi.org/users/tutorial-layouts/)

Run the layout by applying the following settings step by step:

- Group nodes by = “Degree”
- Group nodes by = “Modularity Class”
- Order nodes by = “Degree”

Distribution of nodes by degree inside each community.

- Draw spar/axis as spiral = checked
- Draw spar/axis as spiral = unchecked
- Ascending order = checked

Better show links inside communities
Better show links between communities
An example: ordered by communities

How the Axes/Spars should be ordered around the circle.
To check an attribute’s assortativity:

```
assortivity_val = nx.attribute_assortativity_coefficient(G, "color")
```

The attribute “color” can be replaced by other attributes that your data was tagged with.

If the attribute is “degree” then we obtain degree assortativity:

```
r = nx.degree_assortativity_coefficient(G)
```

If the attribute is “communities” then we obtain modularity:

```
```
What do we measure?
We will study two types of assortative mixing:

1. Based on *enumerative* characteristics (the characteristics don’t fall in any particular order):
   1. Nationality
   2. Race
   3. Gender
   4. Communities

2. Based on *scalar* characteristics, such as:
   1. Age
   2. Income

3. By *degree*: high degree connect to high degree
Type 1: Assortativity based on enumerative characteristics:
1) Nationality
2) Race
3) Gender
4) Communities
A network is **assortative** if there is a significant fraction of edges between same-type vertices. How to quantify the assortativity, $r$, of a network?

**Method 1:**
- Define $c_i$ to be the class of vertex $i$, and tag the nodes to belong to each class $c_i$
- $r_i = \frac{\# \text{ edges within } C_i}{\text{all possible edges}} \implies r = \sum_i r_i$

What is the assortativity if $c_i = V(G)$ as the only class? Does it make sense?

**Method 2:** compare the assortativity of the network to the one of a random graph:
- Compute the fraction of edges in $c_i$ in the **given network**,  
- Compute the fraction of edges in $c_i$ in a **random graph**,  
- $r$ is their difference.

This is the same process used for similarity, rather counting edges between nodes instead of neighbors of pairs of nodes.

Let’s look into this!
Computing the fraction of edges in $c_i$ in the given network

• Let $c_i$ be the class of vertex $i$.
• Let $n_c$ be the total number of classes.
• Let $\delta(c_i, c_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$ be the Kronecker $\delta$ that accounts for vertices in the same class.
• Then the number of edges of the same type is:

$$r = \sum_{i,j \in E(G)} \delta(c_i, c_j) = \frac{1}{2} \sum_{i,j} a_{ij} \delta(c_i, c_j)$$

Checks if vertices are in the same class
Checks for adjacent nodes
Computing the fraction of edges in $c_i$ in the random network

- Construct a random graph with the same degree distrib.  
- Let $c_i$ be the class of vertex $i$  
- Let $n_c$ be the total number of classes  
- Let $\delta(c_i, c_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$ be the Kronecker $\delta$  
- Pick an arbitrary edge in the random graph:  
  - pick a vertex $i \rightarrow$ there are $\deg i$ edges incident with it, so $\deg i$ choices for $i$ to be the 1st end vertex of our arbitrary edge  
  - and then there are $\deg j$ choices for $j$ to be the other end-vertex.  
- If $m = |E(G)|$ edges are placed at random,  
  the expected number of edges between $i$ and $j$ is $\frac{\deg i \cdot \deg j}{2m}$

Checks if vertices are in the same class
Compute the fraction of edges in \( c_i \) in the random network

- Construct a random graph with the same degree distribution.
- Let \( c_i \) be the class of vertex \( i \).
- Let \( n_c \) be the total number of classes.
- Let \( \delta(c_i, c_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \) be the Kronecker delta.
- If \( |E(G)| \) edges are placed at random, the expected number of edges between \( i \) and \( j \) is \( \frac{\deg_i \deg_j}{2|E(G)|} \).
- Then the number of edges between the same class nodes is:
  \[
  r = \frac{1}{2} \sum_{ij \in E(G)} \frac{\deg_i \deg_j}{2|E(G)|} \delta(c_i, c_j)
  \]

Check if vertices are in the same class.

Check if vertices are in the same class.

Check if vertices are in the same class.
Consider their difference

\[ r = \frac{1}{2} \sum_{ij} A_{ij} \delta(c_i, c_j) - \frac{1}{2} \sum_{ij \in E(G)} \frac{\deg i \deg j}{2|E(G)|} \delta(c_i, c_j) \]

\[ r = \frac{1}{2} \sum_{ij} [A_{ij} - \frac{\deg i \deg j}{2|E(G)|}] \delta(c_i, c_j) \]

Checks if vertices are in the same class

And now normalize by \( m = |E(G)| \):

\text{Modularity} = \frac{1}{2m} \sum_{ij} [A_{ij} - \frac{\deg i \deg j}{2m}] \cdot \delta(c_i, c_j)

The same defn as the modularity in community detection, since it measures assortativity based on predefined communities.
Modularity

\[ Q = \frac{1}{2|E(G)|} \sum_{ij} [A_{ij} - \frac{\text{deg} \ i \ \text{deg} \ j}{2|E(G)|}] \cdot \delta(c_i, c_j) \]

• Measure used to quantify the like vertices being connected to like vertices
• \(-1 < Q < 0\) means there are fewer edges between like vertices in a class compared to a random network
  i.e. disassortative network
• \(0 < Q < 1\) means there are more edges between like vertices in a class compared to a random network i.e. assortative network
• \(Q = 0\) means it behaves like a random network.
Enumerative characteristics

• Normalizing the modularity value $Q$, by the maximum value that it can get is realistic
  • Perfect mixing is when all edges fall between vertices of the same type ($Q_{\text{max}} \neq 1$)

$$Q_{\text{max}} = \frac{1}{2m} \sum A_{ij}(2m - \sum_{ij} \frac{k_i k_j}{2m} \delta(c_i, c_j))$$

• Then, the assortativity coefficient, $r = \frac{Q}{Q_{\text{max}}}$, is:

$$-1 \leq \frac{Q}{Q_{\text{max}}} = \sum_{ij} \frac{(A_{ij} - k_i k_j / 2m) \delta(c_i, c_j)}{2m - \sum_{ij} (k_i k_j / 2m) \delta(c_i, c_j)} \leq 1$$
Type 2: Assortativity based on scalar characteristics, such as:
- Age
- Income
Scalar characteristics

• Scalar characteristics: enumerative characteristics taking numerical values, such as age, income

• For example using age: two people are similar if:
  ➢ they are born the same day or
  ➢ within a year or within $x$ years,
  ➢ They are in the same class
  ➢ Same generation
     different granularity based on the data and questions asked.

• If people are friends with others of the same age, we consider the network assortatively mixed by age (or stratified by age)
Assortativity by grade/age

The Social Structure of “Countryside” School District

Points Colored by Grade
When we consider scalar characteristics we basically have an approximate notion of similarity between adjacent vertices (i.e. how far/close the values are)

There is no approximate similarity that can be measured this way when we talk about enumerative characteristics; rather present/absent
Assortativity matrix based on Scalar characteristics

Friendships at the same US high school: each dot represents a friendship (an edge from the network)

Denser along the diagonal (communities)

Sparser as the difference in grades increases
Strongly assortative

Data: 1995 US National Survey of Family Growth

Top figure: A scatter plot of 1141 married couples

Bottom figure: The same data showing a histogram of the age difference

$r = 0.574$
strongly assortative

• How do we measure scalar assortative mixing?
• Would the idea we use for the enumerative assortative mixing work?
• That is to place vertices in bins based on scalar values:
  • Treat vertices that fall in the same bin (such as age) as “like vertices” or “identical”
  • Apply modularity metric for enumerative characteristics
Then the assortativity coefficient \( r = \frac{Q}{Q_{\text{max}}} \) is defined again as:

\[
 r = \frac{\sum_{ij} (A_{ij} - k_i k_j / 2m)x_i x_j}{\sum_{ij} (k_i \delta_{ij} - k_i k_j / 2m)x_i x_j}
\]

Similar to the enumerative one again

\( r=1 \)  \( \rightarrow \) Perfectly assortative network

\( r=-1 \)  \( \rightarrow \) Perfectly disassortative network

\( r=0 \)  \( \rightarrow \) no correlation

Where \( \delta \) is either 0 or 1

Same as Modularity or Pearson correlation coeff.
Computer Science faculty

88 Computer Science faculty:
• vertices are PhD granting institutions in North America
• edge \((i,j)\) captures that a PhD student at \(i\), now faculty at \(j\)
labels are US census regions + Canada

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
& \text{Northeast} & \text{Midwest} & \text{South} & \text{West} & \text{Canada} \\
\hline
\text{Northeast} & 0.119 & 0.053 & 0.074 & 0.055 & 0.022 \\
\text{Midwest} & 0.031 & 0.067 & 0.061 & 0.026 & 0.011 \\
\text{South} & 0.025 & 0.027 & 0.083 & 0.024 & 0.006 \\
\text{West} & 0.049 & 0.033 & 0.043 & 0.073 & 0.011 \\
\text{Canada} & 0.006 & 0.005 & 0.005 & 0.005 & 0.085 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|}
\hline
\text{Northeast} & 0.322 \\
\text{Midwest} & 0.196 \\
\text{South} & 0.166 \\
\text{West} & 0.209 \\
\text{Canada} & 0.107 \\
\hline
\end{array}
\]

\[r = 0.264\]

moderately assortative
A particular scalar characteristic is **the degree**: high degree nodes connect to high degree nodes.
Assortative mixing by degree

A special case is when the characteristic of interest is the degree of the node

• Commonly used in social networks (the most used one of the scalar characteristics)

• More interesting since degree is a topological property of the network (not just a value like age or grade)

• This now reduces to Pearson Correlation Coefficient

\[
\text{degcorr\_coeff} = \frac{\sum_{ij} (A_{ij} - k_i k_j / 2m) k_i k_j}{\sum_{ij} (k_i \delta_{ij} - k_i k_j / 2m) k_i k_j}
\]
## Range of the value $r$ for real networks

### Some statistics about real networks published in 2011

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<th>$\ell_1$</th>
<th>$\ell_1^P$</th>
<th>$C$</th>
<th>$\tilde{C}$</th>
<th>$r$</th>
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[Link to source: https://www.semanticscholar.org/paper/The-unreasonable-effectiveness-of-tree-based-theor-Melnik-Hackett/0ef76143b83257592c4155a648286ba7a70cf4f74]
Assortative mixing by degree

• Assortative network by degree → core of high degrees and a periphery of low degrees (Figure (a) below)

• Disassortative network by degree → uniform: low degree adjacent to high degree (Figure (b) and (c) below)
r = assortativity coefficient


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<th>$m$</th>
<th>$c$</th>
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### Examples (published in 2003)

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References
