CH 6: Applications of Integration

6.1 Areas between curves:

1. Let $f(x)$ be the upper function and $g(x)$ be the lower one. Then

\[ A = \lim_{n \to \infty} \sum_{i=1}^{n} (f(x_i) - g(x_i)) \Delta x, \]

where $x_i$s are the points at which the height of each rectangle is computed. And so we have that:

\[ A = \int_{a}^{b} (f(x) - g(x)) \, dx \]

2. Thus the area between two curves is the difference of the integrals of the upper curve and the lower one. The limits on the integral are the x-values of the intersection of the curves.

3. Note that sometimes the upper function may become the lower one (see page 418), and so in this case we use:

\[ A = \int_{a}^{b} |f(x) - g(x)| \, dx \]

That is to say that if one curve is not always above the other, the integral must be split as in Example 5 page 418. The intersection points of the two curves are needed.

4. If the curves are functions of $y$ versus functions of $x$, then the limits on the integral are the $y$ values of the intersection points, and the integrand is the difference of the curves as functions of $y$ (i.e. the variable inside the integral should be $y$ rather than $x$, and the integral end with $dy$ instead of $dx$). See Example 6.

Homework: 3, 4, 9, 17, 33
6.2 Volumes (Washer method)

1. We now approximate volumes by using little cylinders the same way we used rectangles in approximating areas.

2. The volume of a right cylinder is \( V = A_{\text{base}} \cdot h \), where a right cylinder has the base and the top parallel, and the line segments that are perpendicular to the base and that join the base to the top.

3. In computing the volume of the cylinder, the area of the base is needed. This area is called the cross-section area.

4. Cross-section \( A(x) = \) are of the cross section perpendicular to the \( x \)-axis (if it is a function of \( x \)). Similarly we can have a cross-section \( A(y) = \) are of the cross section perpendicular to the \( y \)-axis (if it is a function of \( y \)).

5. Definition of the volume as the integral of the area of the cross section (notice that if we use \( A(x) \) then we need to have \( dx \) as part of the integral, and the limits on the integral are values of \( x \). Similarly for \( A(y) \) we need to have \( dy \) as part of the integral, and the limits on the integral are values of \( y \).)

\[
V = \int_a^b A(x) \, dx = \int_c^d A(y) \, dy
\]

6. Choose \( A(x) \) if the solid is obtained by rotating the area about the \( x \)-axis, and choose \( A(y) \) if the area is rotated about the \( y \)-axis.

7. Note that the area that is rotated about \( x \)-axis (or the \( y \)-axis) makes a difference in the solid obtained: in finding the area of the washer, you might need to subtract the area of the inner disk (see example 4).

Homework: 3, 5, 7, 9, 17