WIND EFFECT ON FLEXURAL-GRAVITY WAVES

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1. INTRODUCTION

The first navigators to encounter the Arctic and Antarctic ice margins rapidly became aware the penetration of ocean waves into ice-fields. These waves are termed as flexural-gravity waves (or coupled ice-ocean waves) whose dispersion and energy decay are determined by the character of the ice as well as the water beneath. The ultimate energy source for the flexural-gravity wave is the atmospheric pressure fluctuations generated by winds. However, this wind effect has not been included into the flexural-gravity wave theories (e.g., Squire, 1984; Wadhams et al, 1986; Liu and Mollo-Christensen, 1988). The main purpose of this paper is to show the importance of wind forcing on the flexural-gravity waves. Similar to ice compressive stress, the major wind effect on the flexural-gravity wave is to reduce the group velocity. In the area of low compress stress, the wave energy accumulation occurs in the area of high wind forcing.

2. DYNAMICS OF FLEXURAL-GRAVITY WAVE

The flexural-gravity wave has been studied by several investigators: Wadhams (1973), Mollo-Christensen (1983), Squire (1984), and Liu and Mollo-Christensen (1988). The vertical deflection of the ice-water interface η is taken as

$$\eta(x, t) = a \sin(kx - \sigma t) \tag{1}$$

All assumptions concerning water and ice are kept the same as in Wadhams (1973), the water is incompressible, and the water flow is irrotational. The velocity potential ϕ within the water obeys the continuity equation:

$$\nabla^2 \phi = 0 \tag{2}$$

A possible form of the solution is

$$\phi(x, z, t) = b\, e^{kz} \cos(kx - \sigma t). \tag{3}$$

Considering the ice to be a thin ($kh \ll 1$) elastic plate of thickness h, the linearized dynamical relationship among the deflection, the water pressure immediately below the ice, and the atmospheric pressure fluctuation caused by wind is:

$$\left[\frac{Eh^3}{12(1-\nu^2)}\frac{\partial^4}{\partial x^4} + \rho_i h \frac{\partial^2}{\partial t^2} + P h \frac{\partial^2}{\partial x^2}\right]\eta = -\rho_w\left(\frac{\partial \phi}{\partial t} + g\eta\right) - p_a \tag{4}$$

where p_a is the atmospheric pressure fluctuations caused by winds.

The notation used is the following:

- g the acceleration of gravity;
- E Young's modulus of elasticity; for ice, $E = 6 \times 10^9 N/m^2$;
- ν Poisson's ratio: for ice, $\nu = 0.3$;
- P compressive stress in the ice pack;
- ρ_w density of sea water ($1025\ kg/m^3$);
- ρ_i density of sea ice ($= 0.9\rho_w$).

3. WIND-INDUCED ATMOSPHERIC PRESSURE FLUCTUATIONS

As the mean winds $U(z)$ blow over the wavy water surface (Fig.1), Lighthill (1962) indicates that the low (or high) pressure is associated with the wave crest (or trough), as shown in Fig.2.

The atmospheric pressure perturbation is computed by (Lighthill, 1962)

$$p_a = -\rho_a a k\left[\int_0^{\infty} V^2(z) e^{-kz} d(kz)\right] \sin(kx - \sigma t) \tag{5}$$

where a is the wave amplitude, and

$$V(z) \equiv U(z) - U(z_c) \tag{6}$$

z_c is the height where the mean wind equals the phase speed. Assuming that equation (5) is also valid for the atmospheric pressure perturbation over flexural-gravity waves, the last term in righthand-side of (4) is computed by

$$p_a = -\rho_w W \eta \qquad (7)$$

where

$$W \equiv \frac{\rho_a}{\rho_w}\left[\int_0^{\infty} V^2(z) e^{-kz} d(kz)\right] \qquad (8)$$

is the wind forcing on the flexural-gravity waves.

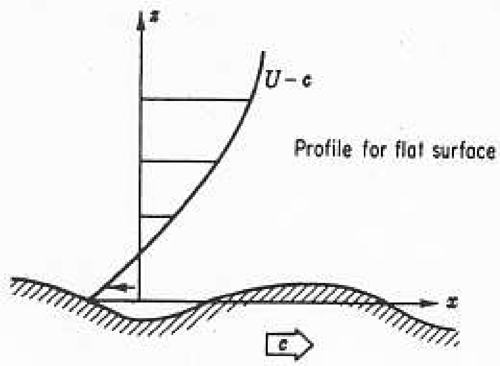

Fig.1. Wind blowing over a wavy ice surface.

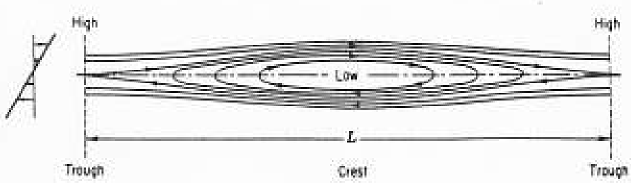

Fig.2. Sheared airflow, pressure perturbation, and ocean surface waves (Lighthill, 1962).

4. DISPERSION RELATION

The kinematic boundary condition at the underside of the ice is

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \qquad (9)$$

to a linear approximation on the plane, $z = 0$. Substituting (1), (3), and (7) into (4) yields the dispersion relation

$$\sigma^2(k) = \frac{Bk^5 - Qk^3 - Wk^2 + gk}{1 + Mk} \qquad (10)$$

The B, Q, W, and M denote the effects that modify the frequency of the flexural-gravity wave due to bending, compression, wind forcing, and inertial of the ice respectively, where B, Q, and M are defined by

$$B = \frac{Eh^3}{12(1-\nu^2)\rho_w},\ Q = \frac{Ph}{\rho_w},\ M = \frac{\rho_i h}{\rho_w}, \qquad (11)$$

The dispersion relation (10) indicates that wind forcing has the similar effect as compressive stress on the flexural-gravity waves. Ice structural failure can be a result of the action of combined wind forcing, stresses due to waves, and structural instability. It is apparent that the frequency can become imaginary for sufficient high wind forcing, or high compressive stresses. The critical compressive stress for the instability known as buckling is

$$P_B = \frac{\rho_w}{hk^2}\left[\frac{Eh^3k^4}{12(1-\nu^2)\rho_w} + g - Wk\right] \qquad (12)$$

which indicates that the wind forcing will reduce the critical compressive stress for the structural instability.

5. GROUP VELOCITY

Liu and Mollo-Christensen (1988) suggest that the high wave amplitudes observed by the R/V Polarstern in the 1986 winter cruise to the Weddell Sea, are caused by the wave energy accumulation due to decrease of group velocity brought on by compressive stress. Inclusion of the wind effect will further enhance the energy accumulation processes and lower the critical value of the compressive stress for zero group velocity. From (10), the group velocity C_g is found to be

$$C_g = \frac{\partial \sigma}{\partial k}$$

$$= \frac{[g - (2 + Mk)Wk - (3 + 2Mk)Qk^2 + (5 + 4Mk)Bk^4]}{[2\sigma(1 + Mk)^2]} \qquad (13)$$

Note that the group velocity takes the value of zero for a compressive stress P in excess of a critical value P_{crit} given by

$$P_{crit} = \frac{\rho_w[g - (2 + Mk)Wk + (5 + 4Mk)Bk^4]}{[h(3 + 2Mk)k^2]} \quad (14)$$

which shows the wind effect on this critical value. Fig.3 indicates the wavelength dependency of P_{crit} for different values of W: (a) $W = 0$, (b) $W = 50m^2/s^2$, (c) $W = 100m^2/s^2$, (d) $W = 150m^2/s^2$, and (e) $W = 250m^2/s^2$. The dashed line indicates the minimum value of P_B ($6.7N/m^2$) for $W = 0$. When $P_{crit} > P_B$, the buckling failure dominates; when $P_{crit} < P_B$, the wave energy accumulation effect (due to reduction of group velocity) dominates. For a given wavelength, the higher the value of W, the smaller the P_{crit}, indicating the wave energy accumulation effect more important for strong wind effect. Furthermore, for a given value of W, the longer the wavelength, the larger the P_{crit}, indicating the buckling failure more important for larger wavelengths.

The dependence of group velocity on wavelength and wind forcing is shown in Fig.4 (for $P = 0$), Fig.5 (for $P = 10^6N/m^2$), and Fig.6 (for $P = 3 \times 10^6N/m^2$). Without any compressive stress (Fig.4), the wind forcing reduces the group velocity for all the wavelengths; e.g., for $\lambda = 250\ m$, the group velocity decreases from 16.5 m/s for $W = 0$ (no wind effect) to 7 m/s for $W = 250\ m^2/s^2$ (strong wind effect). With a given compressive stress (Fig.5 & 6), the wind forcing also reduces the group velocity. This indicates that similar to compress stress, the wind effect also contributes to the local occurrence of large amplitude waves in the pack ice.

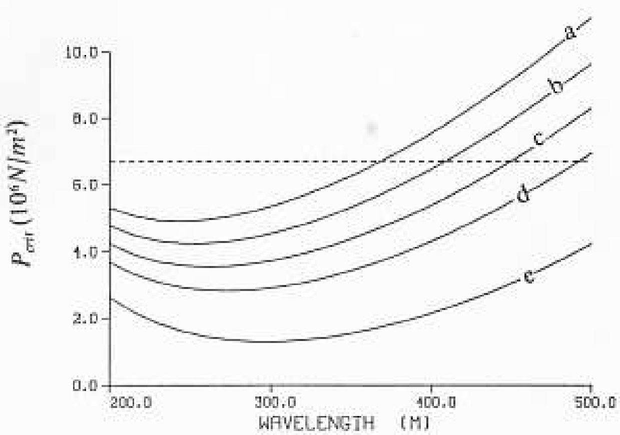

Fig.3. Critical compressive stress of zero group velocity (P_{crit}) for different wind forcing: (a) $W = 0$, (b) $W = 50m^2/s^2$, (c) $W = 100m^2/s^2$, (d) $W = 150m^2/s^2$, and (e) $W = 250m^2/s^2$.

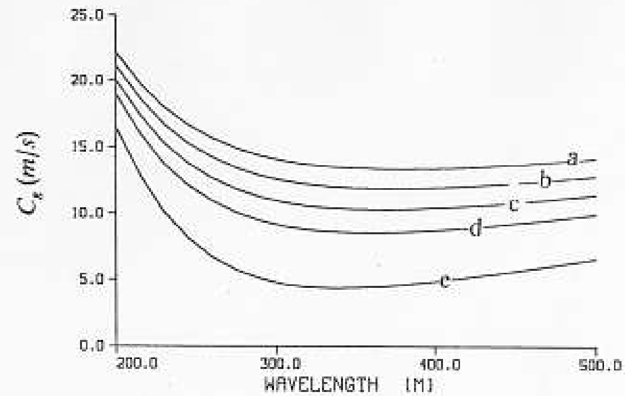

Fig.4. Dependence of group velocity on wavelength at zero compressive stress ($P = 0$) and at different wind forcing: (a) $W = 0$, (b) $W = 50m^2/s^2$, (c) $W = 100m^2/s^2$, (d) $W = 150m^2/s^2$, and (e) $W = 250m^2/s^2$.

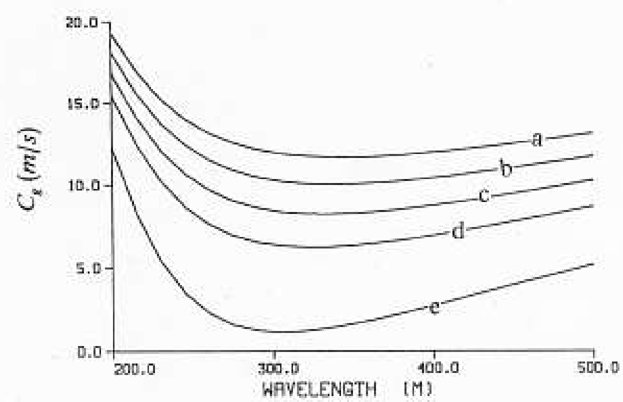

Fig.5. Dependence of group velocity on wavelength at a modest compressive stress ($P = 10^6N/m^2$) and at different wind forcing: (a) $W = 0$, (b) $W = 50m^2/s^2$, (c) $W = 100m^2/s^2$, (d) $W = 150m^2/s^2$, and (e) $W = 250m^2/s^2$.

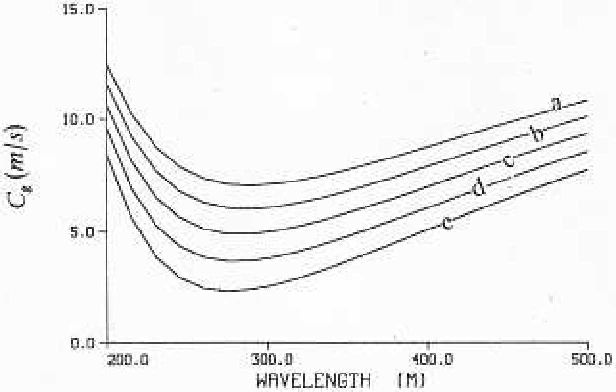

Fig.6. Dependence of group velocity on wavelength at a modest compressive stress ($P = 3 \times 10^6 N/m^2$) and at different wind forcing: (a) $W = 0$, (b) $W = 25m^2/s^2$, (c) $W = 50m^2/s^2$, (d) $W = 75m^2/s^2$, and (e) $W = 100m^2/s^2$.

6. SUMMARY

The wind forcing is an important part for the flexural-gravity waves. It causes the wave energy accumulation by the reduction of the group velocity. For a very small ice compressive stress the wind effect becomes a dominate factor leading to large amplitudes of the flexural-gravity waves

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