

# **Global Ocean Tripole and Climate Variability**

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# Outline

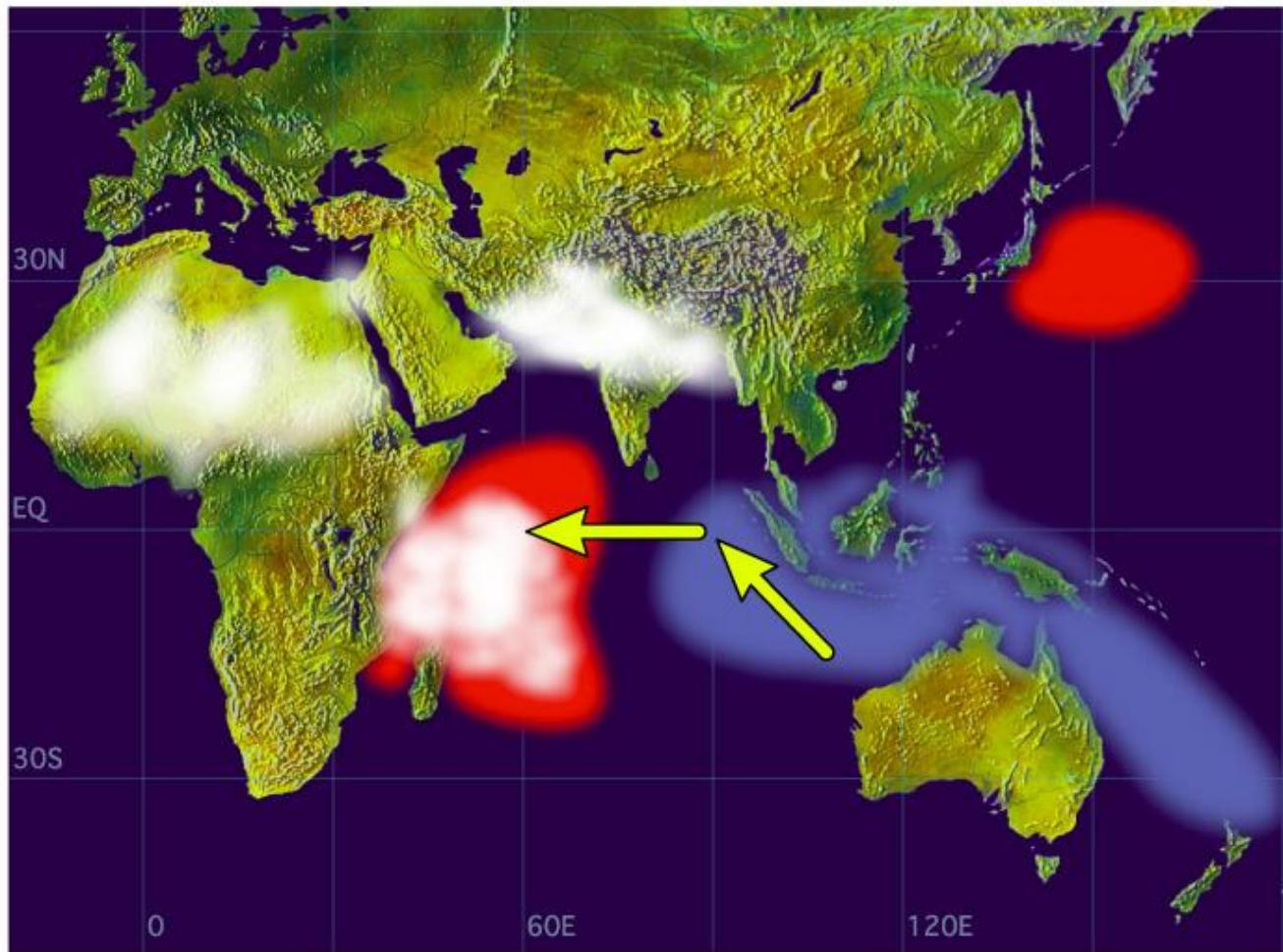
- (1) Recent Development in Short-term Climate Variability
- (2) Data Analysis: (T, S) Profiles → Synoptic Gridded Data with Monthly Increment
- (3) Synoptic Upper Ocean (0-300 m) Heat Content
- (4) Global Tripole → Canonical El Nino, El Nino Modoki, Indian Ocean Dipole, ...

# (1) Recent Development in Climate Variability

Indian Ocean Dipole  
El Nino Modoki

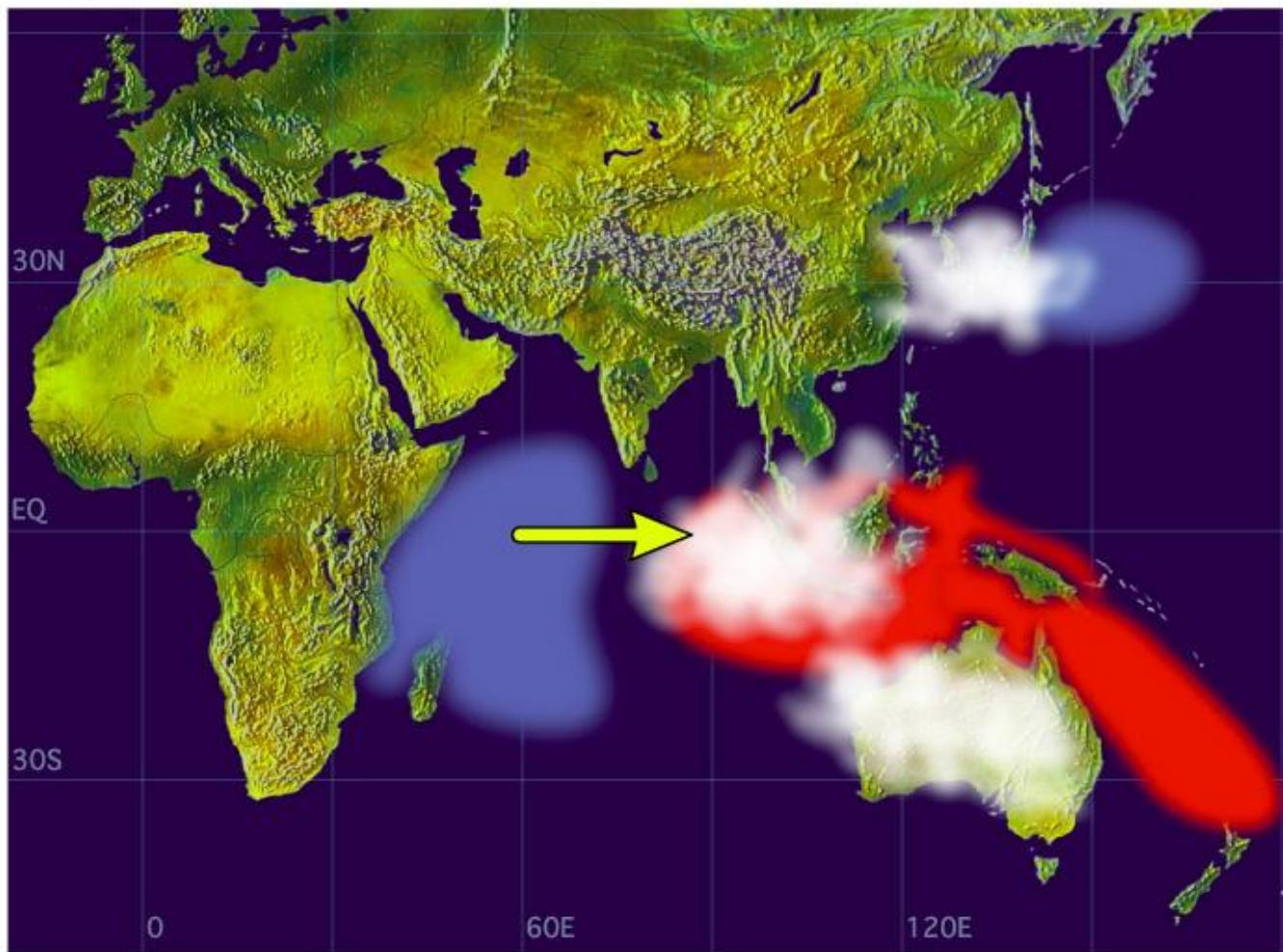
# Indian Ocean Dipole (Saji et al. 1999)

Positive Dipole Mode



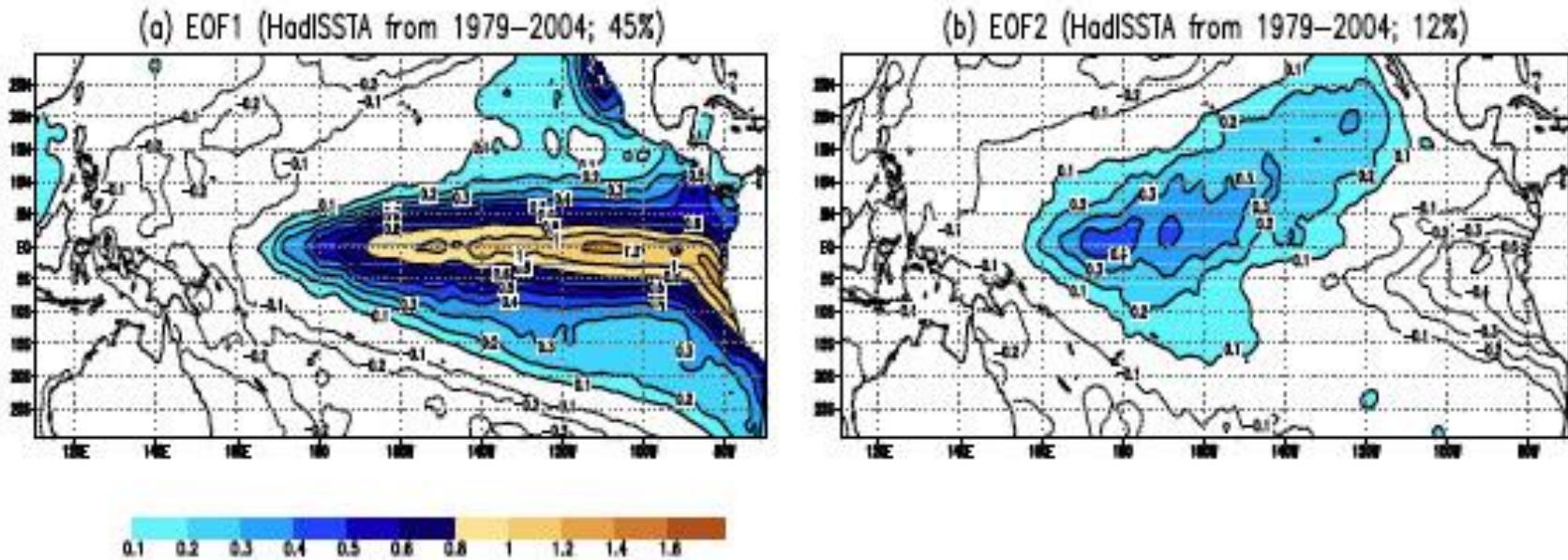
# Indian Ocean Dipole (Saji et al. 1999)

Negative Dipole Mode



# El Nino and El Nino Modoki

(Weng et al. 2007, Ashok et al. 2007)



(Images courtesy Karumuri Ashok, APEC Climate Center)

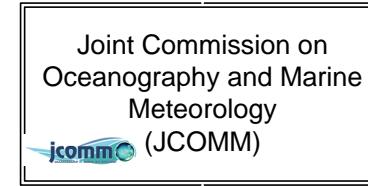
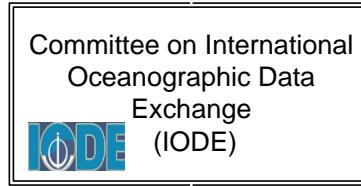
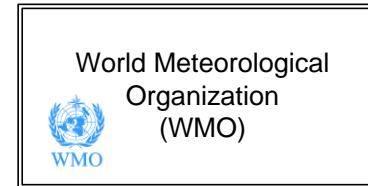
## (2). Data Analysis

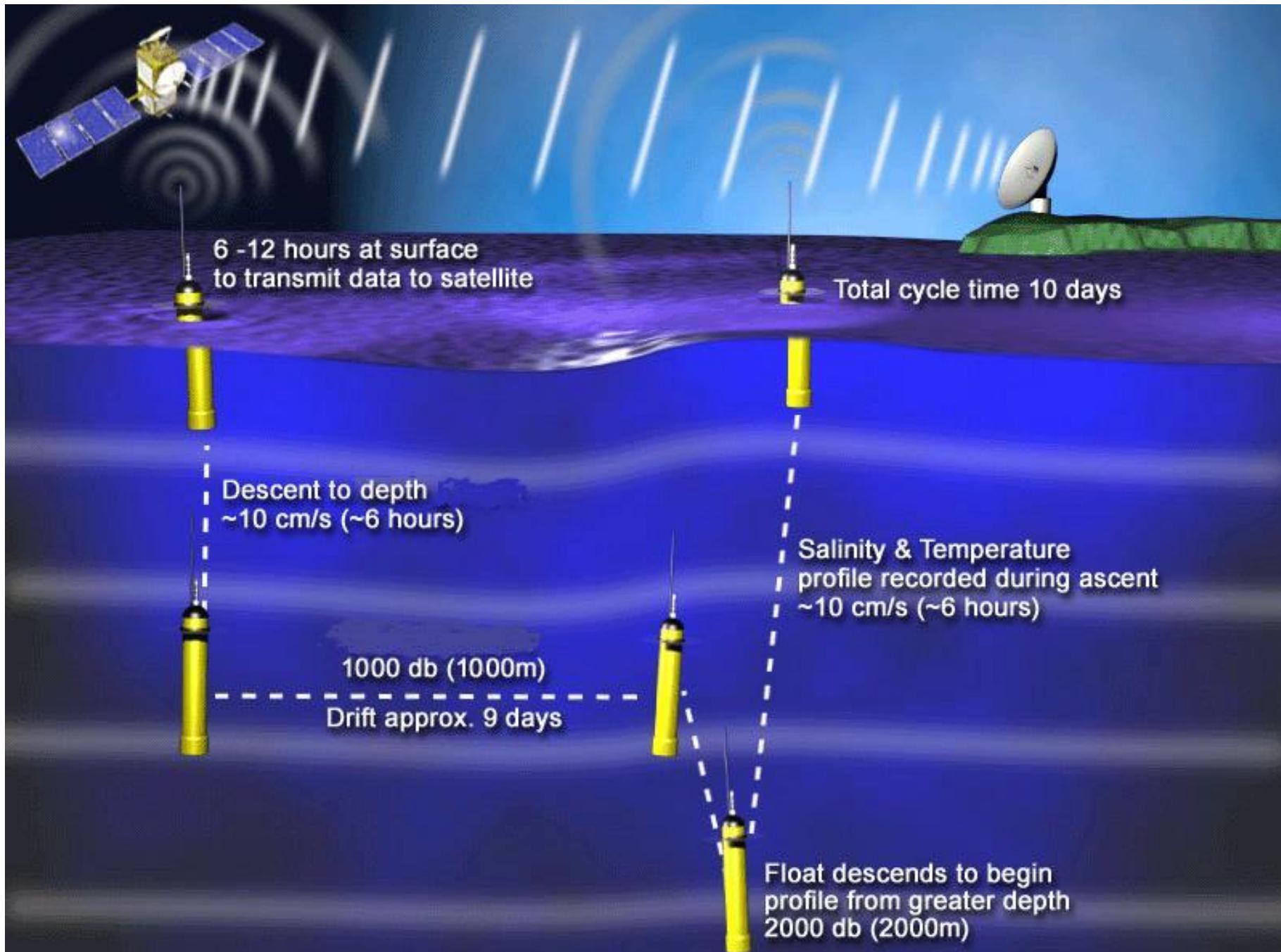
Global Temperature and Salinity Profile  
Program (GTSP)

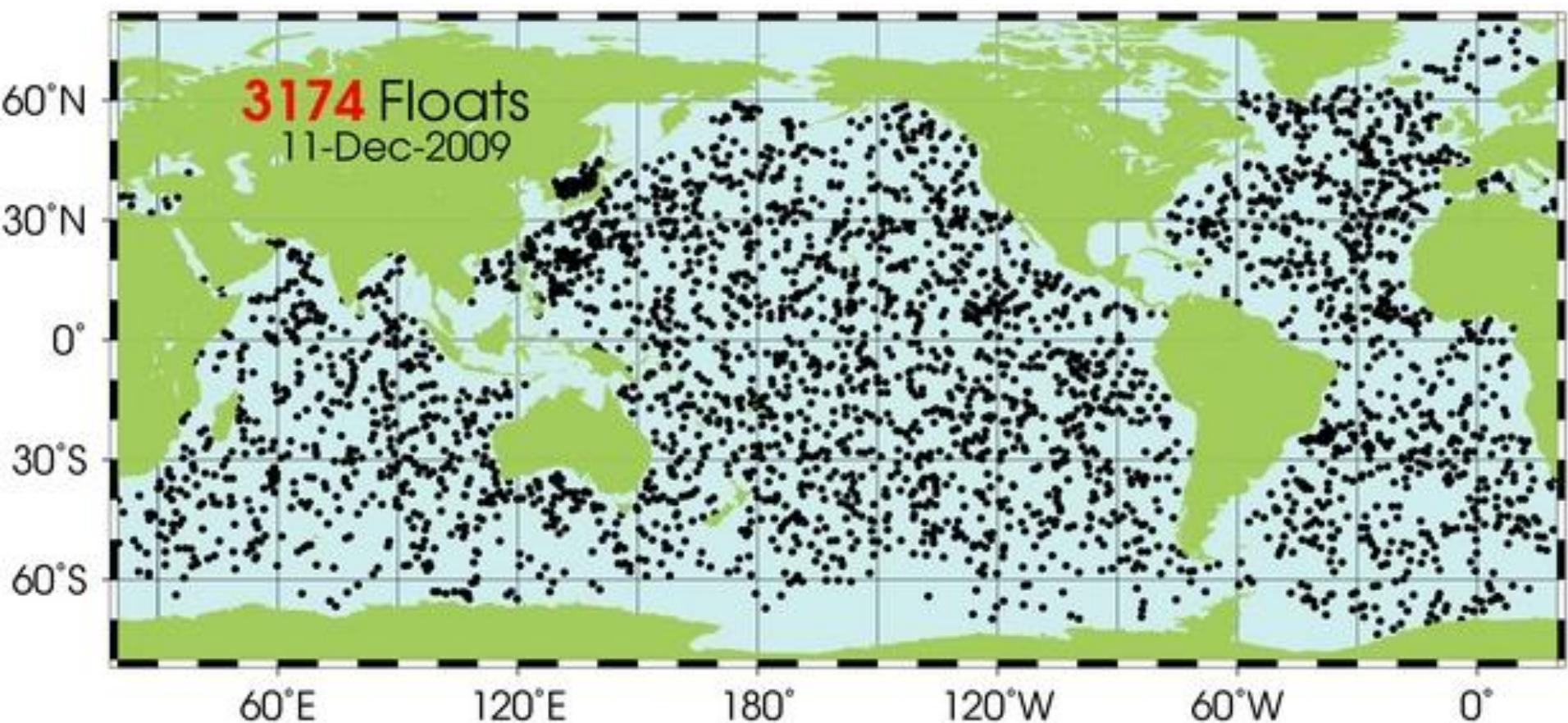
# GTSPP

## **GTSPP = Global Temperature Salinity Profile Program**

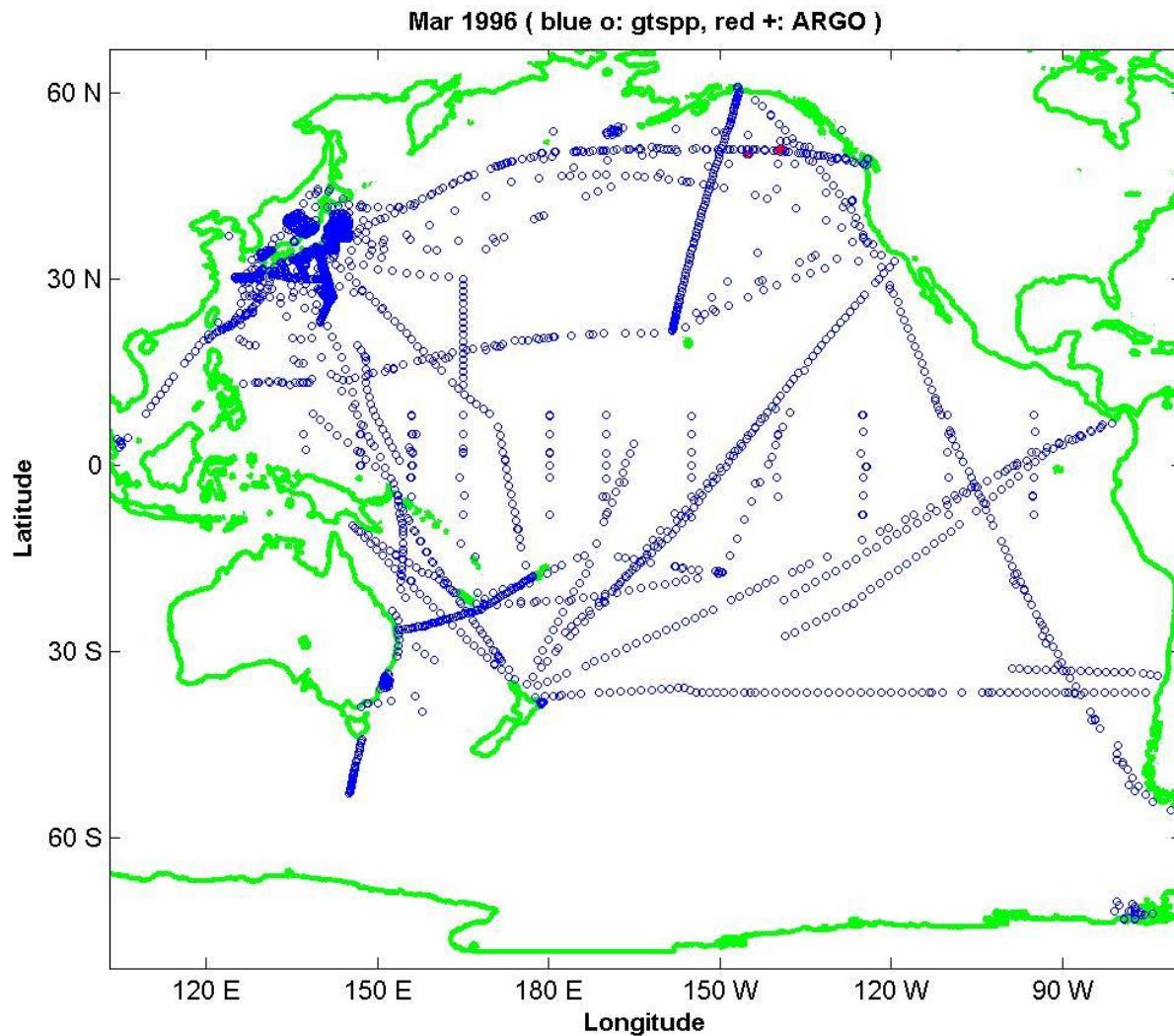
- GTSPP is a joint WMO-IOC program designed to provide improved access to the highest resolution, highest quality data as quickly as possible.
- GTSPP began as an official IODE pilot project in 1989.
- It went into operation in November 1990.



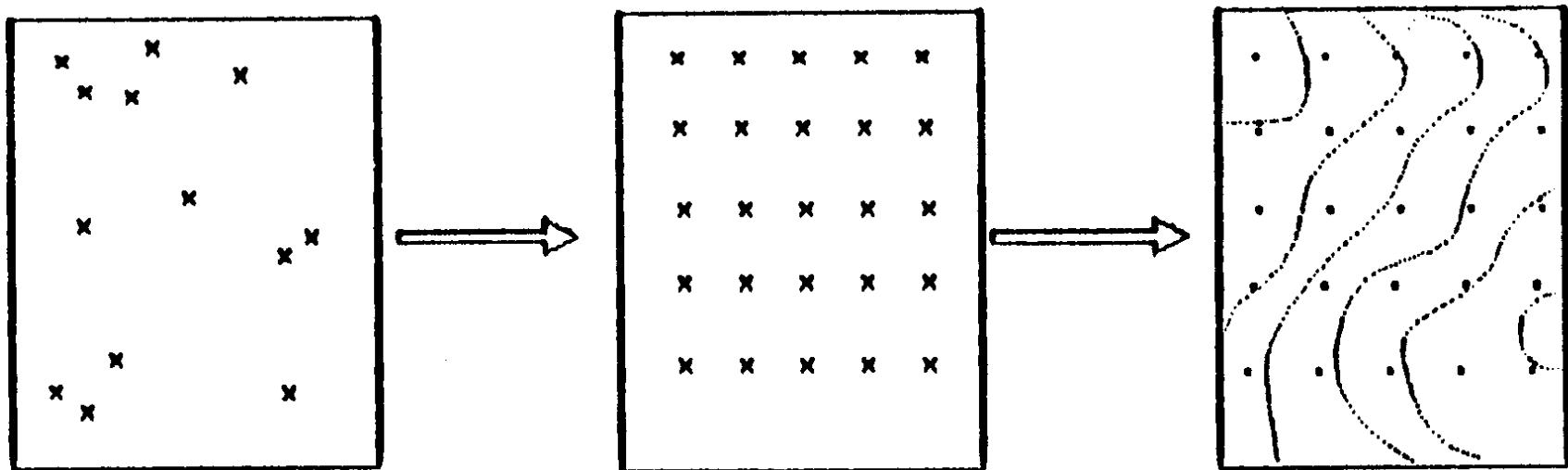




# Example → GTSP Data



# Ocean Data Analysis



**Classical Method → Fourier Series Expansion**

# Joseph Fourier 1768-1830



Fourier was obsessed with the physics of heat and developed the Fourier series and transform to model heat-flow problems.

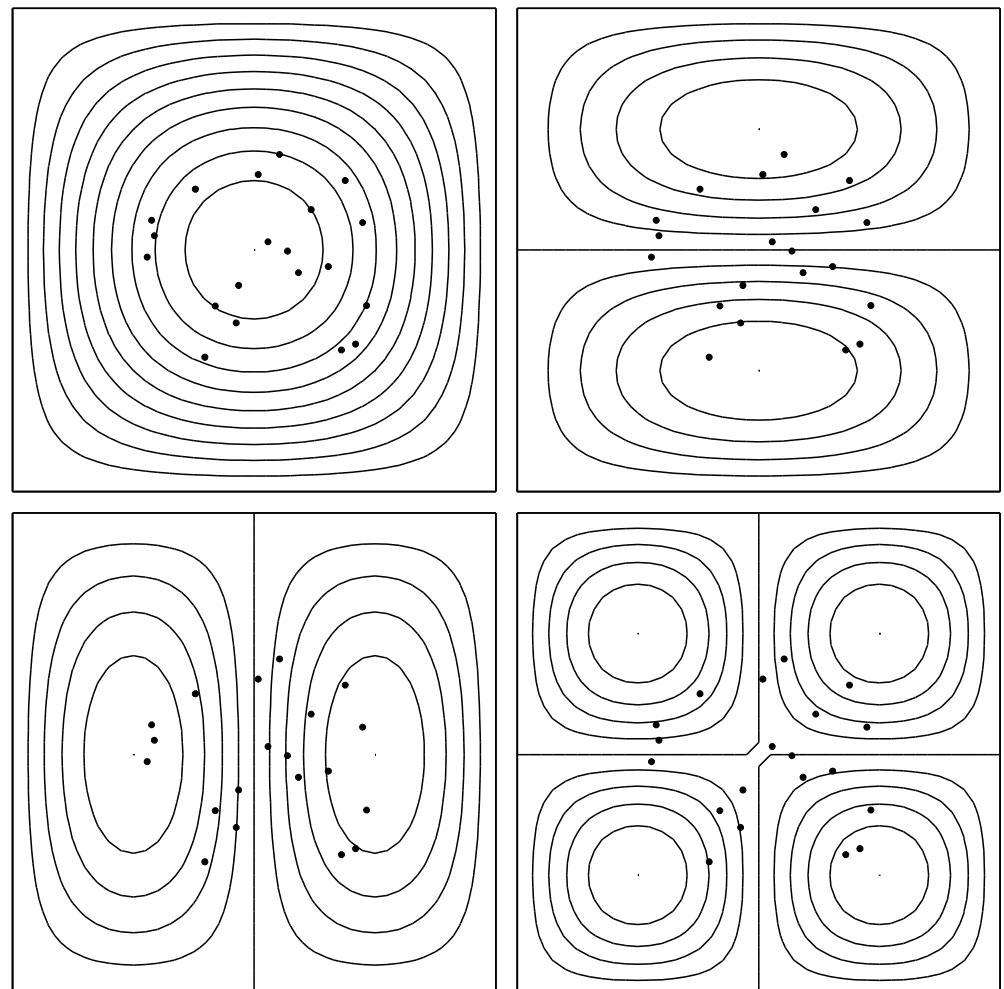
# Fourier Series Expansion

For a rectangular region  $(L_x, L_y)$ , the basis functions are sinusoidal functions.

$$f(x, y) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y}$$
$$+ \sum_i \sum_j b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}$$

For the Dirichlet boundary condition :  $f = 0$  at the boundaries

$$f(x, y) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y}$$



The dots represent the Observations.

# Linear Algebraic Equations for the Coefficients $a_{ij}$

$$f(x_1^{ob}, y_1^{ob}) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x_1^{ob}}{L_x} \sin \frac{j\pi y_1^{ob}}{L_y}$$

$$f(x_2^{ob}, y_2^{ob}) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x_2^{ob}}{L_x} \sin \frac{j\pi y_2^{ob}}{L_y}$$

.....

$$f(x_M^{ob}, y_M^{ob}) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x_M^{ob}}{L_x} \sin \frac{j\pi y_M^{ob}}{L_y}$$

# Determination of Spectral Coefficients (III-Posed Algebraic Equation)

$$\mathbf{A}\hat{\mathbf{a}} = \mathbf{Q}\mathbf{Y},$$

Known  $a_{ij}$  → Analyzed Field

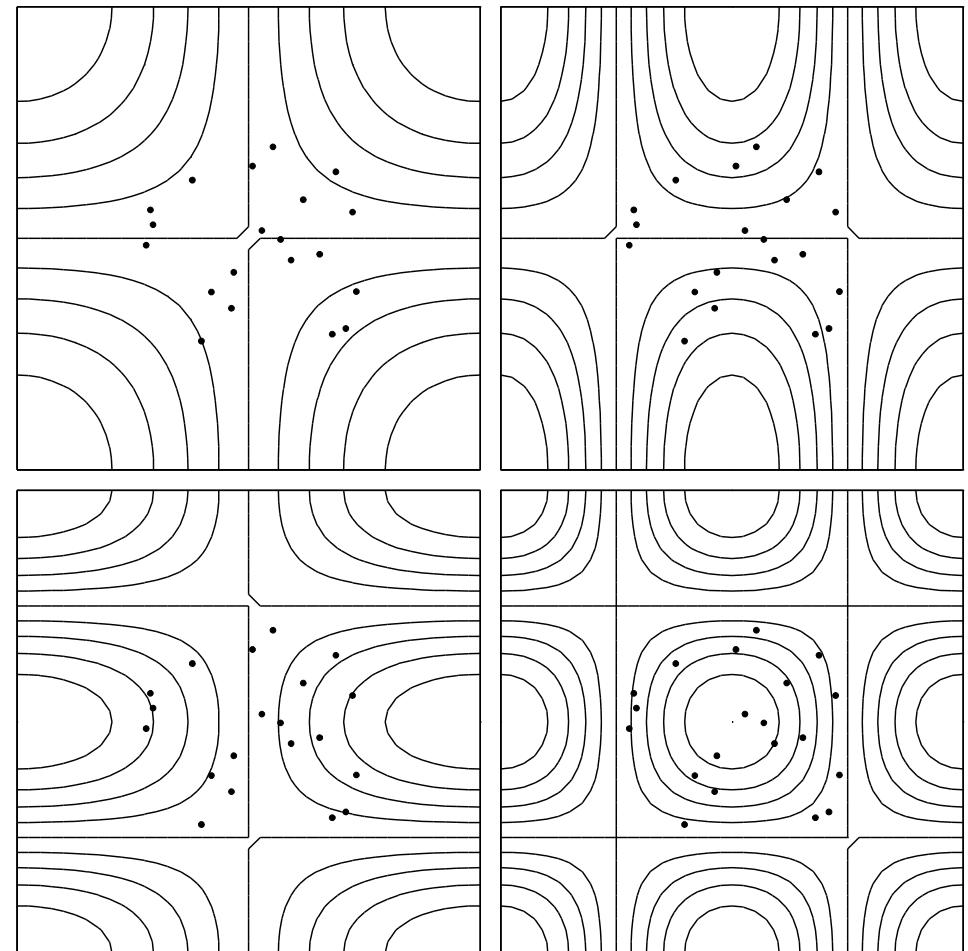
$$f(x, y) = \sum_i \sum_j a_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y}$$

For the Neumann boundary condition  
at the boundaries

$$n \bullet \nabla f = 0$$

$$f(x, y) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}$$

The dots represent the  
Observations.



# Linear Algebraic Equations for the Coefficients $a_{ij}$

$$f(x_1^{ob}, y_1^{ob}) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x_1^{ob}}{L_x} \cos \frac{j\pi y_1^{ob}}{L_y}$$

$$f(x_2^{ob}, y_2^{ob}) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x_2^{ob}}{L_x} \cos \frac{j\pi y_2^{ob}}{L_y}$$

.....

$$f(x_M^{ob}, y_M^{ob}) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x_M^{ob}}{L_x} \cos \frac{j\pi y_M^{ob}}{L_y}$$

Known  $b_{ij}$  → Analyzed Field

$$f(x, y) = \sum_i \sum_j b_{ij} \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y}$$

For General Ocean Basin →  
Generalized Fourier Series  
Expansion

# Spectral Representation Fourier Series Expansion

$$c(\mathbf{x}, z_k, t) = A_0(z_k, t) + \sum_{m=1}^M A_m(z_k, t) \Psi_m(\mathbf{x}, z_k),$$

$\Psi_m \rightarrow$  Basis functions (not sinusoidal)

$c \rightarrow$  any ocean variable

# Determination of Basis Functions

- (1) Eigen Functions of the Laplace Operator  
(Data and Model Independent)
- (2) Empirical Orthogonal Functions (Data or  
Model Dependent)

# Eigen Functions of Laplace Operator → Basis Functions (Closed Basin)

$$\Delta \Psi_k = -\lambda_k \Psi_k, \quad \Psi_k|_{\Gamma} = 0, \quad k = 1, \dots, \infty$$

$$\Delta \Phi_m = -\mu_m \Phi_m, \quad \frac{\partial \Phi_m}{\partial n}|_{\Gamma} = 0, \quad m = 1, \dots, \infty.$$

$\Psi_k \rightarrow$  Streamfunction

$\Phi_m \rightarrow T, S, \text{Velocity Potential}$

# Basis Functions (Open Boundaries)

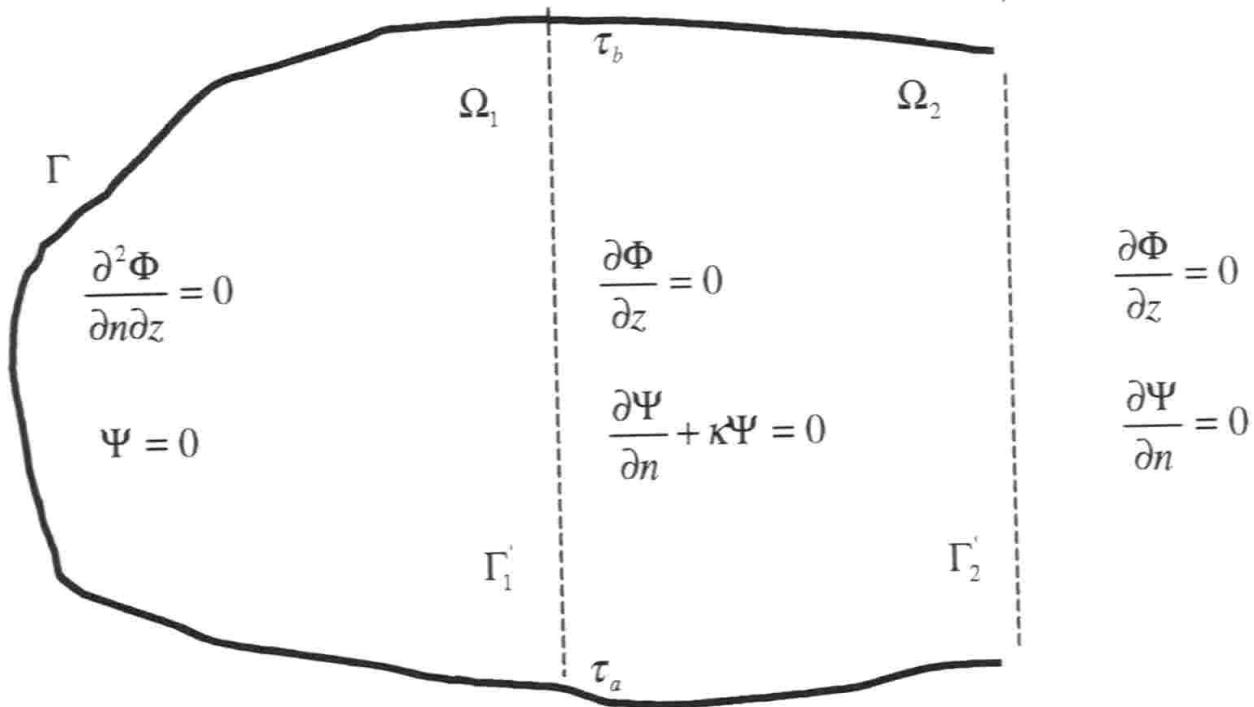
$$\Delta \Psi_k = -\lambda_k \Psi_k,$$

$$\Delta \Phi_m = -\mu_m \Phi_m,$$

$$\Psi_k|_{\Gamma} = 0, \quad \frac{\partial \Phi_m}{\partial n}|_{\Gamma} = 0,$$

$$\left[ \frac{\partial \Psi_k}{\partial n} + \kappa(\tau) \Psi_k \right] |_{\Gamma'_1} = 0, \quad \Phi_m|_{\Gamma'_1} = 0,$$

# Boundary Conditions



# Spectral Decomposition

$$u_{KM} = \sum_{k=1}^K a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial y} + \sum_{m=1}^M b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial x},$$

$$v_{KM} = - \sum_{k=1}^K a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial x} + \sum_{m=1}^M b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial y}$$

$$T(\mathbf{x}, t) = T_0(\mathbf{x}) + \sum_{m=1}^M c_m(t) \Phi_m(\mathbf{x})$$

$$S(\mathbf{x}, t) = S_0(\mathbf{x}) + \sum_{m=1}^M d_m(t) \Phi_m(\mathbf{x})$$

# Benefits of Using OSD

- (1) Don't need first guess field
- (2) Don't need autocorrelation functions
- (3) Don't require high signal-to-noise ratio
- (4) Basis functions are pre-determined before the data analysis. They are independent on the data.

# Optimal Mode Truncation

$$J(a_1, \dots, a_K, b_1, \dots, b_M, \kappa, P) = \frac{1}{2} \left( \|u_p^{obs} - u_{KM}\|_P^2 + \|v_p^{obs} - v_{KM}\|_P^2 \right) \rightarrow \min,$$

# Vapnik (1983) Cost Function

$$J_{emp} = J(a_1, \dots, a_K, b_1, \dots, b_M, \kappa, P).$$

$$\text{Prob} \left\{ \sup_{K,M,S} |\langle J(K, M, S) \rangle - J_{emp}(K, M, S)| \geq \mu \right\} \leq g(P, \mu)$$

$$\lim_{P \rightarrow \infty} g(P, \mu) = 0$$

# Optimal Truncation

- Gulf of Mexico, Monterey Bay, Louisiana-Texas Shelf, North Atlantic
- $K_{\text{opt}} = 40$ ,  $M_{\text{opt}} = 30$

# Determination of Spectral Coefficients (III-Posed Algebraic Equation)

$$\mathbf{A} \hat{\mathbf{a}} = \mathbf{Q} \mathbf{Y},$$

This is caused by the features of the matrix  $\mathbf{A}$ .

# Rotation Method (Chu et al., 2004)

Well-Posed  $\rightarrow$

$$\mathbf{S}\mathbf{A}\hat{\mathbf{a}} = \mathbf{SQY},$$

The matrix S is determined by

$$J_1 = \|\mathbf{A}\|^2 - \frac{\|\mathbf{SQY}\|^2}{\|\mathbf{a}\|^2} \rightarrow \max,$$

# Errors

$$\bar{T}(\mathbf{x}) = T_0 + \underbrace{\sum_{l=1}^{48} D_l \Phi_l(\mathbf{x})}_{\hat{T}} + T'(\mathbf{x})$$

$$\bar{\mathbf{u}}(\mathbf{x}, t) = C\Psi_0 + \underbrace{\sum_{n=1}^{24} A_n \nabla \times \mathbf{k} \Psi_n(\mathbf{x})}_{\hat{\mathbf{u}}} + \tilde{\mathbf{u}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x})$$

$T'$ ,  $\mathbf{u}' \rightarrow$  errors

# Noise-to-Signal Ratio → Error Estimation

$$\eta(\alpha, \beta) = \frac{\|\alpha\|_{(P)}}{\|\beta\|_{(P)}}$$

$$\eta(T', T - T') \sim 0.1$$

# (3) Upper Ocean Heat Content

# Upper Ocean (0-300 m) Heat Content

$$HC = \int_{-h}^0 \rho c T dz$$

$$HC = HC_{\text{mean}} + HC_{\text{seasonal}} + HC_{\text{anomaly}}$$

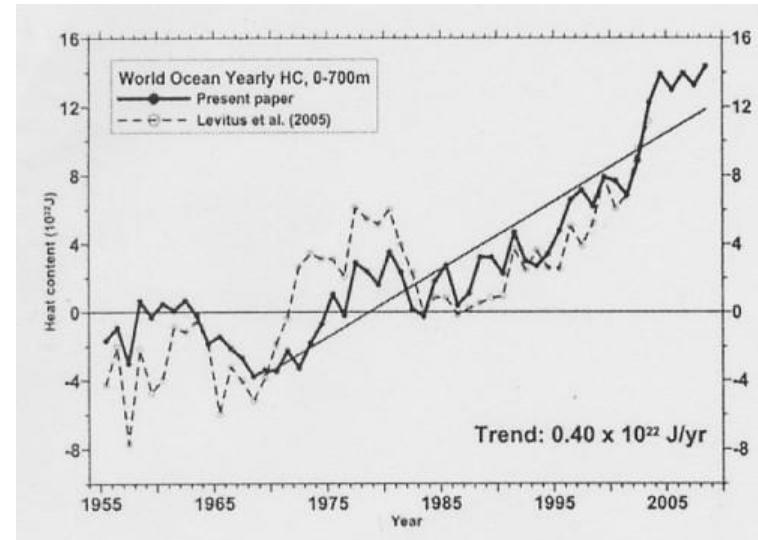
*EOF Analysis* →  $HC_{\text{anomaly}}$

→ Global Ocean Dipole Modes

# Trend of Upper Ocean (0-700 m) Heat Content

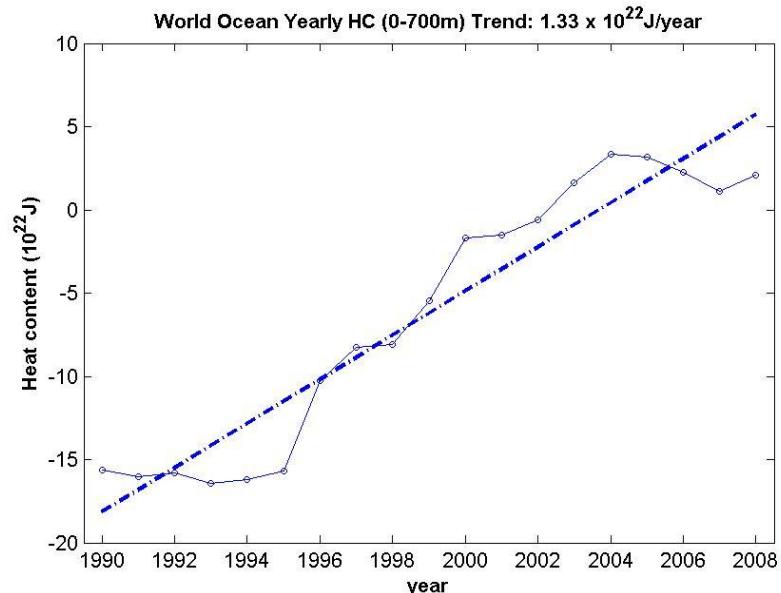
$0.4 \times 10^{22}$  J/yr  
(1958-2008)

(Levitus et al., GRL, 2009)  
Without Argo data

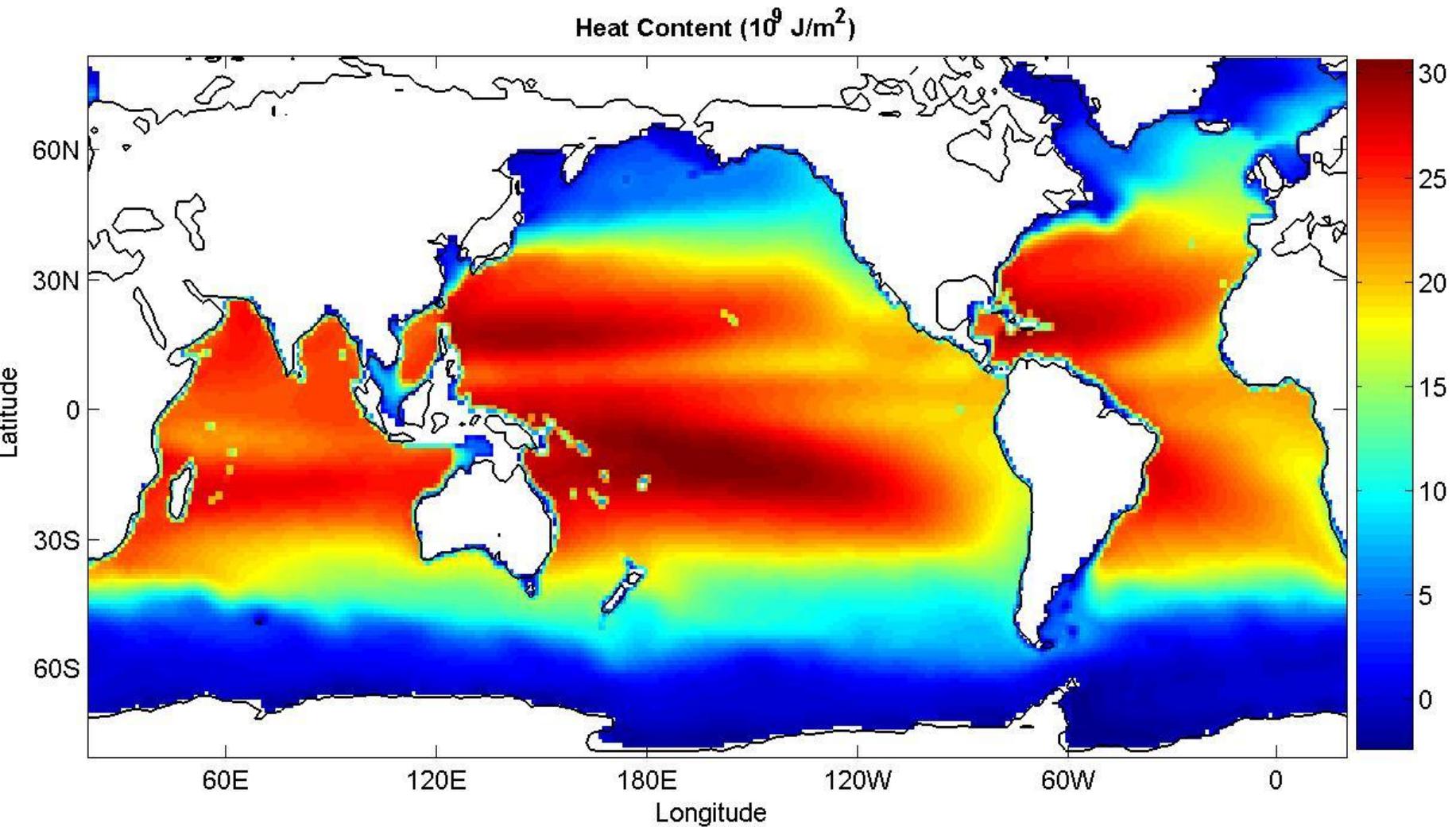


$1.3 \times 10^{22}$  J/yr  
(1990-2008)

With Argo data

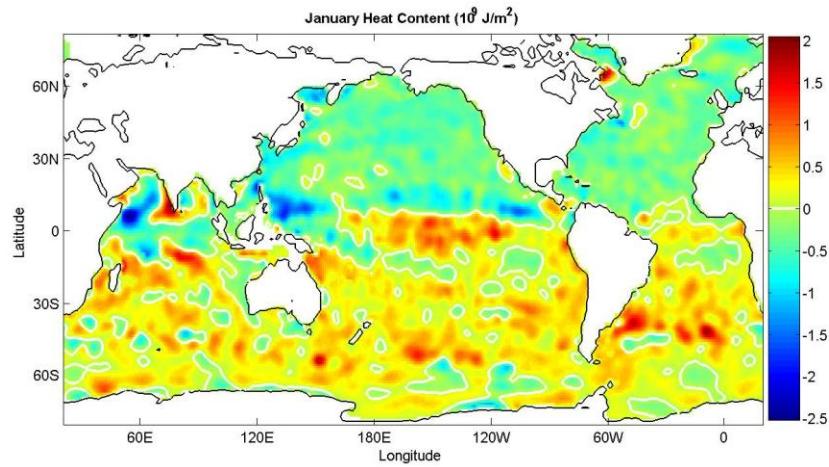


# Upper Ocean (0-300 m) Mean Heat Content (J/m<sup>2</sup>) (1990-2009)

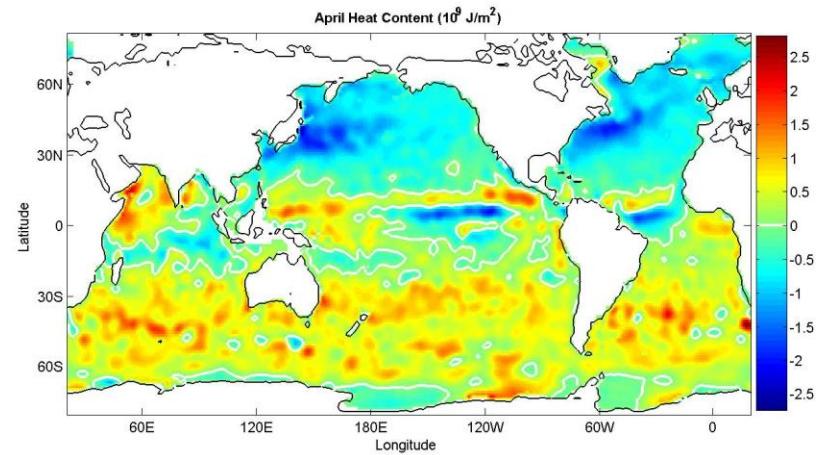


# Seasonal Variability of Upper Ocean (0-300 m) Heat Content ( $\text{J/m}^2$ ) (1990-2009)

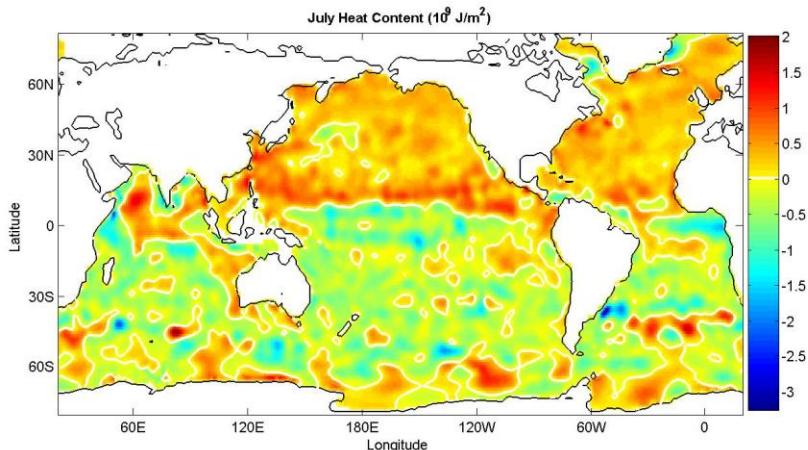
January



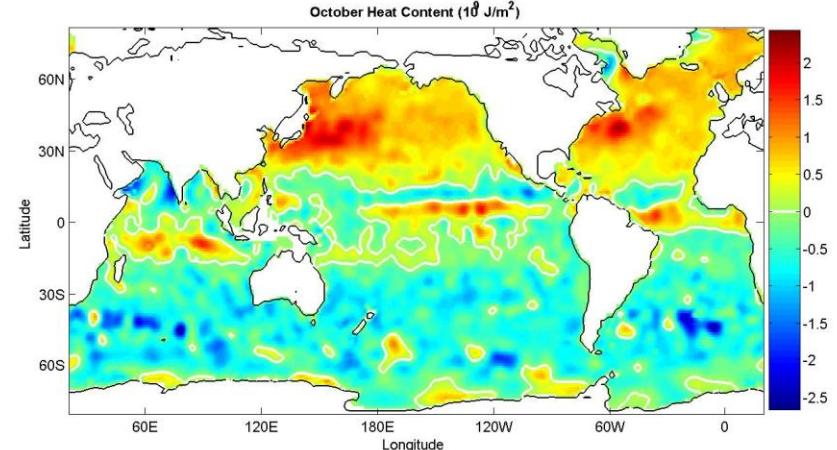
April



July



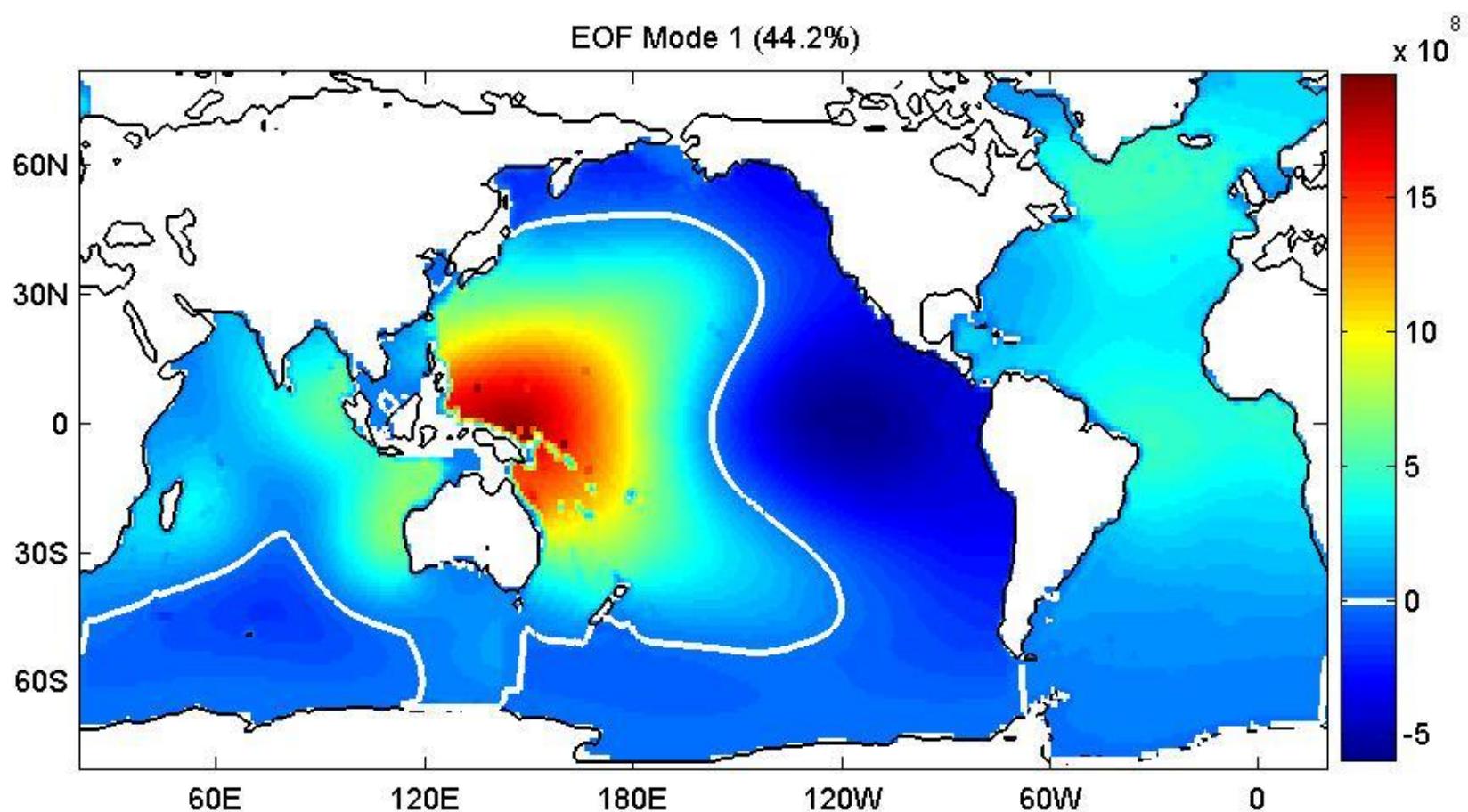
October



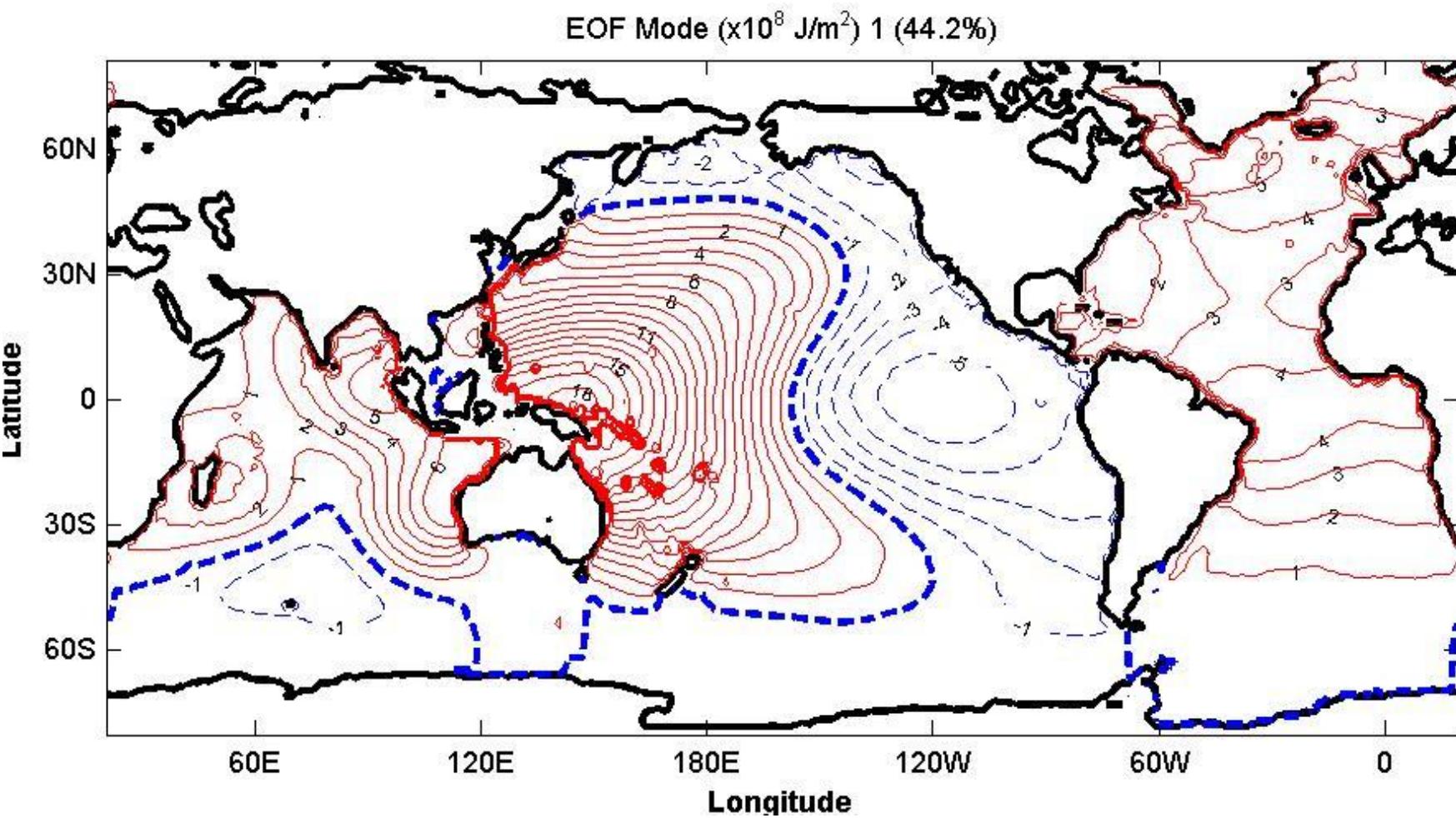
EOF Analysis →

Heat Content Anomaly Relative to  
Seasonal Variation

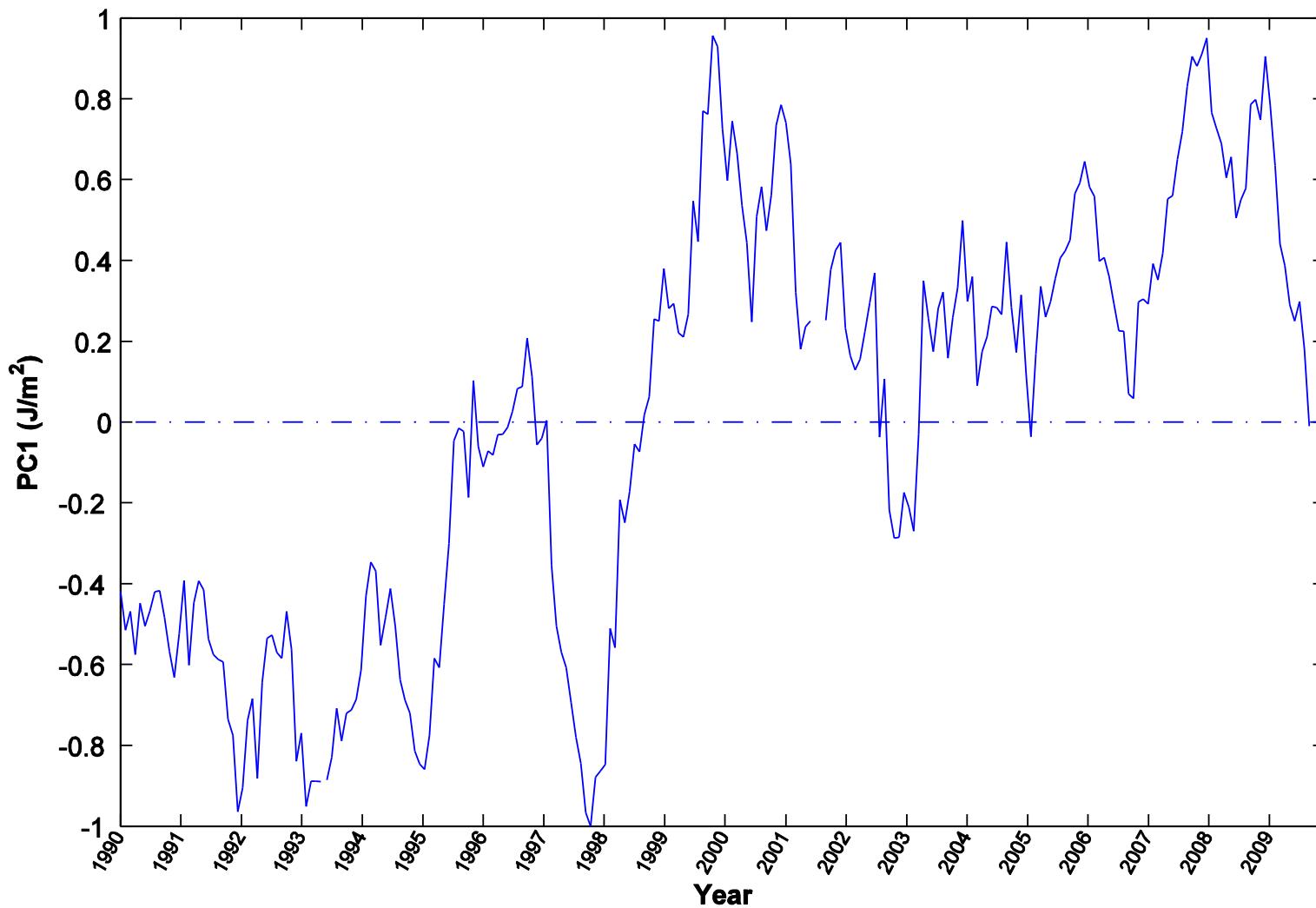
# EOF-1 (in $10^8 \text{ J/m}^2$ )



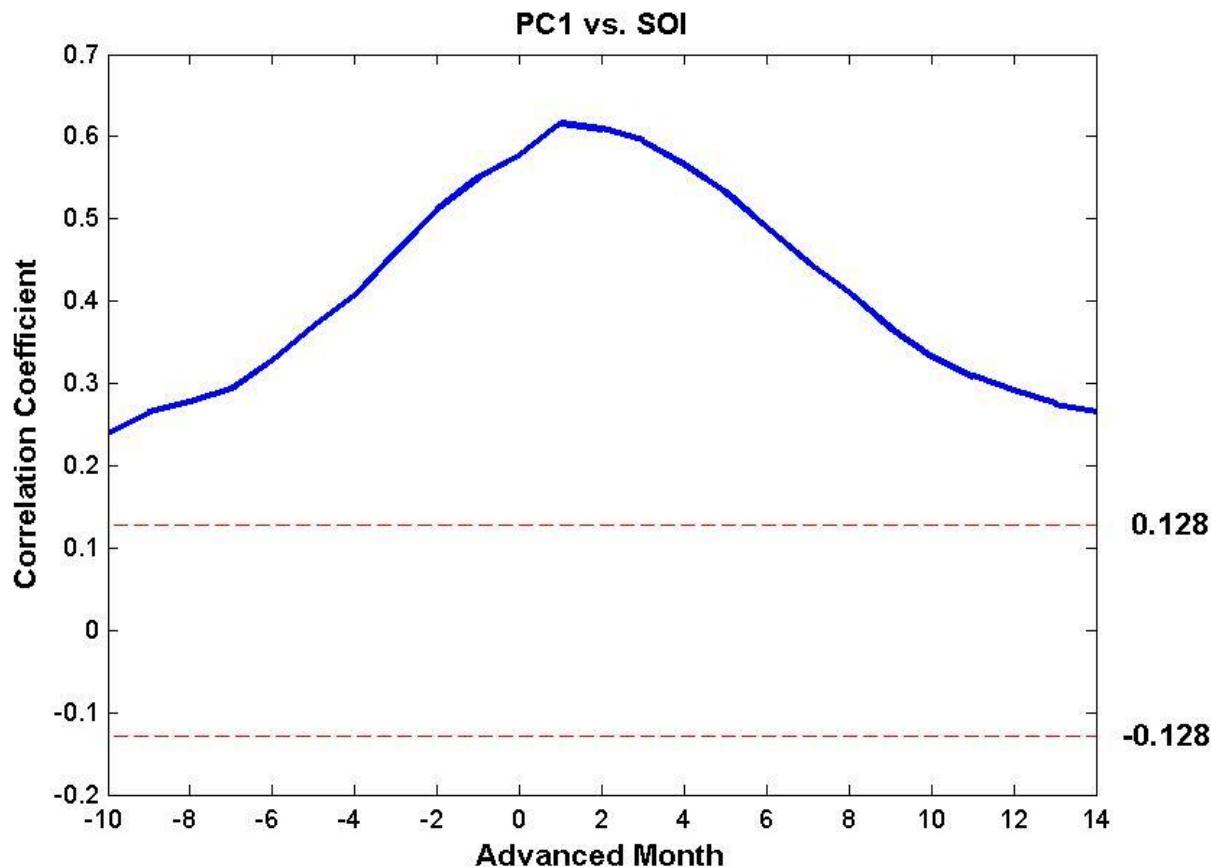
# EOF-1 (in $10^8 \text{ J/m}^2$ )



# $\text{PC}_1$

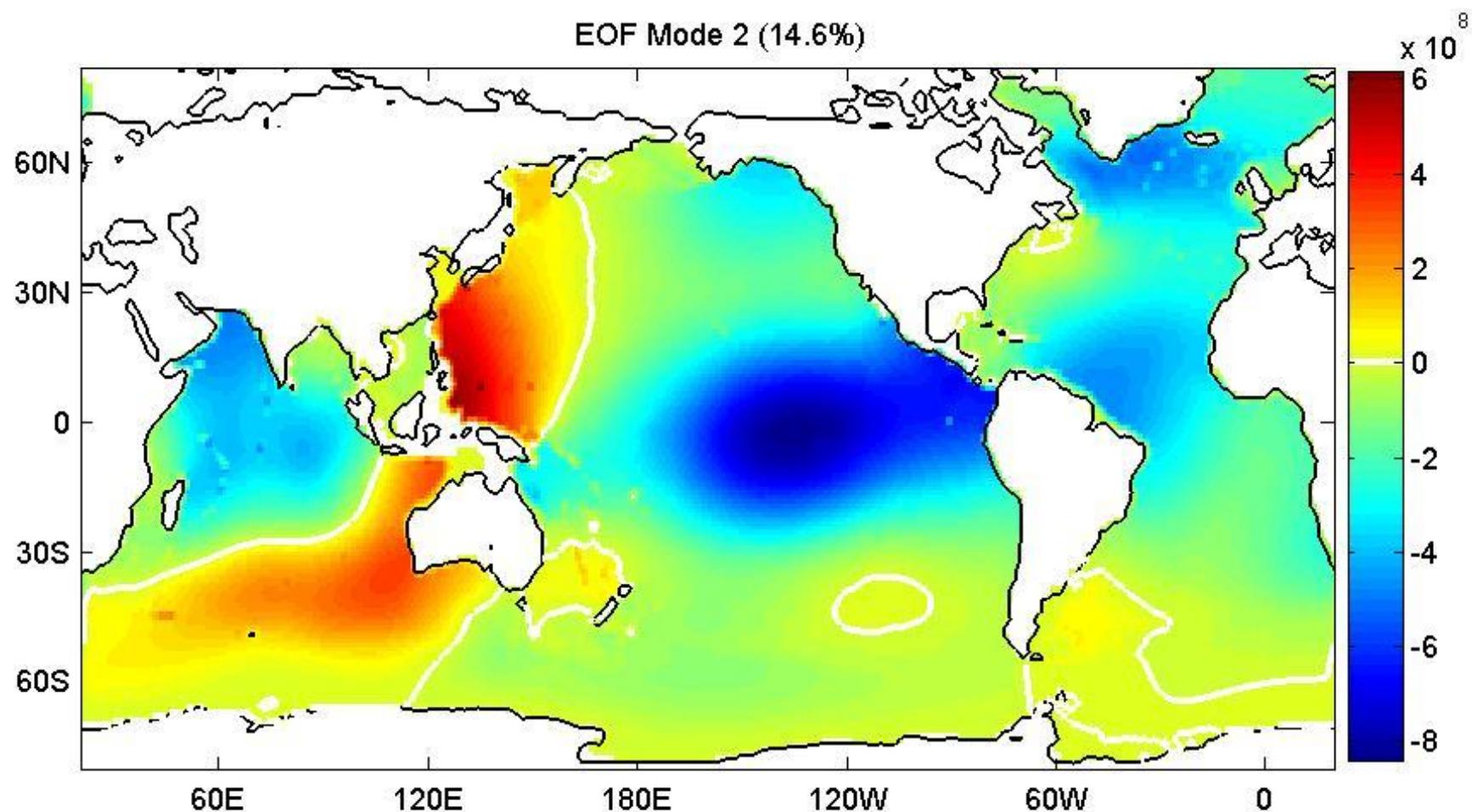


# Lag Correlation between $PC_1$ and SOI

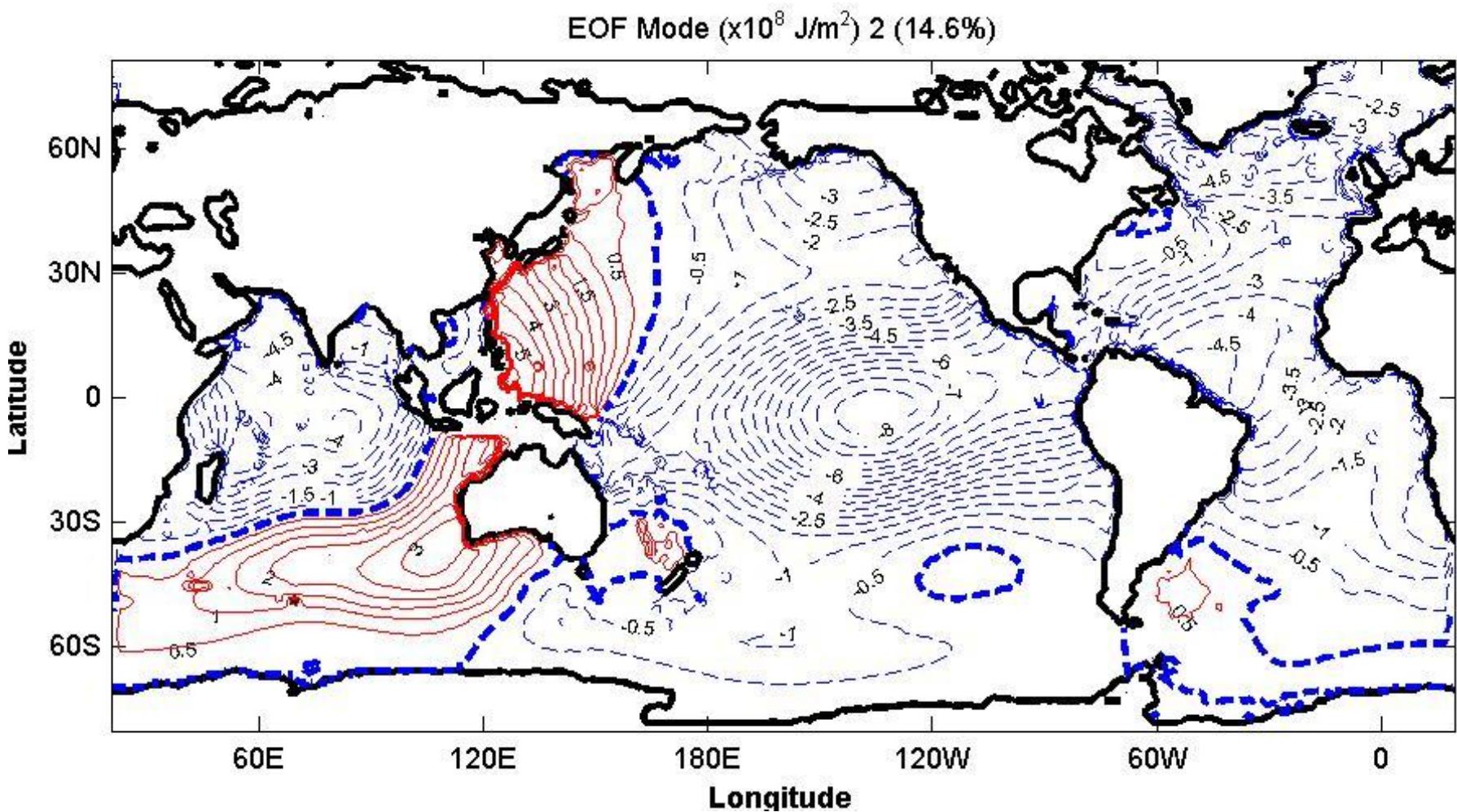


Positive Month →  $PC_1$  advancing SOI

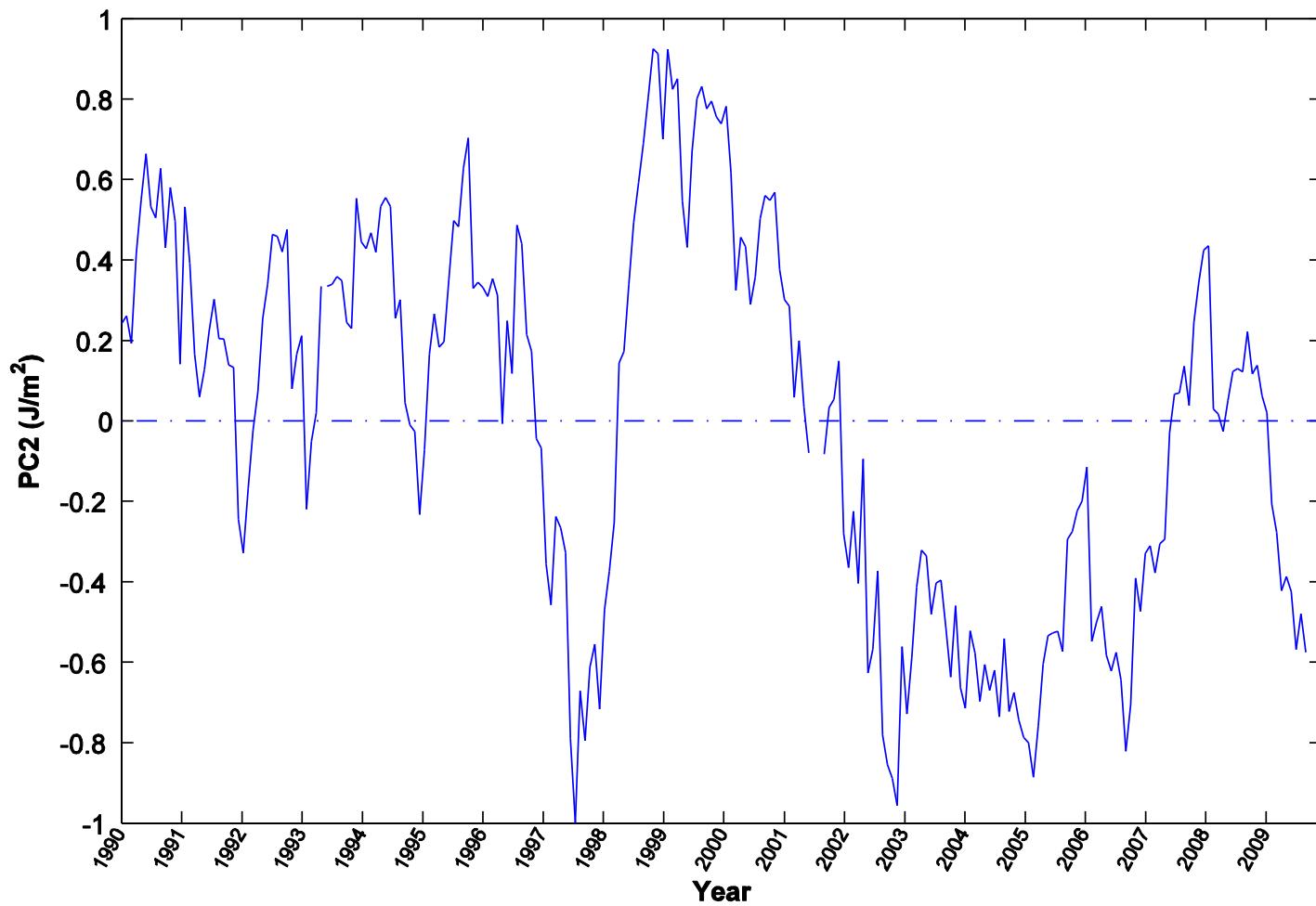
# EOF-2 (in $10^8 \text{ J/m}^2$ )



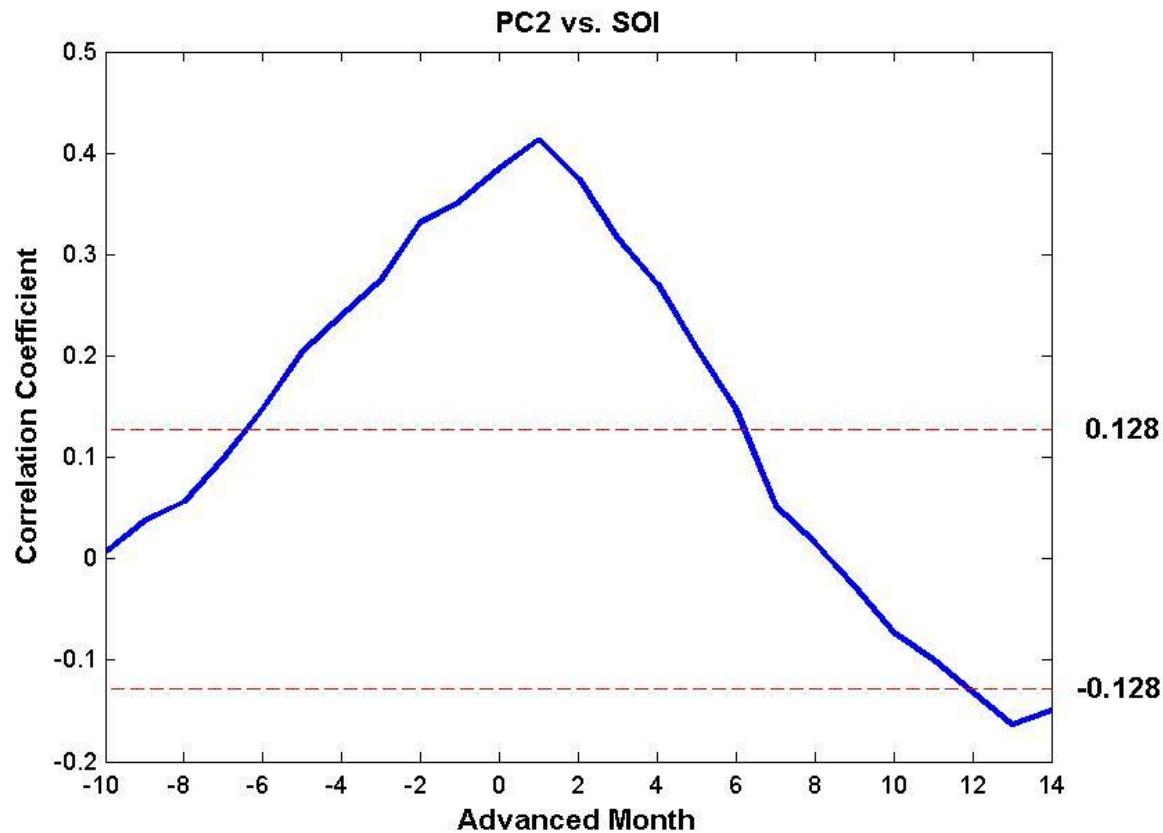
# EOF-2 (in $10^8 \text{ J/m}^2$ )



# PC<sub>2</sub>



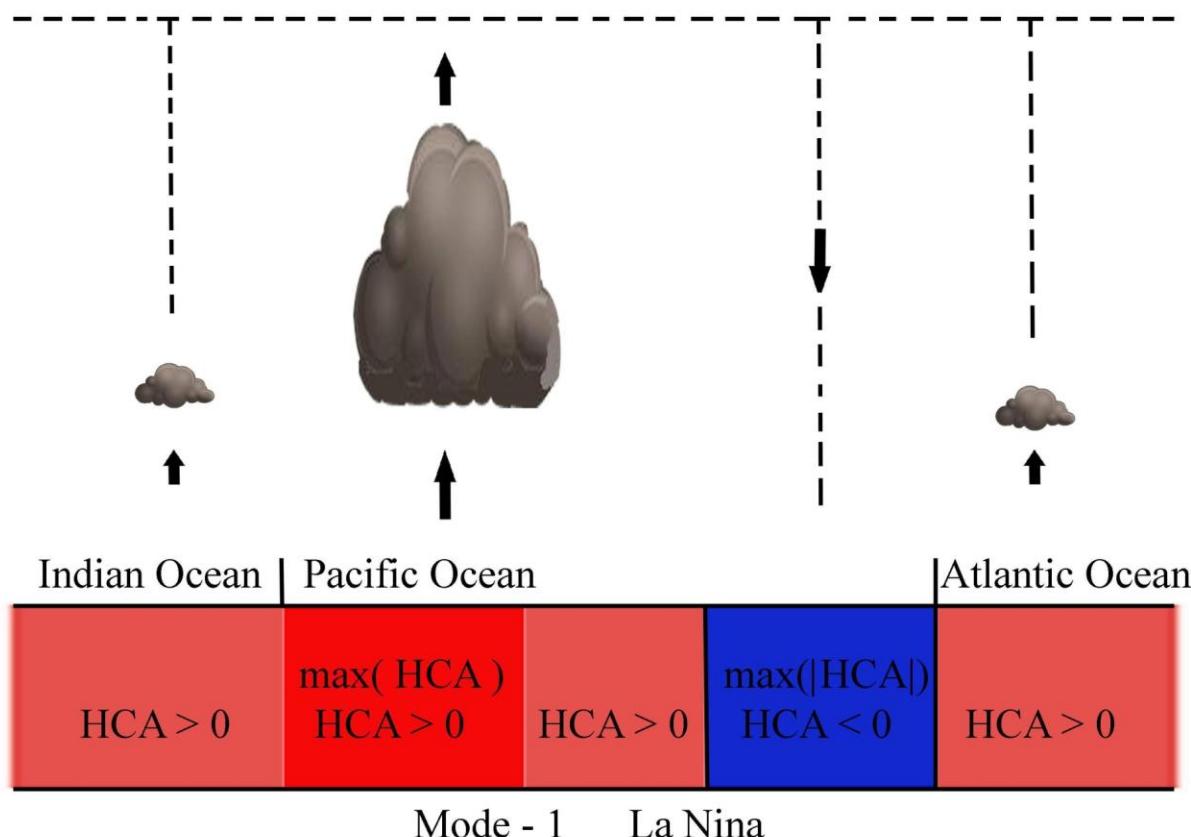
# Lag Correlation between PC<sub>2</sub> and SOI



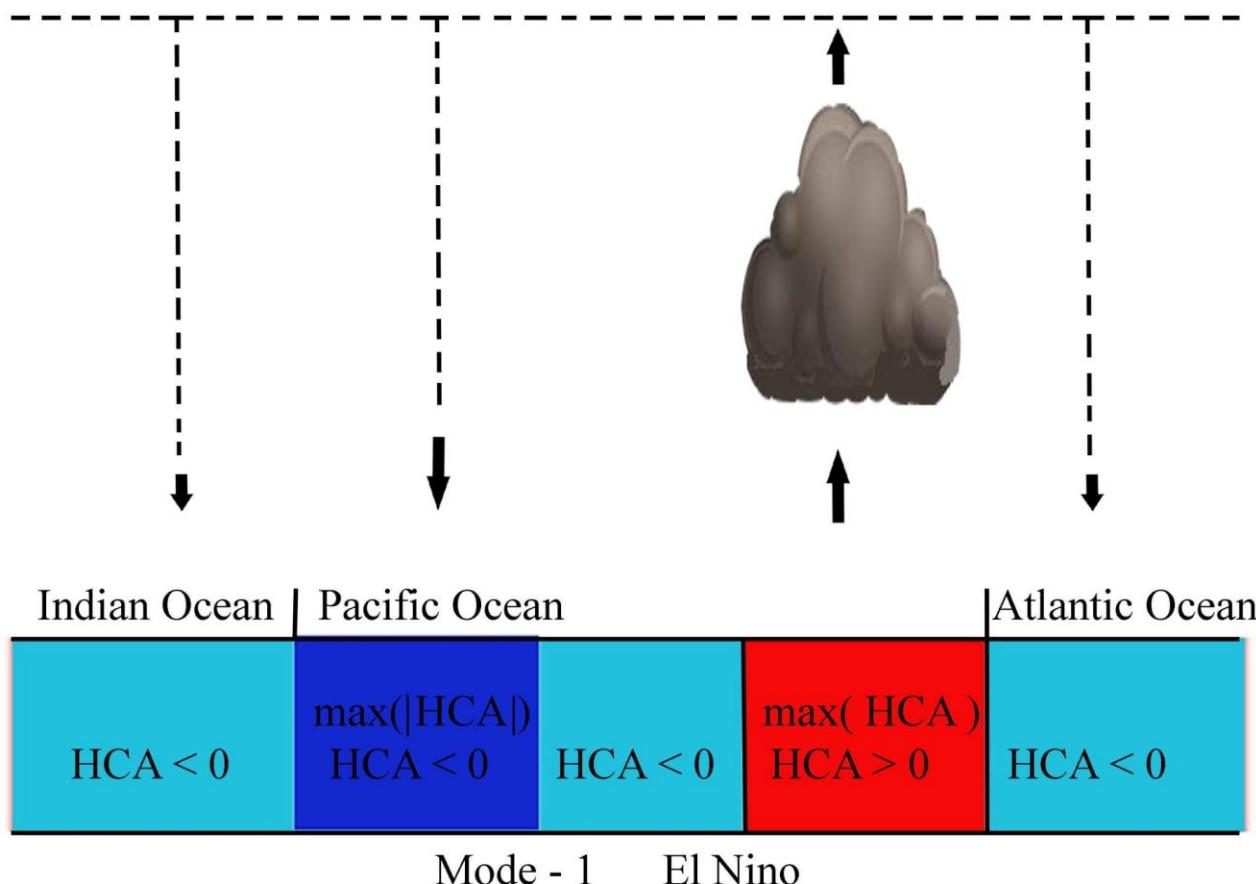
Positive Month → PC<sub>2</sub> advancing SOI

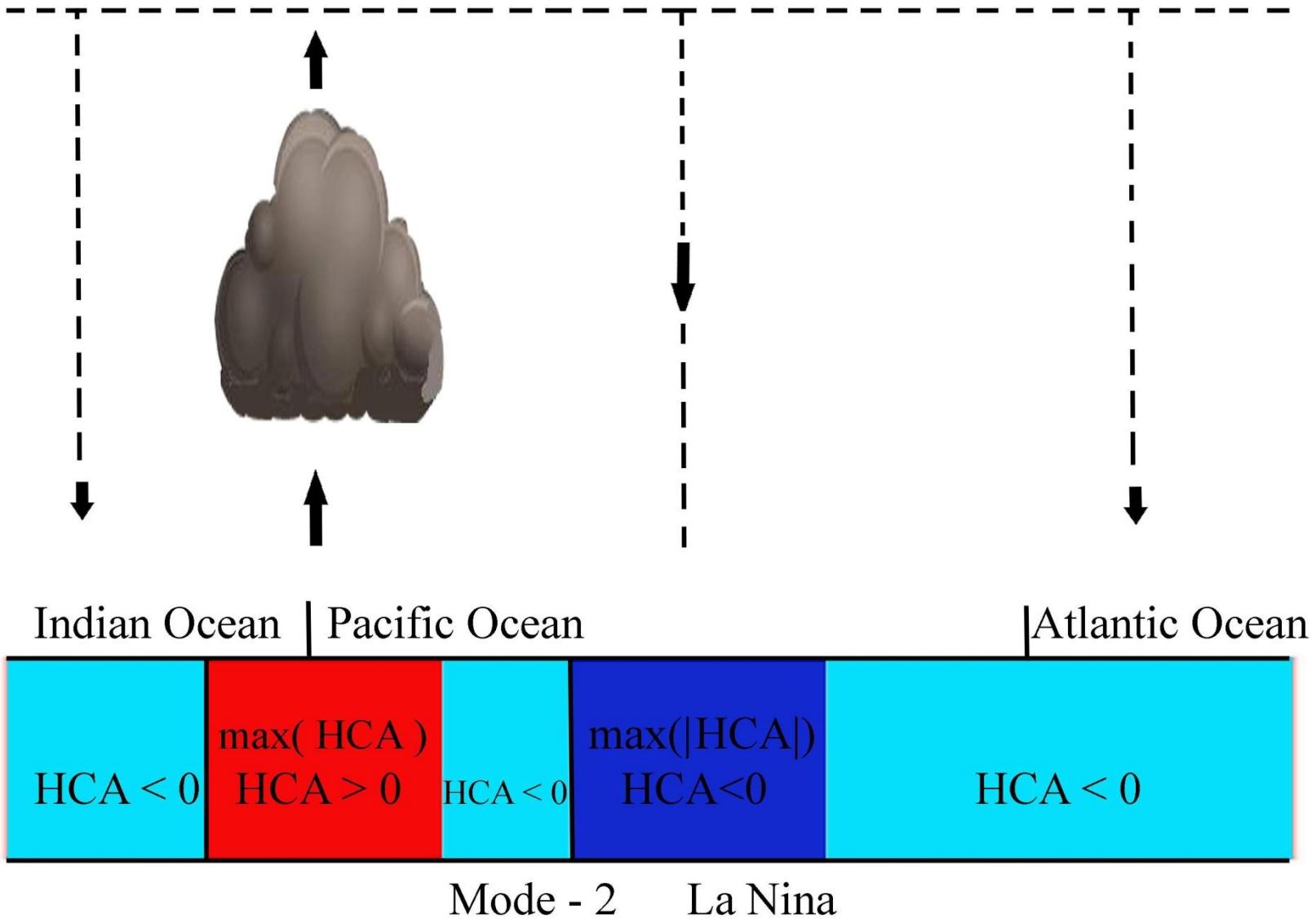
# (4) Global Ocean Tripole

# Canonical La Niña → More and Stronger Hurricanes in Atlantic (Pielke and Landsea, 1999 BAMS)



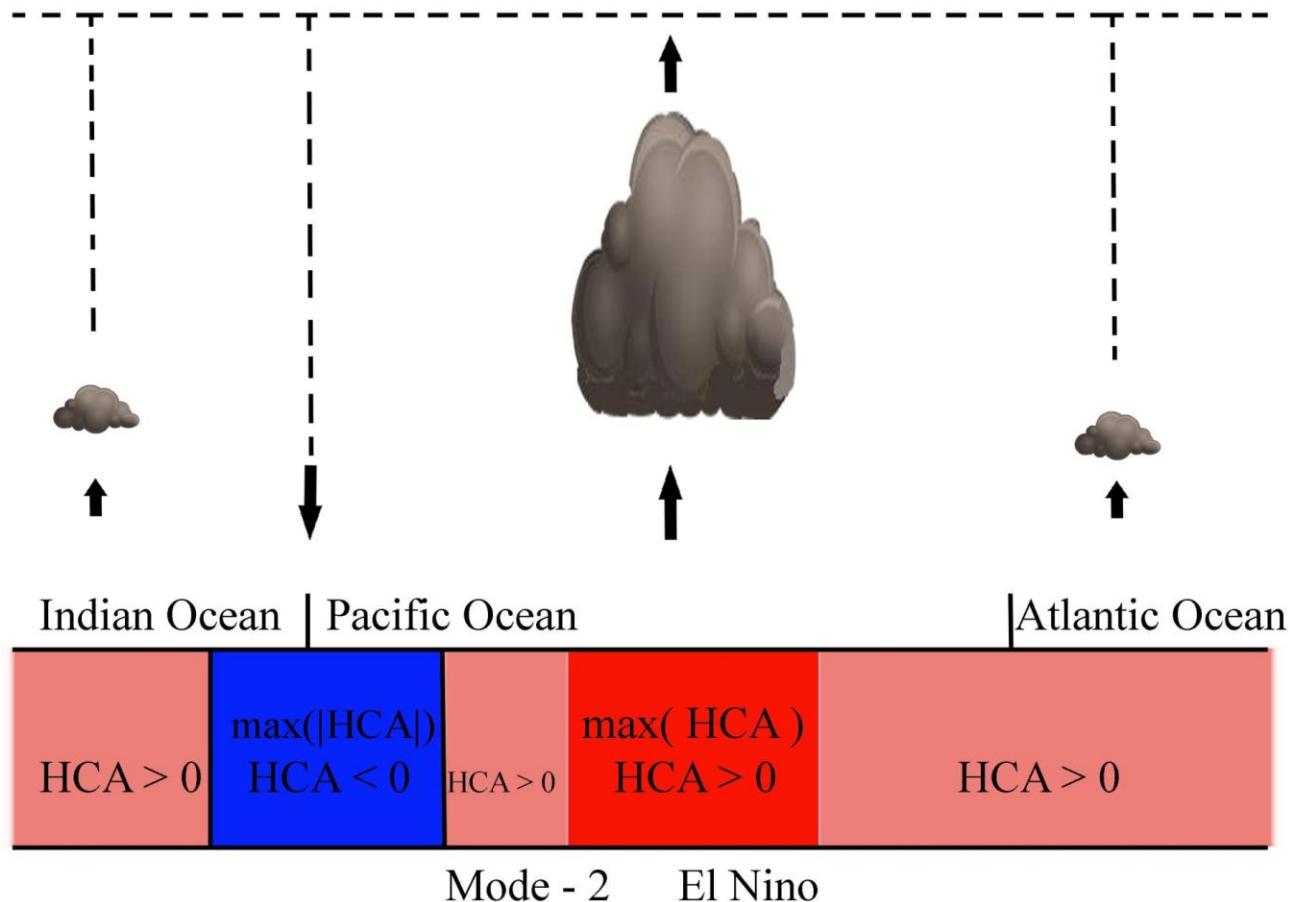
# Canonical El Nino





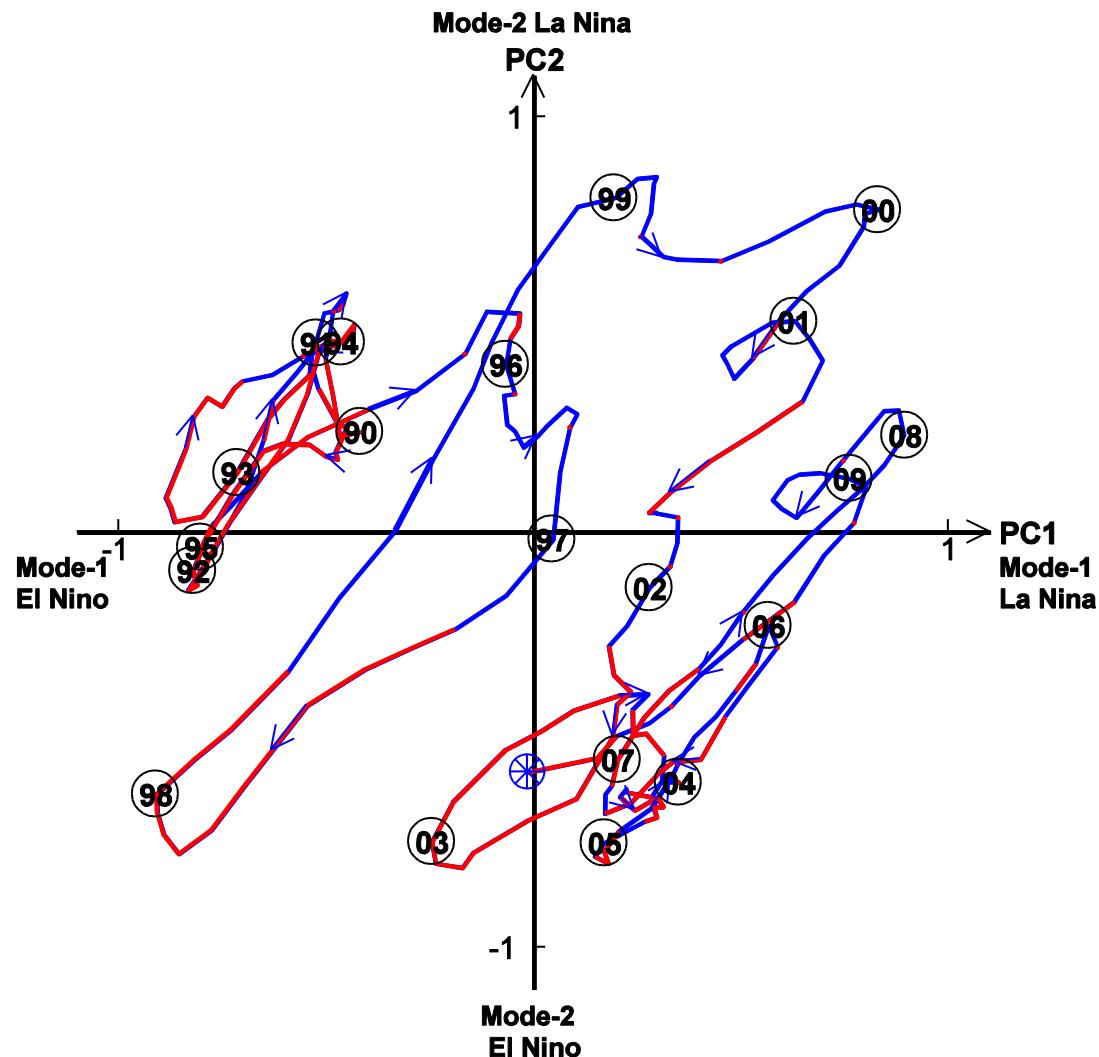
# El Nino Modoki

## More and Stronger Hurricanes in Atlantic



# Trajectory in Phase Space ( $PC_1$ , $PC_2$ )

- Blue Curve → La Nina
- Red Curve → Two Types of El Nino



# Conclusions

- (1) Upper ocean heat content contains the signal for climate change (interannual) → Global Ocean Tripole.
- (2) El Nino, El Nino Modoki, and Indian Ocean Dipole can be unified by Global Ocean Tripole.

# Future Improvement

- Upper ocean heat content **should not be** calculated to a fixed depth such as 300 m in this study.
- Heat content in ocean mixed layer should be most important for the climate change.
- There is no simple, objective and effective method to determine mixed layer depth from profile data.
- My near future work is to develop such a method.