1. (7 points) Prove the following: Let $x \in \mathbb{Z}$. If $x^3 - 1$ is even then $x$ is odd.

Proof: Assume that $x$ is an even integer. Then $x = 2s$, for some $s \in \mathbb{Z}$. Therefore, $x^3 - 1 = (2s)^3 - 1 = 8s^3 - 1 = 2(4s^3 - 1) + 1$. Since $4s^3 - 1 \in \mathbb{Z}$, it follows that $x^3 - 1$ is an odd integer. 

2. (7 points) Prove the following: Let $x, y \in \mathbb{Z}$. If $x$ and $y$ are of opposite parity, then $x^2 + y^2$ is an odd integer.

Proof: Assume that $x$ and $y$ are of opposite parity. Without loss of generality, say $x$ is odd and $y$ is even. Then $x = 2k + 1$ and $y = 2\ell$ for some $k, \ell \in \mathbb{Z}$. Thus

$$x^2 + y^2 = (2k + 1)^2 + (2\ell)^2 = 4k^2 + 4k + 1 + 4\ell^2 = 2(2k^2 + 2\ell^2 + 2k) + 1.$$ 

Since $2k^2 + 2\ell^2 + 2k$ is an integer, $x^2 + y^2$ is an odd integer.
3. (6 points) Prove the following: If \( x \) is an integer, then \( x^2 - 3x - 5 \) is odd.

**Proof:** We consider two cases according to whether \( x \) is even or odd.

**Case 1.** \( x \) is an odd integer. Then \( x = 2k + 1 \) for some \( k \in \mathbb{Z} \). Thus

\[
x^2 - 3x - 5 = (2k + 1)^2 - 3(2k + 1) - 5
= (4k^2 + 4k + 1) - 6k - 8
= 4k^2 - 2k - 7
= 2(2k^2 - k - 4) + 1.
\]

Since \( 2k^2 - k - 4 \) is an integer, \( x^2 - 3x \) is an odd integer.

**Case 2.** \( x \) is an even integer. Then \( x = 2\ell \) for some \( \ell \in \mathbb{Z} \). Thus

\[
x^2 - 3x - 5 = (2\ell)^2 - 3(2\ell) - 5
= (4\ell^2) - 6\ell - 5
= 2(2\ell^2 - 3\ell - 3) + 1.
\]

Since \( 2\ell^2 - 3\ell - 3 \) is an integer, \( x^2 - 3x \) is an odd integer.

\[\blacksquare\]