2.6 Matrices

1. an $m \times n$ matrix $[a_{ij}]$ is a rectangular array of numbers, that has $m$ rows and $n$ columns ($1 \leq i \leq m, 1 \leq j \leq n$)

2. a square matrix is an $n \times n$ matrix

3. for two $m \times n$ matrices $A = [a_{ij}]$ and $B = [b_{ij}]$, we define the sum of the two matrices component wise: $A + B = [a_{ij} + b_{ij}]$.

4. if $A = [a_{ij}]$ is an $m \times t$ matrix and $B = [b_{ij}]$ is a $t \times n$ matrix, we define the product

$$AB = [c_{ij}], \text{ where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{it}b_{tj}.$$ 

Note that the number of columns of $A$ must be equal to the number of rows of $B$ in order for $AB$ to be defined

5. the identity matrix is the $n \times n$ matrix $I$ that has 1s on the main diagonal and zeros everywhere else

6. for an $n \times n$ matrix, we define the power matrix $A^r$ to be $A^r = A A \ldots A$ $r$ times

7. for the $m \times n$ matrix $A = [a_{ij}]$, we define the transpose matrix to be $A^t = [a_{ji}]$

8. a matrix $A$ is symmetric if $A = A^t$

9. a zero-one matrix is a matrix whose elements are either 0 or 1 (this can model discrete structure where the 1 shows a particular relation between the structures, and 0 shows the absence of the relation)

10. define arithmetic for 0-1 matrices: $1 \land 1 = 1, 1 \land 0 = 0, 0 \land 1 = 0, 0 \land 0 = 0$. Also $1 \lor 1 = 1, 1 \lor 0 = 1, 0 \lor 1 = 1$ and $0 \lor 0 = 0$

11. for the $m \times n$ zero-one matrices $A = [a_{ij}]$ and $B = [b_{ij}]$, we define the join of $A$ and $B$ to be the zero-one matrix $A \land B = [c_{ij}]$, where $c_{ij} = a_{ij} \land b_{ij}$, and the meet of $A$ and $B$ to be the zero-one matrix $A \lor B = [c_{ij}]$, where $c_{ij} = a_{ij} \lor b_{ij}$

12. for the $m \times k$ zero-one matrix $A = [a_{ij}]$ and the $k \times n$ zero-one matrix $B = [b_{ij}]$, we define the boolean product of $A$ and $B$ to be the zero-one matrix

$$A \odot B = [c_{ij}], \text{ where } c_{ij} = (a_{i1} \land b_{1j}) \lor (a_{i2} \land b_{2j}) \lor \ldots \lor (a_{ik} \land b_{kj})$$

13. for the $n \times n$ zero-one matrix $A = [a_{ij}]$ we define the $r$th boolean power of $A$ to be the zero-one matrix $A^{[r]} = A \odot A \ldots \odot A$ $r$ times