11.10 Taylor and Maclaurin Series

1. show what functions have power series representation

2. if we know that \( f \) has a power series at \( a \), then

\[
f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots,
\]

where \( f^{(n)}(a) \) is the \( n \)th derivative of \( f \) at the point \( a \)

3. Maclaurin Series of a function is a Taylor series at \( a = 0 \):

\[
f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots
\]

4. so a taylor series \( T_n(x) \) can approximate a function \( f(x) \) with a remainder \( R_n(x) = f(x) - T_n(x) \)

5. When does a function equal its Taylor series? Theorem 8 says that \( \lim_{n \to \infty} R_n(x) = 0 \) if \( |x-a| < R \), so function equal its Taylor series if \( |x-a| < R \).

6. formula 10 is often useful in applying Taylor series: \( \lim_{n \to \infty} \frac{x^n}{n!} = 0 \)

7. Maclaurin series for some nonpolinomial functions with their rates of convergence on page 743

8. Binomial theorem and binomial series (recall that the coefficients come from Pascal’s triangle –MA1113)

9. Taylor’s theorem applications: (1) evaluating integrals as infinite series (Example 10 page 744), and (2) finding limits (example 11 page 744)

Homework: 11, 14, 23, 58