Generation of Unstable Modes of the Iceward-attenuating Swell by Ice Breeze

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ABSTRACT

A thermally forced surface wind (ice breeze) is introduced to Wadhams' model for investigating the effects of ice breeze on iceward-attenuating ocean waves. The wave solutions show that the ice breeze generates growing modes which have the same phase speed as Wadhams' model. The growth rate increases with augmentation of the mean surface temperature gradient and decreases with augmentation of ice thickness. In the frequency spectra, the growth rate has a single peak near 0.1 Hz. It is found that ice thickness affects the location of this peak in the frequency spectra. The thicker the ice, the lower the frequency where the peak is situated. The similarity of the frequency spectra between the growth rate and observed energy density implies that ice breeze may play some role in iceward-propagating waves.

1. Introduction

Observational studies on ocean waves propagating into sea ice (Wadhams, 1975, 1978; Wadhams et al., 1986) show that the frequency spectrum of energy density has a major peak near 0.1 Hz. Some theories developed by Wadhams (1973, 1975, 1978) and Wadhams et al. (1986) discussed in detail the iceward-attenuation of ocean waves, but simply considered the energy peak as being of nonlocal origin—having propagated into the MIZ from the open ocean. In Wadhams' model, the effect of wind stress on the ice surface is neglected. In fact, surface winds, generated by a surface temperature gradient over ice and water, are generally seaward. These winds, acting through stress, in turn exert a force on the ice surface. Therefore, local winds should play some role in the ice–ocean wave system. In this paper, a surface wind stress term, brought on by ice breeze, is added to Wadhams' model (1973). The subsequent sections are intended to depict the influences of ice breeze on the ice–ocean wave system, and to explain the formation of the major energy peak in the frequency spectra by the local forcing.

2. Air–ice–ocean coupled model

The coupled system is indicated in Fig. 1. A semi-infinite sheet of ice of constant thickness $2h$ floats in water of depth D. A monochromatic wave with a plane wave front is propagating into the ice at right angles to the ice edge (Wadhams, 1973). Thermally forced surface winds, blowing from the ice over water, exert an air stress on ice which will affect the properties of the wave in the ice field. The coordinate system is chosen such that the x-axis is in the cross-edge direction, and the y-axis is parallel to the ice edge, as shown in Fig. 2. Spatial variations near the ice edge are presumedly much larger perpendicular to the ice edge than parallel to it, and hence derivatives with respect to y are assumed to be zero.

All assumptions concerning ice and water are kept the same as in Wadhams (1973), i.e., there is no plastic strain of ice in the y-direction, the water is incompressible, and the flow is irrotational due to the wave. The profile of flexure of the sheet of ice, and hence the wave, is given by

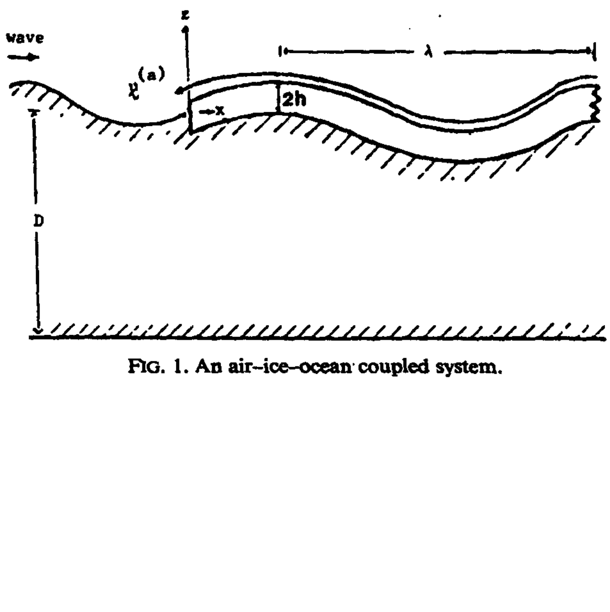

FIG. 1. An air–ice–ocean coupled system.

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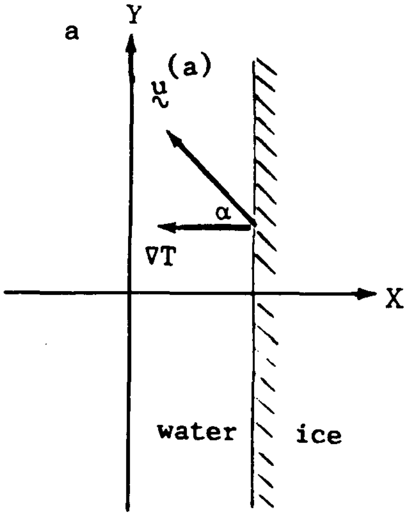

FIG. 2. Thermally forced wind near ice edge. (a) North polar region; (b) South polar region.

$$\xi(x,t) = \mathrm{Re}\{A \exp[i(kx-\omega t)]\}, \quad (1)$$

where A is the amplitude of flexure of the sheet at a penetration distance x and ω is the wave frequency.

The velocity potential ϕ within the water obeys the continuity equation:

$$\nabla^2\phi = 0 \quad (2)$$

with the boundary conditions in the vertical:

$$\partial\phi/\partial z = 0, \qquad z=-D, \quad (3)$$

$$\partial\phi/\partial z = -\partial\xi/\partial t, \quad z=0. \quad (4)$$

Assuming $D \gg h$, a solution to this is

$$\phi(x,z,t) = i(\omega A/k)[\coth k(z+D)/\sinh kD] \times \exp[i(kx-\omega t)]. \quad (5)$$

The equation of motion of the ice is

$$2h\rho_i\partial^2\xi/\partial t^2 = -L\partial^4\xi/\partial x^4 + \delta p - \tau^{(a)}\partial\xi/\partial x, \quad (6)$$

where ρ_i is the ice density, $\tau^{(a)}$ the surface wind stress in the x–z plane,

$$L \equiv 2h^3E/[3(1-\nu^2)]$$

is the flexural rigidity of ice. Here E and ν are Young's modulus and Poisson's ratio for ice, respectively, and δp is the excess of pressure just beneath the ice–water interface over atmospheric pressure. Then by Bernoulli's equation taken to first order

$$\delta p/\rho_w + g\xi - \partial\phi/\partial t = 0, \quad \text{at} \quad z=0, \quad (7)$$

where ρ_w is the water density.

Earlier work (Chu, 1986a,b,c) shows that thermally forced surface winds blow across isotherms from cold ice to warm water with some deflection due to the earth's rotation. The deflection angle α (Fig. 2) depends on the atmospheric stratification (Chu, 1986c), and the wind speed is proportional to the mean surface tem-

TABLE 1. The standard model parameters.

$E = 6\times10^9$ N m^{-2},	$\nu = 0.3$,	$C_a = 3\times10^{-2}$ m s^{-1},
$\rho_w = 10^3$ kg m^{-3},	$\rho_i = 0.9\rho_w$,	$D = 1$ km,
$\alpha = \tan^{-1}(0.6)$,	$b = 2\times10^5$ m^2 s^{-1}/°C.	

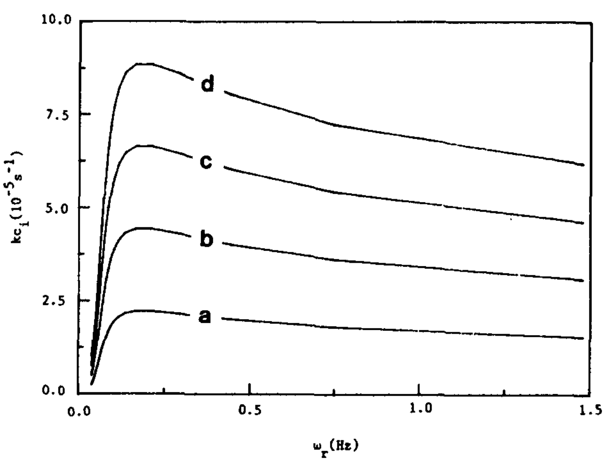

FIG. 3. Growth rate (in 10^{-6} s^{-1}) for $2h$ = 2.5 m: (a) G_T = 5 $\times$ 10^{-5} °C m^{-1}, (b) G_T = 10^{-4} °C m^{-1}, (c) G_T = 1.5 $\times$ 10^{-4} °C m^{-1}, and (d) G_T = 2 $\times$ 10^{-5} °C m^{-1}.

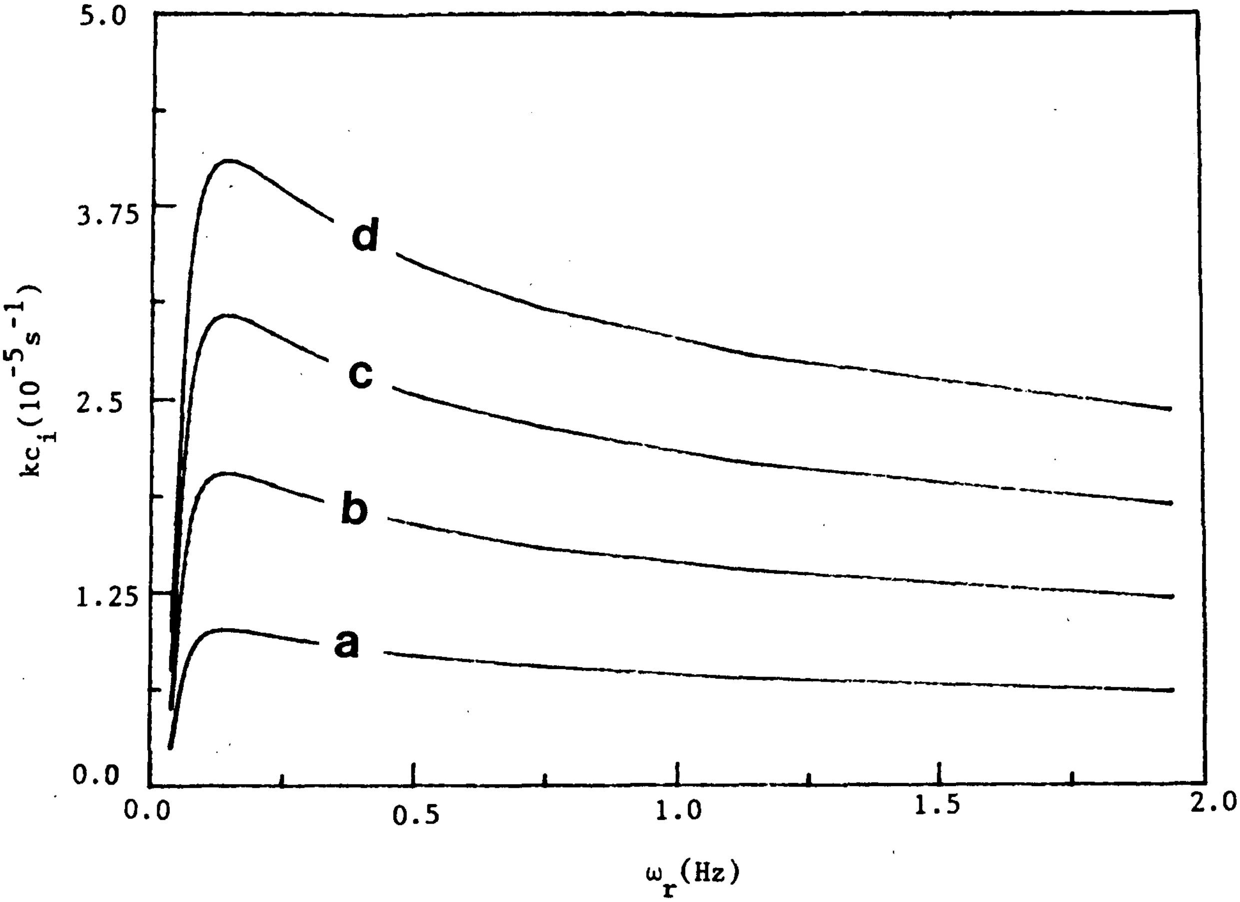

FIG. 4. As in Fig. 3 except for $2h$ = 5 m. Note also that the ordinate has a different scale.

perature gradient G_T. Therefore the surface air speed in the xz-plane is computed by

$$u^{(a)} = bG_T \cos\alpha, \tag{8}$$

where b is a proportionality constant which represents the surface wind thermally forced by a unity of surface temperature gradient. According to Defant (1950), a 1°C/100 km surface temperature gradient can generate a surface wind of about 2 m s^{-1}. Hence we take

$$b = (2\,\mathrm{m\,s^{-1}})/(1\,°\mathrm{C}/100\,\mathrm{km}). \tag{9}$$

The surface wind stress in the x–z plane is obtained by

$$\tau^{(a)} = \rho_a C_a u^{(a)} = \rho_a C_a b G_T \cos\alpha, \tag{10}$$

where C_a is a dimensional (m s^{-1}) air drag coefficient.
Substituting (7) into (6) we have

$$2h\rho_i \partial^2\xi/\partial t^2 = -L\partial^4\xi/\partial x^4 - \rho_w g\xi - \gamma^{(a)}\partial\xi/\partial x + \rho_w \partial\phi/\partial t, \tag{11}$$

where

$$\gamma^{(a)} \equiv \rho_a C_a b G_T \cos\alpha. \tag{12}$$

3. Wave solutions

Substituting (1) and (5) into (11), we obtain the dispersion relation

$$c^2 = \omega^2/k^2 = (Lk^3 + g\rho_w/k + \gamma^{(a)}i)/(\rho_w \coth kD + 2kh\rho_i). \tag{13}$$

If the effect of ice breeze is neglected, $\gamma^{(a)} = 0$, the system is reduced to Wadhams' model. Let

$$c = c_r + ic_i$$

and since only iceward-propagating waves are considered in this paper, $c_r > 0$. From (13) we have

$$c_r = c_0(1 + \{[1 + (\beta_i/\beta_r)^2]^{1/2} - 1\}/2)^{1/2}, \quad c_i = \beta_i/(2c_r), \tag{14}$$

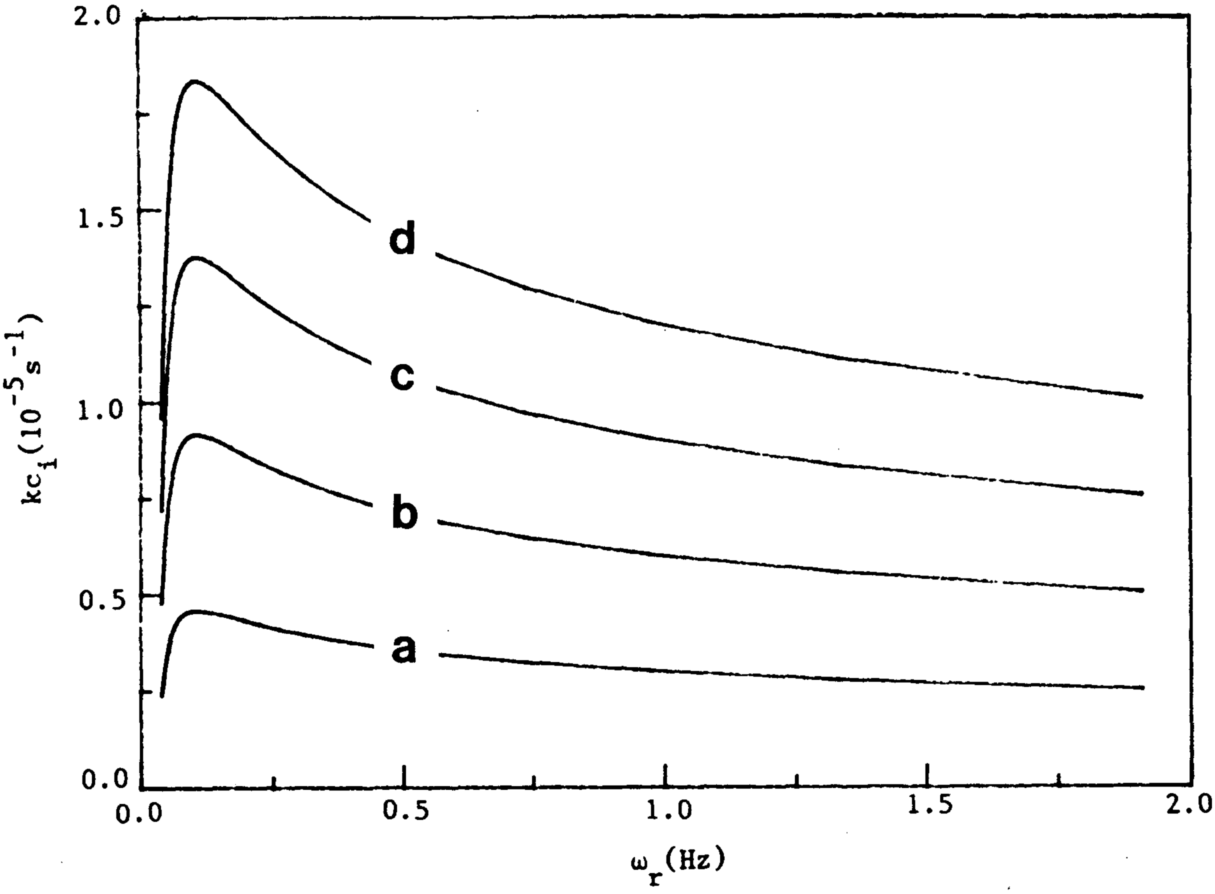

FIG. 5. As in Fig. 3 except for $2h$ = 10 m. Note also that the ordinate has a different scale.

where

$$\beta_r \equiv (Lk^3 + g\rho_w/k)/(\rho_w \coth kD + 2kh\rho_i),$$

$$\beta_i \equiv \gamma^{(a)}/(\rho_w \coth kD + 2kh\rho_i),$$

$$c_0 \equiv \sqrt{\beta_r}. \tag{15}$$

The standard parameter values are given in Table 1. We compute the wave speed c_r and the growth rate kc_i with different values of ice thickness $2h$ (2.5 m, 5 m, and 10 m), mean surface temperature gradient G_T (5°C/100 km, 10°C/100 km, 15°C/100 km, and 20°C/100 km), and wave number k. We found that in all cases the difference between c_r and c_0 is extremely small, which means that the ice breeze doesn't affect the wave speed.

Figs. 3–5 show the growth rate kc_i versus the frequency $\omega_r = kc_r$ with different mean surface temperature gradients and with the different ice thicknesses: 2.5 m (Fig. 3), 5 m (Fig. 4), and 10 m (Fig. 5).

4. Conclusions

This air–ice–ocean model is intended to depict the generation of unstable modes of the iceward-attenuating swell by ice breeze. The model shows the following results (see Figs. 3–5):

1) Ice breeze produces an instability in the ice/ocean wave system. The growth rate increases with augmentation of the mean surface temperature gradient, and decreases with augmentation of the ice thickness. It is quite understandable that the larger the mean surface temperature gradient, the larger the air forcing; and the thicker the ice, the more inertia the air forcing should overcome.

2) The growth rate has a single peak in all cases. The location of the peak in the frequency spectra doesn't depend on the mean surface temperature gradient. It relies on the ice thickness. The thicker the ice, the smaller frequency where the peak is situated. For $2h$ = 2.5 m, the maximum growth rate is located at ω_r

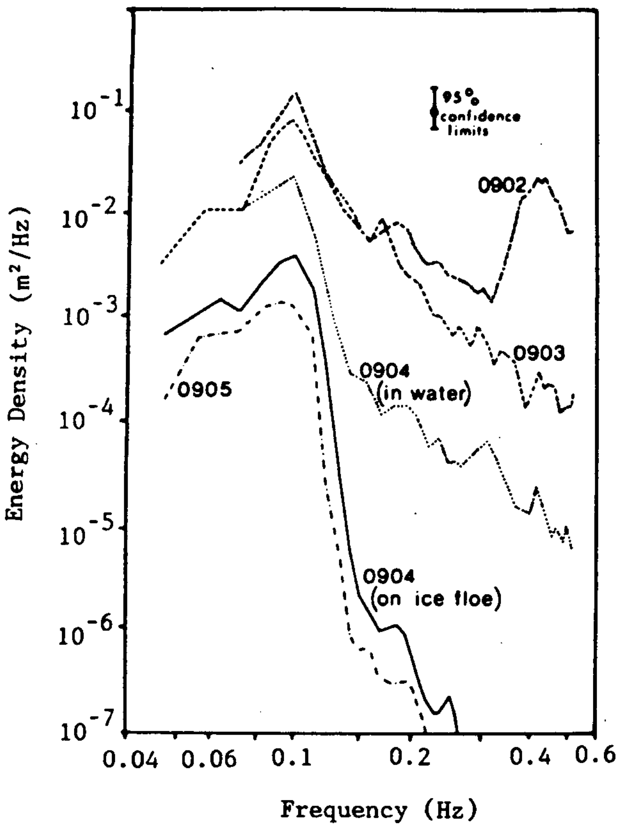

FIG. 6. Observed energy density spectra of ocean wave entering an ice field (from Wadhams et al., 1986).

$= 0.13$ Hz; however, for $2h = 10$ m, it is located at $\omega_r = 0.11$ Hz.

3) Comparing Figs. 3–5 with Fig. 6, which shows the observational results of Wadhams et al. (1986), we find a similarity in the frequency spectra between the growth rate and the energy density. This implies that ice breeze may play some role in the formation of the energy density spectra. We need more observational studies to verify which is important between the remote and local forcings.

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