CH 6: Applications of Integration

6.4 Work

1. total amount of effort required to perform a task.

2. notice the difference when given mass versus weight (multiply by \( g = 9.8 \text{m/s}^2 \)).

3. Newton’s second law of motion: \( F = m \cdot a \) (where \( m \) is the mass of the object, and \( a \) is the acceleration), or mass times the second derivative of the distance.

4. if the force is constant, then \( W = F \cdot d \) (where \( F \) is the force that acts on the object, and \( d \) is the displacement), or \( W = \int F(x) \, dx \), where \( F(x) \) is the constant force.

5. if force is not constant and it given as a function of \( x \), say \( f(x) \), then \( W = \int f(x) \, dx \).

6. Hooke’s law: \( F = k \cdot d \), where \( k \) is Hooke’s constant that depends on each spring, and \( d \) is the displacement. Thus \( W = \int k \cdot d \, dx \), where \( d \) must be expressed in meters (since the unit for force is 1J = 1N \cdot 1\text{m}) or in feet (since the unit for work is ft-lb), but not in cm or inches.

7. gravitational force: \( G = m \cdot g \), where \( g = 9.8 \text{m/s}^2 \) is the gravitational acceleration, and \( m = \rho \cdot V \) is the mass of the object as the product of density and the unit volume.

Homework: 3, 5, 7, 13

6.5 Average Value of a Function

1. the average value function is the function that will give you the average value for each respective function:

\[
 f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx. 
\]

This makes sense since you can think of \( \int_a^b f(x) \, dx = f_{\text{ave}} \cdot (b - a) \).

2. MVT for Integrals gives a value \( c \) at which \( f(c) \) is the average value of \( f \) over the interval: If \( f \) is continuous on \([a, b]\), then there is

\[
 c \in [a, b] \text{ such that } f(c) = f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx 
\]

3. compare MVT for integrals with MVT for derivatives

Homework: 1, 5, 7