1. (18 points) Consider the complex number \( z = -\sqrt{3} + i \).

(4pts) Write \( z \) in polar form

(4pts) Find \( \frac{1}{z} \) in polar form

(4pts) Find \( z^4 \) in polar form

(6pts) Find the 4th roots of \( -\sqrt{3} + i \)
2. (30 points) For the following systems of equations use Gaussian or Gauss-Jordan elimination to find the solution of the system. Indicate whether the system is: inconsistent, consistent, undetermined, determined, homogeneous.

(10pts)

\[-x_1 - 5x_2 = 2\]
\[2x_1 - 3x_2 = 1\]
\[-5x_1 + x_2 = -3\]
(10pts)

\[ x_1 - x_2 + x_3 = 0 \]
\[ 2x_1 - 4x_2 + x_3 = 0 \]
\[ -4x_1 - 2x_2 = 0 \]
(10pts)

\[-3x_1 + x_2 + 2x_3 = 2\]
\[x_1 \quad -5x_3 = 2\]
\[-2x_1 + x_2 - 3x_3 = 4\]
3. (15 points) For the two matrices \( A = \begin{bmatrix} -2 & 3 & 2 \\ 2 & -2 & -1 \\ -4 & 5 & 3 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -2 & -1 \\ 2 & 5 & 3 \end{bmatrix} \).

(3pts) Is \( A \) symmetric? Why or why not.

(3pts) find \((A + B)^T\)

(3pts) find the determinant of \( A \)

(3pts) Is \( A \) nonsingular? Explain.

(3pts) What is the determinant of \( A^{-1} \)? (Hint: Do not compute \( A^{-1} \))
4. (20 points) Solve the following equation: $AX - B = X$, where

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 0 & 1 \\ -3 & 0 & 7 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$
5. (20 points) Let \( A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \). Find all the eigenvalues of \( A \) and their corresponding eigenvectors and eigenspaces.