Reconstruction of Shelf Circulation in Northern Gulf of Mexico from Drifter Buoy Data

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1. Introduction

Ocean observational current data are usually acquired from limited number of stations in domains with open boundaries and contain various errors or noises. It is an important task for physical oceanographers to establish (or to reconstruct) a realistic and complete velocity field from sparse and noisy data.

From a mathematical point of view, the reconstruction requires solving a least square problem without or with a priori information (limit) on\ the circulation characteristics. An a priori limit can be formulated as a set of inequalities that the solutions should satisfy, as a dynamical model applied to the description of circulation dynamics or hypotheses on statistical properties of reconstructed field.

Several techniques are available for fulfilling such a task: various kinds of spline interpolation (Washba and Wendelberger,1980; Smith and Wessel, 1990; Brankart and Brasseur, 1996; and others), optimal interpolation (OI, e.g., Gandin, 1965), fitting models (Cho et al., 1998, Lipphardt et al., 1977, 2000 and others), objective mapping combined with a fitting (e.g., Davis, 1985) and numerous approaches using ocean numerical models, such as the adjoint method, Kalman filter, etc. (e.g., Malonette-Rizzoli and Tziperman,1996).

Several error sources deteriorate the reconstruction skill. One of them is the uncertainty in boundary conditions (Bennett, 1992), especially at open boundaries. Therefore, how to determine open boundary conditions becomes a key issue in the reconstruction process.

The classical OI technique does not allow accounting for any boundary condition as an a priori limitation. To overcome this weakness, Davis (1985) suggested to use a combined OI-spectral fitting model with a priori knowledge of the statistical weights. It remains uncertain how to select the weights for an open domain and how to determine basis functions with a priori non-zero flux at the open boundary.

With velocities given along the open boundary and with an additional boundary condition such as the ``natural'' boundary condition (Courant and Hilbert, 1966), the spline functions can be used as universal basis functions. However, a detailed analysis (Inoue, 1986) shows that the natural boundary condition is more appropriate for rigid than open boundaries.

Without knowing statistical weights and without using ocean numerical models, a kinematical method is proposed for reconstructing a velocity field from noisy and sparse data. For a three-dimensional incompressible flow, two scalar functions, toroidal ( ) and poloidal ( ) potentials, satisfy Poisson equations with the vertical vorticity and vertical velocity as the sources terms, respectively (Moffat,1976; Eremeev et al. 1992a,b; Chu, 1999).

In this study, a new set of basis functions is introduced for reconstructing the ocean circulation in a domain with open boundaries. These functions are the eigenfunctions of Laplacian operator with homogeneous mixed (Robin or Newton) conditions. With known velocities along the open boundary, the mixed boundary conditions are accurate. With unknown velocities along the open boundary, a parameterization scheme is proposed for obtaining approximate open boundary conditions from data. In general, the reconstruction is reduced to linear and nonlinear regression models for known and unknown velocities along the open boundary, respectively. For the latter (without data on the open boundary), the velocity inside the domain and along the boundaries are simultaneously determined.

2. Two Scalar Potentials

2.1. Toroidal and Poloidal Components

In magnetohydrodynamics and astrophysics, it is common to decompose any vector **Q** in arbitrary coordinate system into three parts (Dubrovin et al., 1992). On example, it is in spherical coordinate system written as

$$\mathbf{Q} = \mathbf{r} \quad A_1 + \mathbf{r}\, A_2 + \quad A_3, \qquad (1)$$

where A_1, A_2 and A_3 are scalar functions, **r** is the radius vector from the origin. Borrowing this idea for ocean currents (**Q** =**u**} satisfying the incompressible property,

$$\mathbf{u} = 0 \qquad (2)$$

the three dimensional velocity field at large-, meso- and submeso-scales is represented by

$$u = \quad (\mathbf{r} \quad) + \quad (\mathbf{r} \quad) \qquad (3)$$

where the two terms in the right hand side of (3) are called toroidal and poloidal velocities.

If the velocity is reconstructed on horizontal planes, the radius vector **r** can be replaced by the unit vector in the vertical direction **k** (Moffat, 1978). Thus, the velocity **u** (u,v,w), determined on any horizontal plane, is represented by (Eremeev et al., 1992 a, b)

u= / y + ^{2} / x z,

v= - / x + ^{2} / y z,

w= - (4)

where the Cartesian coordinate system is used with (x, y) and z as the horizontal and vertical coordinates, respectively.

Obviously, both toroidal and poloidal potentials satisfy the Poisson equations

= - , = -w (5)

Here,

= ^{2}/ x^{2} + ^{2}/ y^{2}

is the two-dimensional Laplacian operator, and is the vertical component of vorticity.

In general, the toroidal ( ) and poloidal ( ) potentials are not the same as the geostrophic stream function and velocity potential commonly used in meteorology and oceanography (e.g., Lynch 1988). If the Coriolis parameter varies considerably within the domain, the poloidal potential satisfies Poisson equation with a source term determined by the horizontal velocity and the gradient of the Coriolis parameter even in the pure geostrophic flow. That can be checked out through the direct substitution of (4) into the geostrophic equations.

3. Basis Functions

For a simply-connected open domain (Fig. 1), the normal and tangential velocity components (V_n, V) are usually non-zero at the open boundary '. The potentials have no physical significance themselves (Ladyzhenskaya, 1969). They are meaningful only in representing the circulation. To reduce the degree of freedom without loss of any generality, the poloidal kinetic energy is assumed averaged over the domain including the open boundary segment (') to be minimal and obtain

/ z = 0, [/ n + ( )]=0 at '. (6)

The boundary condition for the rigid segment ( ) is represented by

/ n = 0, =0 at . (7)

The basis functions { $_k$}, { $_m$} can be derived from Poisson equations (5) with boundary conditions (6) and (7). They have the following features:

(a) Each of the two sets of basis functions { $_k$} and { $_m$} is orthonormal and complete (Vladimirov, 1971). To calculate directly these basis functions, it requires a priori knowledge of geometry and velocity components at the boundary (i.e., a known boundary condition). For unknown boundary conditions, a nonlinear regression scheme should be developed.

(b) Three reasons make the basis functions defined here more appropriate than trigonometric polynomials (plane geometry) and spherical harmonics (spherical geometry) in flow reconstruction from noisy and sparse data. First, the trigonometric polynomials and spherical harmonics are not the solutions of (5)-(7) for a domain with complex boundaries and/or with varying along the open boundary. That is to say that the trigonometric polynomials and spherical harmonics cannot formulate a complete set of basis functions in this case. Second, the spectral series usually converges quicker using the basis functions determined by (5)-(7) than using trigonometric polynomials and spherical harmonics since the physical information at the boundary is sufficiently used. This leads to fewer modes needed as the basis functions than using the trigonometric polynomials and spherical harmonics.

(c) If normal and tangential velocities along the open boundary change with time, the coefficient also depends on time. The velocity field should be reconstructed at a particular time. This usually does not add any complexity to the reconstruction.

(d) The approach can be extended to a multiply-connected domain through the methodology originally described by Kamenkovich (1961).

(e) The basis functions { $_k$}, { $_m$} have no physical significance themselves. They are meaningful only in representing the circulation (Ladyzhenskaya, 1969; Lynch, 1988).

4. Five-Step Scheme

A five-step scheme is developed to reconstruct velocity from sparse and noisy data in an open domain: (a) a boundary extension method to determine normal and tangential velocities at an open boundary, (b) establishment of homogeneous open boundary conditions for the two potentials with a spatially-varying coefficient, ( ), (c) spectral expansion of ( ), (d) determination of basis functions for the two potentials for the spectral decomposition using homogeneous boundary conditions, and (e) determination of coefficients in the spectral decomposition of velocity and ( ) using linear or nonlinear regressions. Among them, the first four steps are new.

5. Texas-Louisiana Shelf Circulation Reconstructed from Lagrangian Drifter Data

Two types (Eulerian and Lagrangian) are available in ocean velocity measurements. The

Eulerian-type is to measure the current at a certain location (e.g., current meter, and ADCP). The Lagragian-type is to measure the current through trajectories of drifting buoys. The Lagrangain drifting buoys provide near real-time current information of currents with revealing detailed eddy structures (Davis, 1991, Davis, 1998, Lie et al., 1998; Gartfield et al., 1999 and others) and became popular recently. For example, more than 50 drifters were deployed during Louisiana - Texas Shelf Experiment (LATEX) sponsored by MMS.

From theoretical point of view, the Lagrangian and Eulerian frames should be equivalent in describing fluid dynamics (Landau and Lifshits, 1989), however, they are different in practical. The Lagrangain (drifting buoy) data are more complicated than the Eulerian data. Drifter trajectories show a wide spectrum of oceanic motion including meso and sub-mesoscale eddies, waves, inertial and semidiurnal currents (Thomson et al., 1998, Garfield et al., 1999 and others).

Currently, most ocean hindcast/forecast systems have capability to assimilate observational data in Eulerian frame such as satellite (e.g., sea surface temperature, sea surface height) and hydrographic (e.g., XBT, CTD, …) and occasionally Eulerian velocity data (Malanotte-Rizzoli and Tziperman, 1996). But, no system has capability to assimilate observational data in Lagrangian frame, especially the velocity data. This is caused by the complexity of Lagrangian data. Since the drifter trajectories show a wide spectrum of oceanic motion including meso and sub-mesoscale eddies, waves, inertial and semidiurnal currents, the inherent variability of the ocean current structure can be better represented and forecasted if Lagrangian trajectories of drifting buoys are assimilated into the model.

Thus, two items are crucial for hindcast/forecast of the Gulf of Mexico (GOM) deepwater and shelf circulations: (1) effectively utilizing Lagrangian and Eulerian velocity data in order to make first order estimates of oil spill trajectories, and (2) optimally assimilating Lagrangian and Eulerian velocity data into an ocean numerical model for hindcast/forecast of oil spill trajectories. To do so, we need to transform Lagragian into Eulerian data.

Recently, the principal investigator and his colleagues at NPS developed a new scheme on the base of two-scalar (toroidal and poroidal) flow representation to reconstruct velocity field from sparse and noisy data (Chu et al, 2001a,b). This scheme has a capability to process the drifting buoy data and to transform them into Eulerian frame. For example, we successfully reconstructed the Louisana shelf circulation from LATEX drifting buoy trajectories (Fig. 1). In this proposal, we plan to develop GOM reconstruct and assimilation system for effective utilization of both Lagrangain and Eulerian data.

In Figure 2, the dots denote the location of drifters; the light arrows represent the surface winds. The time bar (indicating different date) is located at the upleft corner of each panel. The reconstructed shelf circulation is closely related to the surface winds. For example, strong northeast winds (three right panels) drive strong southwestward currents (maximum 80 cm/s) along the coast; weak winds (two left bottom panels) cause weak currents.

6. Conclusions

First, a five-step scheme is developed to reconstruct velocity from sparse and noisy data in an open domain.

Second, the homogeneous boundary conditions of (,) at both rigid and open boundary segments make it possible to obtain basis functions for an open domain. The basis functions are the eigenfunctions of the Laplacian operator with homogeneous boundary conditions and depends on the spectrally-varying parameter, , at the open boundary.

Third, the spectra of the two horizontal velocity components and are truncated. The optimal mode truncation is determined through a modified cost function, which is constructed \ on the basis of model capability and data reproduction complexity (penalty). This cost function is also used to verify the model reconstruction skill from sparse and noisy data.

Fourth, the spectral coefficients for horizontal velocity and are determined simultaneously using the stabilized least square (SLS) method. This method does not require a priori knowledge about noise and is robust to the size of observational samples used for the reconstruction.

Fifth, after reconstructing the horizontal velocity field at various depths, the vertical velocity may be reconstructed through solving the integral equation numerically. Since the coefficient matrix is square, the minimum sensitivity of solution is used to determine the regularization parameter and then use Tikhonov's approach to reconstruct the vertical velocity.

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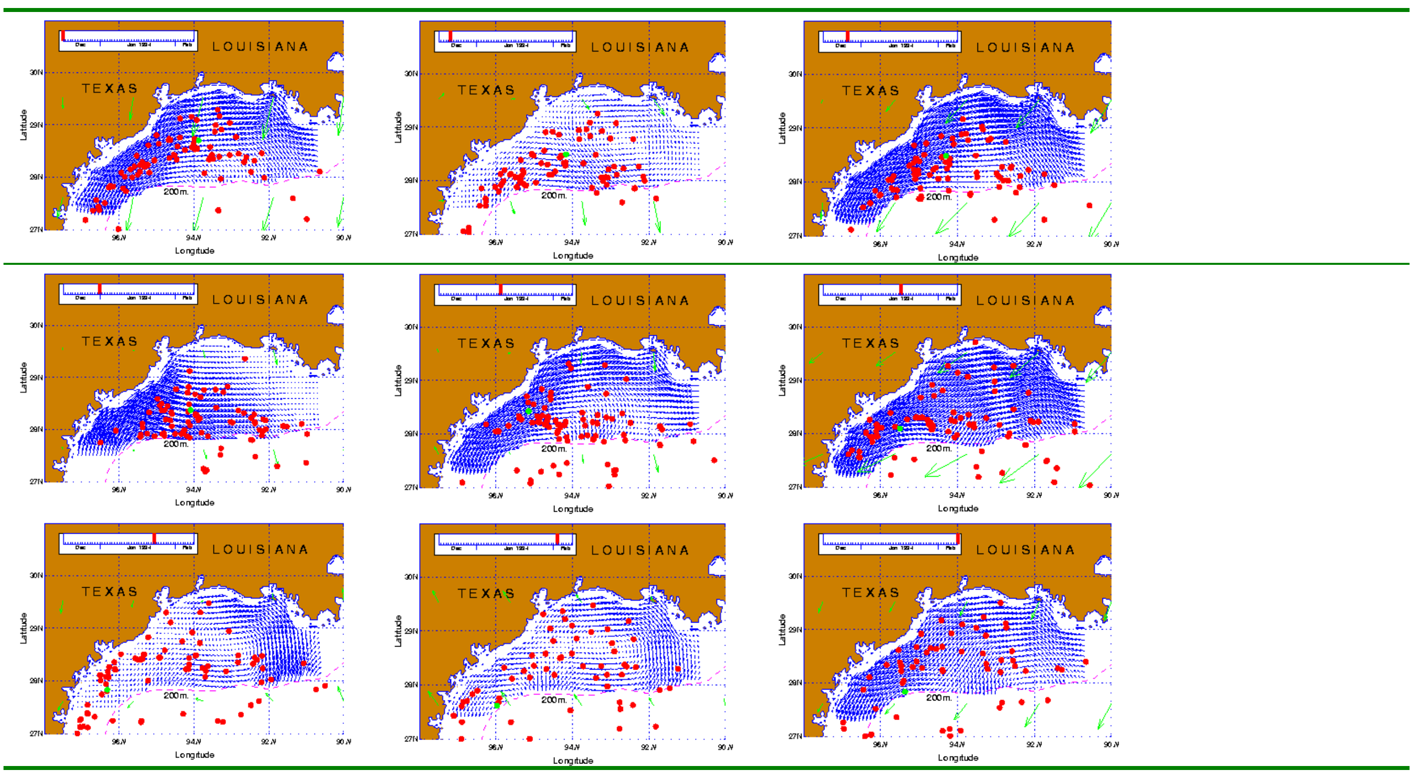

Fig. 2. Temporal variation of Texas-Louisiana shelf surface currents (Dec 15, 1993 – March 15, 1994) reconstructed from LATEX SCULP-1 (surface floats) data using two-scalar potential method. The dots denote the position of drifters and light arrows indicate the surface wind vectors.