INSTRUCTIONS: Answer all 5 of 5 questions below. Use calculator to verify answers. Show your work and justify your steps. Please work on your own. Time limit is 2 hours. There are two extra credit problems.

1. (20 points) Use the definition of a Riemann Sums to evaluate \( \int_0^3 (5x^2 + 4) \, dx \) (no credit will be given for not using the Riemann sums.)

Solution: \( \Delta x = \frac{3}{n} \) and \( x_i = 0 + \frac{3i}{n} \).

\[
\int_0^3 (5x^2 + 4) \, dx = \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} f \left( 0 + \frac{3i}{n} \right)
\]

\[
= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left( 5 \frac{9i^2}{n^2} + 4 \right)
\]

\[
= \lim_{n \to \infty} \frac{3}{n} \left( \frac{45n(n+1)(2n+1)}{6n^2} + 4n \right)
\]

\[
= \frac{3 \cdot 45 \cdot 2}{6} + 12 = 57
\]
2. (20 points) Evaluate the following integral: \[ \int \left( \frac{1}{x - 1} + \frac{1}{x^2} + \frac{x}{x^2 + 1} \right) \, dx \]

Solution: \[ \int \left( \frac{1}{x - 1} + \frac{1}{x^2} + \frac{x}{x^2 + 1} \right) \, dx = \ln |x - 1| - \frac{1}{x} + \frac{1}{2} \ln (x^2 + 1) + C, \]

using \( u = x^2 + 1 \) for the 3rd integral above.

3. (20 points) Evaluate the following integral: \[ \int_0^1 \frac{x}{e^{3x^2}} \, dx \]

Solution: Let \( y = 3x^2 \) \( \Rightarrow \) \( dy = 6x \, dx \) \( \Rightarrow \) \( \frac{dy}{6} = x \, dx \). Then

\[ \int_0^1 \frac{x}{e^{3x^2}} \, dx = \frac{1}{6} \int_0^3 \frac{1}{e^{y}} \, dy = \frac{1}{6} \int_0^3 e^{-y} \, dy = \frac{1}{6} \left( -e^{-y} \right)_0^3 = \frac{-1}{6} \left( \frac{1}{e^3} - 1 \right). \]
4. (20 points) Evaluate the following integral: \( \int \frac{\ln x}{\sqrt{x}} \, dx \)

Solution: \( u = \ln x \quad \Rightarrow \quad du = \frac{1}{x} \, dx \)

\( dv = \frac{1}{\sqrt{x}} \, dx \quad \Rightarrow \quad v = \frac{x^{1/2}}{1/2} = 2\sqrt{x} \)

\[
\int \frac{\ln x}{\sqrt{x}} \, dx = 2\sqrt{x} \ln x - \int \frac{1}{x} \cdot 2\sqrt{x} \, dx
\]

\[
= 2\sqrt{x} \ln x - 2 \int x^{-1/2} \, dx
\]

\[
= 2\sqrt{x} \ln x - 2 \cdot \frac{x^{1/2}}{1/2} + C
\]

\[
= 2\sqrt{x} \ln x - 4\sqrt{x} + C
\]

5. (20 points) Use Newton’s method to approximate the negative root of \( f(x) = \frac{x^3+2}{x-1} \) correct to 3 decimal places with \( x_1 = -1 \).

Solution: First notice that \( f'(x) = \frac{3x^2(x-1) - (x^3+2)}{(x-1)^2} = \frac{2x^3 - 3x^2 - 2}{(x-1)^2} \)

\( x_n = x_{n-1} - \frac{x_n^3 + 2}{2x_n^3 - 3x_n^2 - 2} \)

\( x_n = x_{n-1} - \frac{(x_{n-1}^3 + 2)(x_{n-1}^3 - 3x_{n-1}^2 - 2)}{2x_{n-1}^3 - 3x_{n-1}^2 - 2} \), where the solutions are:

\( x_1 = -1, \)
\( x_2 = -1.285714, \)
\( x_3 = -1.260152, \)
\( x_4 = -1.259921, \)

Thus the approximation is \( x = -1.2599 \) (or \( x = -1.26 \))
Extra credit 1. (2 points—all or nothing) Evaluate and show work: \[ \int \sin^2 x + \cos^2 x \, dx \]

Solution: \[ \int \sin^2 x + \cos^2 x \, dx = \int 1 \, dx = x + C \]

Extra credit 2. (10 points) Evaluate and show work: \[ \int e^{-\pi x} \cos(\pi x) \, dx \]

Solution: \[ u = e^{-\pi x} \quad \Rightarrow \quad du = -\pi e^{-\pi x} \, dx \]

\[ dv = \cos(\pi x) \, dx \quad \Rightarrow \quad v = \frac{\sin(\pi x)}{\pi} . \]

\[ \int e^{-\pi x} \cos(\pi x) \, dx = \frac{e^{-\pi x} \sin(\pi x)}{\pi} + \int e^{-\pi x} \sin(\pi x) \, dx \]

\[ u_1 = e^{-\pi x} \quad \Rightarrow \quad du_1 = -\pi e^{-\pi x} \, dx \]

\[ dv_1 = \sin(\pi x) \, dx \quad \Rightarrow \quad v_1 = -\frac{\cos(\pi x)}{\pi} . \]

\[ \int x \sin(\pi x) \, dx = \frac{e^{-\pi x} \sin(\pi x)}{\pi} + \left( -\frac{e^{-\pi x} \cos(\pi x)}{\pi} - \int e^{-\pi x} \cos(\pi x) \, dx \right) \]

\[ 2 \int x \sin(\pi x) \, dx = \frac{e^{-\pi x} \sin(\pi x)}{\pi} + \frac{e^{-\pi x} \cos(\pi x)}{\pi} + C \]

\[ \int x \sin(\pi x) \, dx = \frac{e^{-\pi x} (\sin(\pi x) - \cos(\pi x))}{2\pi} + C \]