FOURTEENTH INTERNATIONAL CONFERENCE ON FIBONACCI NUMBERS AND THEIR APPLICATIONS

Mathematical Institute UNAM, Morelia, Mexico
July 5–9, 2010

ABSTRACTS OF TALKS

in the order in which they are presented

MONDAY, JULY 5

9:30–9:55 Curtis Cooper *The k-Zeckendorf array*

Let $k \geq 2$ be an integer. We define the k-generalized Fibonacci sequence, the k-Zeckendorf representation of a positive integer, and the k-Zeckendorf array. When $k = 2$ these definitions are the Fibonacci sequence, the Zeckendorf representation of a positive integer, and the Zeckendorf array defined by Kimberling. The 3-Zeckendorf array is

1	2	4	7	13	24	44	81	149	274	504	$\cdots$
3	6	11	20	37	68	125	230	423	778	1431	
5	9	17	31	57	105	193	355	653	1201	2209	
8	15	28	51	94	173	318	585	1076	1979	3640	
10	19	35	64	118	217	399	734	1350	2483	4567	
12	22	41	75	138	254	467	859	1580	2906	5345	
14	26	48	88	162	298	598	1008	1854	3410	6272	
16	30	55	101	186	342	629	1157	2128	3914	7199	
18	33	61	112	206	379	697	1282	2358	4337	7977	
$\vdots$											

We prove that each of these k-Zeckendorf arrays is an interspersion.

9:55–10:10 Clark Kimberling *The Wytoff triangle and unique representation of positive integers*

Each row of the left-justified Wythoff array begins with a pair a, b, where $a = 1, 2, 3, \ldots$ and $b = 0, 1, \ldots, a-1$. Let $n(a, b)$ be the number of the row which starts with a, b. When arranged in increasing order, the numbers $n(a, b)$ form row a of the Wythoff triangle. Underlying its interesting properties is an integer $m(a, b)$ which, aside from its usefulness in connection with the Wythoff triangle, leads to various unique representations of positive integers. The methods apply to the dual Wythoff array and dual Wythoff triangle, as well as other Stolarsky arrays.

10:20–10:45 Heiko Harborth *Crossing numbers for Fibonacci distance graphs*

Distance graphs $D_n(d_1, d_2, \ldots, d_r)$ have as vertices the natural numbers $1, 2, \ldots, n$ and edges occur for all pairs of vertices (a, b) with $|a - b| = d_i$, $1 \le i \le r$. For Fibonacci numbers as distances d_i, some crossing numbers $\mathrm{cr}(D_n)$ are determined, where $\mathrm{cr}(D_n)$ is the minimum number of crossings of edges in a realization of D_n in the plane.

10:45–11:10 Rebecca A. Hillman *Some specific Binet forms for higher-dimensional Jacobsthal and other recurrence relations*

Recurrence relations for generalizations of the Jacobsthal, Perrin/Padovan and Pell numbers are considered and Binet style formulas for generating values are obtained. Based on joint work with Charles K. Cook and Gerarld E. Bergum.

11:10–11:35 Peter G. Anderson *Tetrabons: Fundamental regions of a three space Zeckendorf representation*

We present three-dimensional models of the "fundamental regions", $S_k \subset \mathbb{Z}^3$, which can be expressed using at most k terms in a space generalization of Zeckendorf representation. The underlying number sequence for this representation is the *tetrabonacci numbers*: $T_{-2} = T_{-1} = T_0 = 0$, $T_1 = 1$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4}$ for all n. Our regions correspondingly satisfy: S_n is the union of translates of S_{n-1}, S_{n-2}, S_{n-3}, and S_{n-4}. Zeckendorf's representation of positive integers as sums of distinct Fibonacci numbers generalizes to representation of integer n-tuples as sums of distinct vectors based on negatively subscripted $(n+1)$-bonacci numbers.

12:00–12:25 Russell J. Hendel *Continued fractions consisting of alternating string pattterns*

We show that the numerators of the sequence of continued fractions of the form $[1^{a_1 n}, b_1, 1^{a_2 n}, b_2, \ldots, 1^{a_m n}, b_m]$ with $a_i,\ b_i \in \mathbb{N}$, $1 \le i \le m$, $m \in \mathbb{N}$, and n a positive integer variable - with the notation $1^{a_j n}$ using string exponentiation to signify a concatenation of $a_j n$ ones - has an annihilator that can be explicitly factored as a product of quadratic factors.

12:25–12:50 Pante Stănică *Nonoverlap property of the Thue-Morse sequence*

In this talk, we provide a new proof for the nonoverlap property of the Thue-Morse sequence using a Boolean functions approach and investigate other patterns that occur in a generalization of the Thue-Morse sequence. Based on joint work with Thomas W. Cusick.

12:50–1:15 Nathan Hamlin *Representing positive integers as a sum of linear recurrence sequences*

The Zeckendorf representation, using sums of Fibonacci numbers, is widely known. Fraenkel generalized it to recurrence sequences $u_n = a_1 u_{n-1} + \cdots + a_h u_{n-h}$ provided $a_1 \ge a_2 \ge \cdots \ge a_h > 0$. We remove this restriction but do assume that $a_i \ge 0$ and show that a unique representation of every positive integer is possible with digit strings composed of certain blocks which are lexicographically less than $a_1 a_2 \cdots a_h$. Based on joint work with William A. Webb.

3:00–3:25 William A. Webb *Cryptography Using Recurrence Sequence Bases*
One of the first public key codes to be proposed was the knapsack code. However, within a few years, two different types of attacks were discovered which were successful at breaking this code. We look at how replacing the ordinary base 2 representation with a more complex base using recurrence sequences, may create a secure code which cannot be broken by these known attacks. Based on joint work with Bala Krishnamoorty and Nathan Moyer.

3:25–3:50 Santos H. Hernández *Fibonacci numbers which are sums of three factorials*
In 2002, Grossman and Luca have shown that if $k \geq 1$ is any fixed positive integer, then the Diophantine equation $F_n = m_1! + m_2! + \cdots + m_k!$ has at most finitely many effectively computable positive integer solutions $(n, m_1, \ldots, m_k)$. In this talk, we will present all the solutions to the case $k = 3$. Based on joint work with Mark Bollman and Florian Luca.

3:50–4:10 V. Janitzio Mejía Huguet *The sequence* $\phi(F_n)/F_n$ *is dense in* $[0,1]$
In this talk, we will show that the sequence $\{\phi(F_n)/F_n\}_{n\geq 1}$ is dense in $[0,1]$. Based on joint work with Florian Luca and Florin Nicolae.

4:40–5:05 Thomas J. Barrale *Tojaaldi sets: An original idea applicable to first digits in both the Fibonacci and Lucas series*
We present some experimental results and formulate some conjectures regarding the leading digits of Fibonacci and Lucas numbers.

5:05–5:30 Karl Dilcher and Curtis Cooper *The electronic Fibonacci Quarterly*
Official launch of the electronic version of *The Fibonacci Quarterly.*

TUESDAY, JULY 6

9:30–9:55 William A. Webb *Matrices with forbidden submatrices*

We show that the number of $k \times n$ matrices, with k fixed, which do not contain any submatrices from a given set of forbidden matrices, is given by a linear, constant coefficient recurrence in n.

9:55–10:20 Paul K. Stockmeyer *Slicing the Menger Sponge*

A surprising new fractal is displayed when the Menger Sponge is sliced by a certain plane. While investigating this new fractal we will explore an associated integer sequence, one that doesn't currently appear in the *Online Encyclopedia of Integer Sequences.*

10:20–10:45 Christian Ballot *Lucas sequences with discriminant -3 times a square*

We will emphasize some properties specific to Lucas sequences whenever the associated discriminant is of the from $-3F^2$, where F is an integer.

10:45–11:10 Mohand-Ouamar Hernane *Large values of some arithmetic function*

In this work, we establish some results concerning the large values of functions of factorization, which count the number of solutions of the diophantine equation

$$x_1 x_2 \cdots x_r = n, \quad r \geq 1.$$

Denoting by $h(n)$ the function which counts the number of solutions of the above equation with x_1, x_2, …, x_r prime numbers, then there exist two constants c_1 and c_2 such that

$$\log h(n) \leq \lambda \log n - c_1 \frac{(\log n)^{1/\lambda}}{\log\log n} \qquad \text{for all} \quad n \geq 3.$$

Furthermore, for all $n \geq 3$, there exists a positive integer $m \leq n$, such that

$$\log h(m) \geq \lambda \log n - c_2 \frac{(\log n)^{1/\lambda}}{\log\log n}.$$

Also, if $K(n)$ denotes the number of the solutions of the above equation with integers $x_i \geq 2$, then there exist two constants c_3 and c_4 such that for all sufficiently large integers n the inequality

$$\log K(n) \leq \rho \log n - c_3 \frac{(\log n)^{1/\rho}}{\log\log n}$$

holds, while also for all sufficiently large integers n there exists a positive integer $m \leq n$, such that

$$\log K(m) \geq \rho \log n - c_4 \frac{(\log n)^{1/\rho}}{\log\log n} \geq \rho \log m - c_4 \frac{(\log m)^{1/\rho}}{\log\log m}.$$

In the above, λ and ρ are the real numbers defined by $\zeta_{\mathcal{P}}(\lambda) = 1$, $\zeta(\rho) = 2$, $\mathcal{P}$ is the set of prime numbers, and

$$\zeta_{\mathcal{P}}(s) = \sum_{p \in \mathcal{P}} \frac{1}{p^s}.$$

Based on joint work with Jean-Luis Nicolas.

11:10–11:35 Alejandra Alvarado *Arithmetic progressions in the y-coordinates of certain elliptic curves*

We consider arithmetic progressions in the y-coordinate on the elliptic curve $y^2 = x^3 + k$ whose coefficients are rational. We investigate lengths three, four, and five.

12:00–12:25 Akos Pínter *A new characterization of Fibonacci numbers*

We present a new characterization of Fibonacci numbers. Based on joint work with Volker Ziegler.

12:25–12:50 Kálmán Liptai *About (a, b)-type balancing numbers*

A positive integer n is called a balancing number if

$$1 + 2 + \cdots + (n-1) = (n+1) + (n+2) + \cdots + (n+r)$$

for some positive integer r. We study a generalization of balancing numbers which is called (a, b)-type balancing numbers. We give effective finiteness theorems for the polynomial values of (a, b)-type balancing numbers. Specially, we investigate the power values of (a, b)-type balancing numbers. Moreover, we give effective finiteness theorem for combinatorial numbers in the sequence of (a, b)-type balancing numbers.

12:50–1:15 Mátyás Ferenc *Further generalizations of the Fibonacci-coefficient polynomials*

The aim of this talk is to investigate the location of the zeros of general polynomials

$$q_n^{(i,t)}(x) = R_i x^n + R_{i+t} x^{n-1} + \cdots + R_{i+(n-1)t} x + R_{i+nt},$$

where $i \geq 1$ and $t \geq 1$ are fixed integers and the second order linear recursive sequence

$$R = \{R_n\}_{n=0}^{\infty}$$

is defined by the following manner: let $R_0 = 0$, $R_1 = 1$, A and B be fixed positive integers. Then for $n \geq 2$

$$R_n = AR_{n-1} + BR_{n-2}.$$

3:00–3:25 Takao Komatsu *On the nearest integer of the sum of reciprocals of Fibonacci numbers*

Let G_n be the generalized Fibonacci numbers, defined by $G_n = aG_{n-1} + G_{n-2}$ $(n \geq 2)$ with $G_0 = 0,\ G_1 = 1$, where a is a positive integer. We discuss the nearest integer of the reciprocal of the sum of reciprocal generalized Fibonacci numbers $(\sum_{k=n}^{\infty}(-1)^k G_k^{-1})^{-1}$ and $(\sum_{k=n}^{\infty}(-1)^k G_k^{-2})^{-1}$. These results are analogous to those of Holliday and Komatsu who considered the integer part of $(\sum_{k=n}^{\infty} G_k^{-1})^{-1}$ and $(\sum_{k=n}^{\infty} G_k^{-2})^{-1}$, generalizing the results of Ohtsuka and Nakamura.

3:25–3:50 Kiyota Ozeki *On arithmetic properties of a generalized difference operator*
In this paper, we introduce a generalized difference operator,

$$\Delta(\alpha;\beta)f(x) = f(\alpha x + \beta) - f(x),$$

and develop arithmetic properties for it.

3:50–4:15 Darren Glass *Optimal strategy for defending a chain in Risk*
When playing the board game of *Risk*, one is faced with a decision of how to distribute your armies among the territories you control in order to best defend them from attacks from your opponent. In this talk, we will consider how to distribute armies along a chain of territories. In particular, we develop a Markov Chain model for the game and give a conjecture that the numerical results of this model predict. Moreover, we are able to use recurrence relations to prove special cases of this conjecture as well as related results.

4:45–5:10 Casey Mongoven *Musical compositions with Zeckendorf representations* (*Auditorium*)
Three contrasting polyphonic musical compositions based on Zeckendorf representations in the style of music characterized by Fibonacci numbers and the golden ratio are presented and analyzed. Based on joint work with Ron Knott.

THE EDOUARD LUCAS LECTURE

Combinatorial trigonometry (and a method to DIE for)

Arthur T. Benjamin

5:30–6:30

Auditorium

Many trigonometric identities, including the Pythagorean theorem, have combinatorial proofs. Furthermore, some combinatorial problems have trigonometric solutions. All of these problems can be reduced to alternating sums, and are attacked by a technique we call D.I.E. (Description, Involution, Exception). This technique offers new insights to identities involving binomial coefficients, Fibonacci numbers, derangements, and Chebyshev polynomials.

WEDNESDAY, JULY 7

9:30–9:55 Art Benjamin *The combinatorialization of linear recurrences*

We provide an original combinatorial proof of Binet's formula for Fibonacci numbers. Naturally, any k-th order linear recurrence with constant coefficients has a closed form solution, obtainable by factoring its (k-th degree) characteristic polynomial. We extend our proof of Binet's formula to show that these closed form solutions can also be given a combinatorial interpretation, even in the repeated roots situtation. Based on joint work with Halcyon Derks and Jennifer Quinn.

9:55–10:20 Aynur Yalçiner *Block tridiagonal matrices and Fibonacci numbers*

In this study, we show that the determinants of block tridiagonal matrices can be computed using recurrence relations. We will present some connections between determinants of block tridiagonal matrices and Fibonacci sequences. Thus, we obtain a generalization of the results on determinants of tridiagonal matrices.

10:20–10:45 Pante Stănică *Generating matrics of C-nomial coefficients and their spectra*

In this talk, we consider a generalization of binomial coefficients, called C–nomial coefficients, dependent upon a sequence $\{u_n\}_n$, with indices in arithmetic progressions. We obtain a general recurrence relation and a generating matrix, and point out some new relationships between these coefficients and the generalized Pascal matrices. Further, we obtain generating functions, combinatorial representations, and many new interesting identities and properties of these coefficients. Based on joint work with Emrah Kilic.

10:45–11:10 Claudio Pita *More on Fibonomials*

We obtain the Z transform of products of powers of Fibonacci sequences, and show how some new interesting identities involving Fibonomials can be derived from it.

11:10–11:35 Peter G. Anderson *Tiling Rectangles, Cylinders, Tori, Möbius Strips, and Klein Bottles*

We count the number of ways an $M \times N$ rectangle can be tiled (i.e., subdivided along lines with integer coordinates parallel to the coordinate axes) using a limited repertoire of tile sizes and various spacing and orientation rules. As in the one-dimensional case (board tiling), we also consider the various ways the edges of rectangles can be identified: cylinders, tori, Möbius strips, and Klein bottles. We count tilings by counting the paths in transition networks using powers of the networks' transition matrices, thus we achieve linear recurrences for the counting sequences (albeit of order exponential in the width of the rectangles). The matrices follow a recurrence pattern, and thus give rise to new and old fractal patterns.

12:00–12:25 Paul Thomas Young *Generalizations of Bernoulli Numbers and factorial sums*

We prove a pair of identities expressing Bernoulli numbers and Bernoulli numbers of the second kind as sums of generalized falling factorials. These are derived from an expression for the Mahler coefficients of degenerate Bernoulli numbers. As corollaries several unusual identities and congruences are derived, involving the Bernoulli numbers, Bernoulli numbers of the second kind, degenerate Bernoulli numbers, and Norlund numbers.

12:25–12:50 Calvin Long *Extending the* GCD *Star of David Theorem to more than two* GCD*'s.*

In this talk, we present some results showing how to divide arrays of 3, 4, or 5 diamonds in Pascal's triangle into equnumerous subsets with 3, 4 and 5 equal GCD's. Also, we present a construction for an arbitrarily large triangle divisible into n subsets with n equal GCD for arbitrary $n \geq 2$.

12:50–13:15 Marjorie Bicknell–Johnson *Hexahexaflexagons: a Mathematical ramble*

Flexagons are objects created by folding a strip of paper along certain lines to form loops. By manipulating the folds, it is possible to hide and reveal different faces. A hexaflexagon is a flexagon in the shape of a hexagon made by folding a strip into adjacent equilateral triangles. This talk discusses the history and construction, and some mathematical properties, of the six-faced hexagonal figure, the hexahexaflexagon. Based on joint work with Colin Paul Spears. (*This talk will be followed by a* 25 *minute workshop for those participants who wish to learn how to fold a flexagon by Marjorie Bicknell-Johnson and Frank Johnson*).

THURSDAY, JULY 8

9:30–9:55 Karl Dilcher *Mod p^3 analogues of theorems of Gauss and Jacobi on binomial coefficients*

The theorem of Gauss that gives a modulo p evaluation of a certain central binomial coefficient was extended modulo p^2 by Chowla, Dwork, and Evans. In this talk, I present a further extension to a congruence modulo p^3, with a similar extension of a theorem of Jacobi. This is done by first obtaining congruences to arbitrarly high powers of p for certain quotients resembling binomial coefficients and related to the p-adic gamma function. These congruences are of a very simple form and involve Catalan numbers as coefficients. As another consequence we obtain complete p-adic expansions for certain Jacobi sums. Based on joint work with John B. Cosgrave.

9:55–10:20 Paul Thomas Young *On the binary expansion of the odd Catalan numbers and p-adic congruences for p^q-Catalan numbers*

Let $c_n = \dfrac{1}{n+1}\dbinom{2n}{n}$ denote the nth Catalan number. In the first part of this talk, we look at some of the properties of the binary digits of c_n. When c_n is odd we get good information on the binary digits from both the left and the right, and in particular determine all instances where c_n is a binary palindrome. In the second part of this talk, we consider the sequence of s-Catalan numbers $C_s(n) = \dfrac{1}{(s-1)n+1}\dbinom{sn}{n}$ for integers $s > 1$, which coincide with the usual Catalan numbers when $s = 2$. When $s = p^q$ is a power of a prime p we derive several congruences for $C_s(n)$ modulo powers of p, including a generalization of Wolsenholme's theorem. Based on joint work with Florian Luca.

10:20–10:45 Charles K. Cook *The "magicness" of powers of some magic squares*

Powers of matrices whose elements form semimagic or magic squares are investigated and powers of several examples of classical magic squares are computed. Conditions that guarantee their magic properties (magicness) are retained or lost are explored. Based on joint work with Michael R. Bacon and Rebecca A. Hillman.

10:45–11:10 Augustine O. Munagi *Large alternating subsets and successions*

We present a unified extension of alternating subsets to k-combinations of $\{1, 2, \ldots, n\}$ containing a prescribed number of sequences of elements of the same parity. Enumeration formulas for both linear and circular combinations are obtained by direct combinatorial arguments. The results are applied to the enumeration of binary sequences.

11:10–11:35 Luis H. Gallardo *On odd perfect numbers of special forms*

We give necessary conditions for perfection of some families of odd numbers with special multiplicative forms. Extending earlier work of Steuerwald, Kanold, McDaniel et al. Based on joint work with Olivier Rahavandrainy.

12:00–12:25 Leonardo Bardomero *On triunitary and tetraunitary numbers*

We report on recent progress on triunitary and tetraunitary numbers. Based on joint work with Douglas Iannucci.

12:25–12:50 Douglas Iannucci *On triunitary and tetraunitary numbers II*

We will give some more details related to the results presented in the prebious talk. Based on joint work with Leonardo Bardomero.

12:15–1:15 Florian Luca *Multiperfect Fibonacci numbers*

We report on a recent result showing that there are no Fibonacci numbers > 1 which divide the sum of their divisors. Based on joint work with Kevin A. Broughan, Marcos González, Ryan Lewis, V. Jantzio Mejía Huguet and Alain Togbé.

3:00–3:25 Karyn McLellan *Growth rates of recurrence sequences with periodic coefficients*

This talk will extend some ideas from Viswanath's work on random Fibonacci sequences by looking at non-random cases. Specifically, I will look at second order linear recurrence sequences whose coefficients belong to the set $\{1, -1\}$ and form periodic cycles. I will analyze the growth of such sequences and develop criteria for determining whether a given sequence is bounded, grows linearly or grows exponentially. Also, I will introduce an equivalence relation on the sequences such that each equivalence class has a common growth rate, and consider the number of such classes for a given cycle length.

3:25–3:50 Elizabeth M. Magargee *Asymptotic behavior of solutions to min-max recurrences of higher-order*

In this talk, we consider asymptotic behavior of solutions to min-max recurrences of the form

$$y_n = \min\{\max\{y_{n-k_1}, y_{n-k_2}\}, \max\{y_{n-k_3}, y_{n-k_4}\}\}, \tag{1}$$

We are particularly interested in the behavior of such solutions to such higher-order recurrences (i.e. periodicities, convergence, boundedness, etc.) in terms of the vector of delays $[k_i]$. Based on joint work with Kenneth S. Berenhaut and Scott M. Rabidoux.

3:50–4:15 Bennett J. Stancil *Fibonacci-type piecewise linear recurrences and generalized Ramanujan-Nagell equations*

In this talk, we consider asymptotic behavior of solutions to piecewise linear recurrences of Fibonacci-type. A connection to generalized Ramanujan-Nagell equations is employed to obtain results on equations for which all solutions are eventually periodic. Based on joint work with Kenneth S. Berenhaut and Elizabeth M. Magargee.

4:40–4:05 Ross Hilton *An application of recursive sequences to expansions for distributions of sums of random variables*

In this talk we consider improvements of local Edgeworth expansions for probability distributions of sums of independent identically distributed (i.i.d.) random variables.

Suppose X is a random variable with finite variance, and $X_1, X_2, \ldots$ is a sequence of i.i.d. random variables each with the same distribution as X. In addition, suppose that X has probability function p, where p is either a density or a probability mass function. Let $p^{(i)}$ be the probability function for the partial sum $S_i = X_1 + X_2 + \cdots + X_i$.

We are interested in local expansions for $p^{(n)}$ in terms of powers of $1/\sqrt{n}$. The well-known Edgeworth expansion is of this form. The coefficients in this expansion involve cumulants of X, as well as Hermite polynomials. Our new expansion makes direct use of the probability function p in computing a rival sequence of coefficients with no need to obtain cumulants or employ Hermite polynomials. The expansion is particularly advantageous when X is a discrete random variable on a small finite support. Oftentimes, our expansion produces much better asymptotic results, for instance, in the case where X is a symmetric random variable. The work involves properties of Hermite polynomials as well as binomial coefficients and Stirling numbers. Based on joint work with Kenneth S. Berenhaut.

5:05–5:30 Austin H. Jones *Asymptotic behavior of solutions to symmetric rational recurrences*

In this talk, we consider generalization of several recent results regarding asymptotic behavior of solutions to symmetric rational recurrences. In particular, we are interested in asymptotic behavior of solutions to recurrences involving ratios of sums of elementary symmetric polynomials. Based on joint work with Kenneth S. Berenhaut.

FRIDAY, JULY 9

9:30–9:55 Victor Cuahutemoc García *Waring type problem involving Fibonacci numbers in fields of prime order*

In this talk, I am planing to show that is there a positive integer k such that for almost all primes p, every residue class can be written as

$$F_{n_1} + \cdots + F_{n_k} \pmod p.$$

Based on joint work with Florian Luca and V. Janitzio Mejía Huguet.

9:55–10:20 Juan José Alba González *Fibonacci numbers divisible by their indices*

In this talk, we will report about some properties of the positive integers n dividing the Fibonacci number F_n. Based on joint work with Florian Luca and Carl Pomerance.

10:20–10:45 V. Janitzio Mejía Huguet *Fibonacci, Riesel and Sierpiński*

In 1960, W. Sierpiński showed that there are infinitely many odd positive integers k such that $2^n k + 1$ is composite for all n. In 1962, J. Selfridge showed that $k = 78557$ is a Sierpiński number. This is now believed to be the smallest Sierpiński number. In a similar vein, a Riesel number is an odd positive integer k such that $2^n k - 1$ is composite for all nonnegative integers n. They were first investigated by H. Riesel in 1956, four years before Sierpiński's paper. There are infinitely many such and it is believed that the smallest Riesel number is 509203. In this talk, we prove that there are infinitely many Fibonacci numbers which are Riesel numbers. We also show that there are infinitely many Fibonacci numbers which are Sierpiński numbers. Based on joint work with Florian Luca.

10:45–11:10 Sergio Guzman *Smooth values of* $x^2 \pm 2$

The largest positive integer x such that one of $x^2 \pm 2$ has no prime factor > 100 is $x = 340064590$. In fact,

$$340064590^2 + 2 = 2 \cdot 3^4 \cdot 11^4 \cdot 17^2 \cdot 19^2 \cdot 59 \cdot 89^2.$$

In my talk, I will explain how to justify the above claim. The technique uses Primitive Divisors for Lehmer sequences and some computations.

11:10–11:35 Saul D. Alvarado *Fibonacci numbers which are sums of two redigits*

In my talk, I will show that $F_{20} = 6666 + 99$ is the largest Fibonacci number which is the sum of two repdigits. The proof uses Baker's bounds for linear forms in logarithms of algebraic numbers. Based on joint work with Florian Luca.

12:00–12:25 Florian Luca *Fibonacci integers*

Let G be the multiplicative group generated by the Fibonacci numbers. Since $L_m = F_{2m}/F_m$, the Lucas numbers belong to G. In my talk, I will present upper and lower bounds for the counting function of the positive integers $n \le x$ which belong to G. Based on joint work with Carl Pomerance and Stephan Wagner.