Main Concepts:

1. DETERMINANT OF A MATRIX:
   (a) of $2 \times 2$ : $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

   (b) of a $3 \times 3$ : Use the cofactor expansion: The cofactor $A_{ij}$ is defined by $A_{ij} = (-1)^{i+j}M_{ij}$, a signed minor. $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = a_{11}M_{11} - a_{12}M_{12} - a_{13}A_{13}$. It can be expanded by any row or column.

   (c) of a $4 \times 4$ : Elimination Method: Put the determinant into row-echelon form using row operations iteratively (keep track of sign changes). Then use the determinant of a triangular matrix.

   (d) methods in (b) and (c) work for any size matrix, these were just recommendations.

2. INVERSE OF A MATRIX:
   (a) of $2 \times 2$ : $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

   (b) of a $3 \times 3$ or $4 \times 4$ : Row Elimination method:

   $[A \mid I] \rightarrow \ldots \rightarrow [I \mid A^{-1}]$

   (c) any size: Adjoint Rule: $A^{-1} = \frac{1}{\det(A)}. \begin{pmatrix} A_{11} & A_{21} & \ldots & A_{n1} \\ A_{12} & A_{22} & \ldots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1n} & A_{2n} & \ldots & A_{nn} \end{pmatrix}$

   where $A_{i,j}$ is $(-1)^{i+j}$ times the determinant of the matrix $A_{i,j}$, where $A_{i,j}$ is $A$ taking away row $i$ and column $j$

3. SOLVE $Ax = b$:
   (a) by finding $A^{-1}$ and then $x = A^{-1}b$

   (b) Cramer’s rule: $x_i = \frac{\det A_i}{\det A}$ $i = 1, 2, \ldots, n$, where $x = (x_1, x_2, \ldots, x_n)$

   (c) using the elementary row operations on the augmented matrix to reduce it to a strict triangular form, followed by back substitution.