Global Ocean Tripole and Climate Variability

Peter C. Chu
Naval Postgraduate School

EAPS, MIT June 2, 2010
Outline

• (1) Recent Development in Short-term Climate Variability
• (2) Data Analysis: (T, S) Profiles ➔ Synoptic Gridded Data with Monthly Increment
• (3) Synoptic Upper Ocean (0-300 m) Heat Content
• (4) Global Tripole ➔ Canonical El Nino, El Nino Modoki, Indian Ocean Dipole, ...
(1) Recent Development in Climate Variability

Indian Ocean Dipole
El Nino Modoki
Indian Ocean Dipole (Saji et al. 1999)
Indian Ocean Dipole (Saji et al. 1999)
El Nino and El Nino Modoki
(Weng et al. 2007, Ashok et al. 2007)

(Images courtesy Karumuri Ashok, APEC Climate Center)
(2). Data Analysis

Global Temperature and Salinity Profile Program (GTSPP)
GTSSPP = **Global Temperature Salinity Profile Program**

- GTSSPP is a joint WMO-IOC program designed to provide improved access to the highest resolution, highest quality data as quickly as possible.
- GTSSPP began as an official IODE pilot project in 1989.
- It went into operation in November 1990.
6-12 hours at surface to transmit data to satellite

Total cycle time 10 days

Descent to depth
~10 cm/s (~6 hours)

1000 db (1000 m)
Drift approx. 9 days

Salinity & Temperature profile recorded during ascent
~10 cm/s (~6 hours)

Float descends to begin profile from greater depth
2000 db (2000 m)
Example → GTSPP Data
Ocean Data Analysis

Classical Method $\rightarrow$ Fourier Series Expansion
Joseph Fourier 1768-1830

Fourier was obsessed with the physics of heat and developed the Fourier series and transform to model heat-flow problems.
Fourier Series Expansion

For a rectangular region \((L_x, L_y)\), the basis functions are sinusoidal functions.

\[
 f(x, y) = \sum_{i} \sum_{j} a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y} \\
+ \sum_{i} \sum_{j} b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}
\]
For the Dirichlet boundary condition: $f = 0$ at the boundaries

\[ f(x, y) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y} \]

The dots represent the Observations.
Linear Algebraic Equations for the Coefficients $a_{ij}$

\[
f(x_1^{ob}, y_1^{ob}) = \sum_ia_{ij}\sin\frac{i\pi x_1^{ob}}{L_x}\sin\frac{j\pi y_1^{ob}}{L_y}
\]

\[
f(x_2^{ob}, y_2^{ob}) = \sum_ia_{ij}\sin\frac{i\pi x_2^{ob}}{L_x}\sin\frac{j\pi y_2^{ob}}{L_y}
\]

\[
\ldots\ldots
\]

\[
f(x_M^{ob}, y_M^{ob}) = \sum_ia_{ij}\sin\frac{i\pi x_M^{ob}}{L_x}\sin\frac{j\pi y_M^{ob}}{L_y}
\]
Determination of Spectral Coefficients
(Ill-Posed Algebraic Equation)

\[ A \hat{a} = QY, \]
Known $a_{ij} \rightarrow$ Analyzed Field

$$f(x, y) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y}$$
For the Neumann boundary condition $n \cdot \nabla f = 0$
at the boundaries

$$f(x, y) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}$$

The dots represent the Observations.
Linear Algebraic Equations for the Coefficients \( a_{ij} \)

\[
f(x_{1}^{ob}, y_{1}^{ob}) = \sum_{i} \sum_{j} b_{ij} \cos \frac{i \pi x_{1}^{ob}}{L_{x}} \cos \frac{j \pi y_{1}^{ob}}{L_{y}}
\]

\[
f(x_{2}^{ob}, y_{2}^{ob}) = \sum_{i} \sum_{j} b_{ij} \cos \frac{i \pi x_{2}^{ob}}{L_{x}} \cos \frac{j \pi y_{2}^{ob}}{L_{y}}
\]

\[
\cdots
\]

\[
f(x_{M}^{ob}, y_{M}^{ob}) = \sum_{i} \sum_{j} b_{ij} \cos \frac{i \pi x_{M}^{ob}}{L_{x}} \cos \frac{j \pi y_{M}^{ob}}{L_{y}}
\]
Known $b_{ij} \rightarrow$ Analyzed Field

$$f(x, y) = \sum_{i} \sum_{j} b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}$$
For General Ocean Basin →
Generalized Fourier Series Expansion
Spectral Representation
Fourier Series Expansion

\[ c(x, z_k, t) = A_0(z_k, t) + \sum_{m=1}^{M} A_m(z_k, t) \Psi_m(x, z_k), \]

\( \Psi_m \rightarrow \text{Basis functions (not sinusoidal)} \)

\( c \rightarrow \text{any ocean variable} \)
Determination of Basis Functions

• (1) Eigen Functions of the Laplace Operator (Data and Model Independent)

• (2) Empirical Orthogonal Functions (Data or Model Dependent)
Eigen Functions of Laplace Operator $\rightarrow$ Basis Functions (Closed Basin)

\[ \triangle \Psi_k = -\lambda_k \Psi_k, \quad \Psi_k|_\Gamma = 0, \quad k = 1, \ldots, \infty \]

\[ \triangle \Phi_m = -\mu_m \Phi_m, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0, \quad m = 1, \ldots, \infty. \]

$\Psi_k \rightarrow$ Streamfunction

$\Phi_m \rightarrow$ T, S, Velocity Potential
Basis Functions
(Open Boundaries)

\[ \triangle \Psi_k = -\lambda_k \Psi_k, \]

\[ \triangle \Phi_m = -\mu_m \Phi_m, \]

\[ \Psi_k|_{\Gamma} = 0, \quad \frac{\partial \Phi_m}{\partial n}|_{\Gamma} = 0, \]

\[ \left[ \frac{\partial \Psi_k}{\partial n} + \kappa(\tau) \Psi_k \right]|_{\Gamma_1} = 0, \quad \Phi_m|_{\Gamma_1} = 0, \]
Boundary Conditions

\[
\frac{\partial^2 \Phi}{\partial n \partial z} = 0
\]
\[
\Psi = 0
\]

\[
\frac{\partial \Phi}{\partial z} = 0
\]
\[
\frac{\partial \Psi}{\partial n} + \kappa \Psi = 0
\]

\[
\frac{\partial \Phi}{\partial n} = 0
\]
\[
\frac{\partial \Psi}{\partial n} = 0
\]
Spectral Decomposition

\[ u_{KM} = \sum_{k=1}^{K} a_k(z, t^o) \frac{\partial \Psi_k(x, y, z, \kappa^o)}{\partial y} + \sum_{m=1}^{M} b_m(z, t^o) \frac{\partial \Phi_m(x, y, z)}{\partial x}, \]

\[ v_{KM} = -\sum_{k=1}^{K} a_k(z, t^o) \frac{\partial \Psi_k(x, y, z, \kappa^o)}{\partial x} + \sum_{m=1}^{M} b_m(z, t^o) \frac{\partial \Phi_m(x, y, z)}{\partial y}, \]

\[ T(x, t) = T_0(x) + \sum_{m=1}^{M} c_m(t) \Phi_m(x), \]

\[ S(x, t) = S_0(x) + \sum_{m=1}^{M} d_m(t) \Phi_m(x). \]
Benefits of Using OSD

• (1) Don’t need first guess field

• (2) Don’t need autocorrelation functions

• (3) Don’t require high signal-to-noise ratio

• (4) Basis functions are pre-determined before the data analysis. They are independent on the data.
Optimal Mode Truncation

\[ J(a_1, \ldots, a_K, b_1, \ldots, b_M, \kappa, P) = \frac{1}{2} \left( \| u_p^{obs} - u_{KM} \|_P^2 + \| v_p^{obs} - v_{KM} \|_P^2 \right) \rightarrow \min, \]
Vapnik (1983) Cost Function

\[ J_{\text{emp}} = J(a_1, \ldots, a_K, b_1, \ldots, b_M, \kappa, P). \]

\[
\text{Prob} \left\{ \sup_{K,M,S} |\langle J(K,M,S) \rangle - J_{\text{emp}}(K,M,S) | \geq \mu \right\} \leq g(P, \mu)
\]

\[
\lim_{P \to \infty} g(P, \mu) = 0
\]
Optimal Truncation

- Gulf of Mexico, Monterey Bay, Louisiana-Texas Shelf, North Atlantic

- $K_{opt} = 40$, $M_{opt} = 30$
Determination of Spectral Coefficients
(Ill-Posed Algebraic Equation)

\[ A \hat{\alpha} = QY, \]

This is caused by the features of the matrix \( A \).
Rotation Method (Chu et al., 2004)

Well-Posed $\rightarrow$ 

$$SA\hat{a} = SQY,$$

The matrix $S$ is determined by

$$J_1 = \|A\|^2 - \frac{\|SQY\|^2}{\|a\|^2} \rightarrow \text{max},$$
Errors

\[
\tilde{T}(x) = T_0 + \sum_{l=1}^{48} D_l \Phi_l(x) + T'(x)
\]

\[
\tilde{u}(x, t) = C \Psi_0 + \sum_{n=1}^{24} A_n \nabla \times k \Psi_n(x) + \tilde{u}(x) + u'(x)
\]

\[T', u' \rightarrow \text{errors}\]
Noise-to-Signal Ratio \( \rightarrow \)

Error Estimation

\[
\eta(\alpha, \beta) = \frac{\|\alpha\|_{(P)}}{\|\beta\|_{(P)}}
\]

\[
\eta(T', T - T') \sim 0.1
\]
(3) Upper Ocean Heat Content
Upper Ocean (0-300 m) Heat Content

\[ HC = \int_{-h}^{0} \rho c T dz \]

\[ HC = HC_{\text{mean}} + HC_{\text{seasonal}} + HC_{\text{anomaly}} \]

EOF Analysis \(\rightarrow HC_{\text{anomaly}}\)

\(\rightarrow\) Global Ocean Dipole Modes
Trend of Upper Ocean (0-700 m) Heat Content

0.4 \times 10^{22} \text{ J/yr} 
(1958-2008) 
(Levitus et al., GRL, 2009) 
Without Argo data

1.3 \times 10^{22} \text{ J/yr} 
(1990-2008) 
With Argo data
Upper Ocean (0-300 m) Mean Heat Content (J/m²) (1990-2009)
Seasonal Variability of Upper Ocean (0-300 m) Heat Content (J/m$^2$) (1990-2009)

January

April

July

October
EOF Analysis ➔

Heat Content Anomaly Relative to Seasonal Variation
EOF-1 (in $10^8 \text{ J/m}^2$)
EOF-1 (in $10^8$ J/m$^2$)
Lag Correlation between $PC_1$ and SOI

Positive Month $\Rightarrow$ $PC_1$ advancing SOI
EOF-2 (in $10^8$ J/m$^2$)
EOF-2 (in $10^8$ J/m$^2$)
$PC_2$
Lag Correlation between $\text{PC}_2$ and SOI

Positive Month $\rightarrow$ $\text{PC}_2$ advancing SOI
(4) Global Ocean Tripole
Canonical La Nina $\rightarrow$
More and Stronger Hurricanes in Atlantic
(Pielke and Landsea, 1999 BAMS)
 Canonical El Nino

Indian Ocean | Pacific Ocean | Atlantic Ocean
HCA < 0 | max(|HCA|) | max(HCA)
HCA < 0 | HCA < 0 | HCA > 0
HCA < 0 | HCA < 0 | HCA < 0

Mode - 1 El Nino
El Nino Modoki
More and Stronger Hurricanes in Atlantic
Trajectory in Phase Space (PC₁, PC₂)

• **Blue Curve** →
  La Nina

• **Red Curve** →
  Two Types of
  El Nino
Conclusions

• (1) Upper ocean heat content contains the signal for climate change (interannual) → Global Ocean Tripole.

• (2) El Nino, El Nino Modoki, and Indian Ocean Dipole can be unified by Global Ocean Tripole.
Future Improvement

• Upper ocean heat content should not be calculated to a fixed depth such as 300 m in this study.

• Heat content in ocean mixed layer should be most important for the climate change.

• There is no simple, objective and effective method to determine mixed layer depth from profile data.

• My near future work is to develop such a method.