4.1 Maximum and Minimum value

1. the local extrema of a functions are the local minimums and local maximums
2. a function \( f(x) \) has an **absolute maximum** at \( x = c \) if \( f(c) \geq f(x) \) for all values \( x \in \text{Domain}(f) \)
3. a function \( f(x) \) has an **absolute minimum** at \( x = c \) if \( f(c) \leq f(x) \) for all values \( x \in \text{Domain}(f) \)
4. a function \( f(x) \) has a **local maximum** at \( x = c \) if \( f(c) \geq f(x) \) for all values \( x \) in some open interval around \( c \)
5. a function \( f(x) \) has a **local minimum** at \( x = c \) if \( f(c) \leq f(x) \) for all values \( x \) in some open interval around \( c \) (open interval around \( c \) means that the immediate values to the left and to the right of \( c \) are in that open interval)

6. **Extreme Value Theorem:**

   \( f \) is continuous on \([a, b]\), then \( f \) has an absolute max at \( c \) and and absolute min at \( d \), where \( c, d \in [a, b] \)

7. **Fermat’s Theorem:** If \( f' \) exists at a local/global maximum or minimum, then \( f' = 0 \) at that point.

8. a **critical number** of a function \( f \) is a number \( c \) in the domain of \( f \) such that either \( f'(c) = 0 \) or \( f'(c) \) does not exist (particularly, local extrema are critical numbers)

9. **Closed interval method:** if \( f \) is continuous on a closed interval \([a, b]\), then the absolute min/max occur at the critical points or at the end points \( a \) or \( b \)

4.2 The Mean Value Theorem

1. Rolle’s Theorem (helps find a root of the derivative on a given interval): If
   \begin{itemize}
   \item[(a)] \( f \) is continuous on \([a, b]\),
   \item[(b)] \( f \) is differentiable on \((a, b)\), and
   \item[(c)] \( f(a) = f(b) \)
   \end{itemize}
   then \( \exists c \in (a, b) \) such that \( f'(c) = 0 \)

2. Mean Value Theorem (shows the existence of a point \( c \) where the slope of the tangent line to the function matches the slope of the secant line joining the end points of the interval): If
   \begin{itemize}
   \item[(a)] \( f \) is continuous on \([a, b]\), and
   \item[(b)] \( f \) is differentiable on \((a, b)\),
   \end{itemize}
   then there is a number \( c \in (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b-a} \)

3. \( f \) is the constant function on \((a, b) \iff f'(x) = 0 \) for all values \( x \in (a, b) \)

4. if \( f(x)' = g(x)' \) then \( f(x)' - g(x)' = 0 \) and so \( f(x) - g(x) = \text{constant} \), say \( f(x) = g(x) + c \)
4.3 How derivatives affect the shape of a graph

In this section we will learn how to use the limits at infinity and the derivatives to sketch the graph of a function

1. **The first derivative** helps find local extrema and it tells if the function is increasing or decreasing.

2. **INCREASING/DECREASING TEST:**
   - if \( f' > 0 \) on an interval, then \( f \) is increasing on that interval
   - if \( f' < 0 \) on an interval, then \( f \) is decreasing on that interval

3. **FRIST DERIVATIVE TEST:**
   - if \( f' \) changes from positive to negative, then \( f \) has a local maximum on that interval
   - if \( f' \) changes from negative to positive, then \( f \) has a local minimum on that interval
   - if \( f' \) does not change sign on an interval, then \( f \) has no local extrema on that interval

4. **The second derivative** gives the concavity of the function

5. **CONCAVITY TEST:**
   - if \( f'' > 0 \) on an interval, then \( f \) is concave up on that interval
   - if \( f'' < 0 \) on an interval, then \( f \) is concave down on that interval

6. an **inflection point** is a point where \( f \) changes concavities.

7. **SECOND DERIVATIVE TEST:**
   - if \( f'(c) = 0 \) and \( f''(c) > 0 \), then \( f \) has a local minimum at \( c \)
   - if \( f'(c) = 0 \) and \( f''(c) < 0 \), then \( f \) has a local maximum at \( c \)
   - if \( f'(c) = 0 \) and \( f''(c) = 0 \), then the test is inconclusive at \( c \)
   - if \( f'' \) changes sign, then \( f \) has an inflection point