Appendix A: Numbers, Inequalities and Absolute Values

In this section we review the numbers, sets, inequalities, and absolute values for a smoother refresher.

1. sets of numbers: natural numbers, integers, rationals, irrationals, reals

2. infinity (symboled by \( \infty \)) is not a number

3. intervals (they are subsets of the real numbers): open or closed intervals

4. rules for inequalities (pay particular attention to rule 5 page A4: if \( 0 < a < b \), then \( \frac{1}{a} > \frac{1}{b} \).)

5. multiplying both sides of an inequality by a negative number changes the direction of the inequality

6. absolute value:

\[
|a| = \begin{cases} 
  a & \text{if } a \geq 0 \\
  -a & \text{if } a < 0.
\end{cases}
\]

Most people have a hard time accepting this definition since it may look like you would get a negative value for the case that \( a < 0 \), but that is not the case. Try for yourself the case when \( a = -3 \): \( |-3| = -(−3) = 3 \).

7. \( \sqrt{a^2} = |a| \), i.e. the square root of a nonnegative number is always nonnegative (compare this to \( x^2 = 4 \Rightarrow \sqrt{x^2} = \sqrt{4} \Rightarrow |x| = 2 \Rightarrow \pm x = 2 \) usually written as \( x = \pm 2 \) )

8. properties of absolute value:

(a) \( |ab| = |a| \ |b| \);

(b) \( \left| \frac{a}{b} \right| = \frac{|a|}{|b|}, \ b \neq 0 \);

(c) \( |a|^n = |a^n| \)

(d) \( |x| = a \iff x = \pm a \)

(e) \( |x| \leq a \iff -a \leq x \leq a \)

(f) \( |x| \geq a \iff x \leq -a \) or \( x \geq a \)

(g) triangle inequality: \( |a + b| \leq |a| + |b| \) (note that it doesn’t work with the negative sign, unless used this way: \( |a - b| = |a + (-b)| \leq |a| + |(-b)| = |a| + |b| \).)