Pseudocylinder Parametrization For Mine Impact Burial Prediction

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1 Introduction

The United States military has undergone numerous changes since the end of the Cold War. Specifically, the U.S. Navy experienced a shift in the area of engagement from the "blue water" (water depth greater than 100 m) to littoral regions of the world. The sea mines become the big threat in the naval operations. Within the past 15 years three U.S. ships, the USS Samuel B. Roberts (FFG-58), Tripoli (LPH-10), and Princeton (CG-59) have fallen victim to mines. The total ship damage was $125 million while the mines cost approximately $30 ,000 [1].

Let (x,y) be the horizontal coordinates and z the vertical coordinate. A two-dimensional model (called IMPACT28) was developed to predict the mine's movement in the (x,z) cross section [2]. The model contains two momentum equations (in x and z directions) and one moment of the momentum equation (in the y direction), and predicts the mine's center of mass (COM) position in the (x,z) plane and the rotation (i.e., the mine's orientation) around the y axis. Since the mine's movement in IMPACT28 is strictly in the (x,z) plane, it is very hard to include the motion of fluid in the two-dimensional model, because it is impossible to lay a mine in the same direction of the fluid velocity. In the littoral zone, the water velocity is not negligible. The application of the two-dimensional model for the operational use is limited.

Recently, a three-dimensional recursive model (IMPACT35) has been developed to predict the rigid cylinder's (or cylindrical mine's) translation velocity and orientation in fluid involving nonlinear dynamics, fluid–structure interaction, and instability theory [3–5]. However, the Navy operational mines are usually not cylindrical. The existing model should be extended from the cylindrical mines to more general shapes of mines with nose and tail.

2 Mine's Location and Orientation

For an axially symmetric cylinder, the centers of mass (COM) $\mathbf{X}$ and center of volume (COV) $\mathbf{B}$ are on the mine's main axis (Fig. 1). Let (L,R,χ) represent the mine's length, radius, and the distance between the two points $(\mathbf{X},\mathbf{B})$. The positive χ values refer to the nose-down case, i.e., the point $\mathbf{X}$ is lower than the point $\mathbf{B}$. Let $F_{\mathrm{E}}(\mathrm{O},\mathbf{i},\mathbf{j},\mathbf{k})$ be the earth-fixed coordinate (E coordinate) with the origin "O," and three axes: x, y axes (horizontal) with the unit vectors $(\mathbf{i},\mathbf{j})$ and z axis (vertical) with the unit vector $\mathbf{k}$ (upward positive). The position of the cylinder is represented by the position of the COM,

$$\mathbf{X} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \tag{1}$$

which is a translation of the cylinder. The translation velocity is given by

$$\frac{d\mathbf{X}}{dt} = \mathbf{V}, \quad \mathbf{V} = (u,v,w). \tag{2}$$

Let the orientation of the mine's main axis (pointing downward) is given by $\mathbf{i}_{\mathrm{M}}$. The angle between $\mathbf{i}_{\mathrm{M}}$ and $\mathbf{k}$ is denoted by $\psi_2 + \pi/2$. The projection of the vector $\mathbf{i}_{\mathrm{M}}$ onto the (x,y) plane creates an angle (ψ_3) between the projection and the x axis (Fig. 2). The mine rotates around the main axis (i.e., $\mathbf{i}_{\mathrm{M}}$) with an angle of ψ_1. The three angles (ψ_1,ψ_2,ψ_3) determine the mine's orientation.

Three coordinate systems are used to calculate the forces and torques: earth-fixed coordinate (E coordinate), the cylinder's main axis following the coordinate (M coordinate), and the hydrodynamic force following the coordinate (F coordinate) [3]. The origin of both M and F coordinates is at COM. The hydrodynamic forces and torques are easily computed using the F coordinate. The cylinder's moments of gyration are simply represented using the M coordinate.

3 Pseudocylinder Parametrization

For a near-cylindrical mine with a nose and tail falling through a single medium or multiple media, the buoyancy force and torque are relatively easy to calculate. But, the hydrodynamic forces (lift, drag) and torques are difficult to compute. A feasible way is to transform a mine with nose and tail to a cylindrical mine (i.e., called the pseudocylinder parametrization). An axially symmetric mine usually consists of three parts: cylindrical body with radius of R, nose, and tail (Fig. 3). The lengths of the mine, nose, and tail are L, L_n, and L_t. A pseudocylinder is defined with the following features: the same radius (R) of the mine's cylindrical body and the same volume as the original mine (Fig. 4). It consists of three parts: original cylindrical body, and equivalent cylinders for nose and tail. Let (Π,Π_n,Π_t) be the volumes of the mine, nose, and tail. The equivalent cylinder has length

$$L_{ne} = \frac{\Pi_n}{\pi R^2}, \tag{3}$$

for the nose, and

$$L_{te} = \frac{\Pi_t}{\pi R^2}, \tag{4}$$

for the tail. Let (c_c,c_m) be the mine's midpoint on the main axis and the COM position, and let c_{ev} be the COV of the pseudocylindrical mine (Fig. 4). The gravity is downward and passing through c_m. The buoyancy force is upward and passing through c_{ev}. Let ε_1 be the distances between c_c and c_m,

$$\varepsilon_1 = \frac{L_n - L_{ne}}{2} - \frac{L_t - L_{te}}{2}. \tag{5}$$

Let ε_2 be the displacement from c_c to c_m that is easy to determine if COM is given. Let χ be the displacement from c_{ev} to c_m that is calculate

$$\chi = \varepsilon_1 + \varepsilon_2. \tag{6}$$

Both χ and ε_2 can be positive and negative. The positive values refer to the nose-down case, i.e., the point c_m is lower than the point c_{ev} for positive χ and the point c_c is lower than the point c_{ev} for positive ε_2.

4 Impact Burial Prediction

4.1 Two-Dimensional Modeling. Let the mine be moving in the (x,z) cross section. The mine's orientation is represented by the angle (ψ_2) rotating around the y axis. The two-dimensional model (called IMPACT28) consists of two momentum equations [for (x,z)] and one moment of momentum equation (for ψ_2) (see [2])

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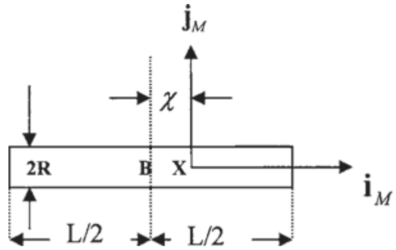


Fig. 1 Axially symmetric cylindrical mine. Here, χ is the distance between the COV (B) and COM (X), and (L,R) are the cylinder's length and radius.

$$\frac{d^2x}{dt^2} \equiv \frac{du}{dt} = \frac{F_h^x}{\rho\Pi}, \tag{7}$$

$$\frac{d^2z}{dt^2} \equiv \frac{dw}{dt} = -g + \frac{F_{nh}^z + F_h^z}{\rho\Pi}, \tag{8}$$

$$J_2\frac{d^2\psi_2}{dt} = F_{nh}^z\chi\cos\psi_2 + M_h^y, \tag{9}$$

where (F_h^x, F_h^z, F_{nh}^z) are the components of hydrodynamic and nonhydrodynamic forces; J_2 is the moment of inertia and M_h^y is the hydrodynamic torque in the y direction; and g is the gravitational acceleration. Since the mine's movement is strictly in the (x,z) plane, it is very hard to include the motion of fluid in the two-dimensional model, because it is impossible to lay a mine in the same direction of the fluid velocity. In the littoral zone, the water velocity is not negligible. The application of the two-dimensional model for the operational use is limited.

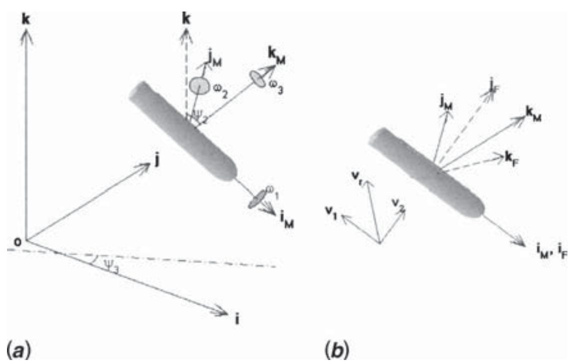


Fig. 2 Three coordinate systems

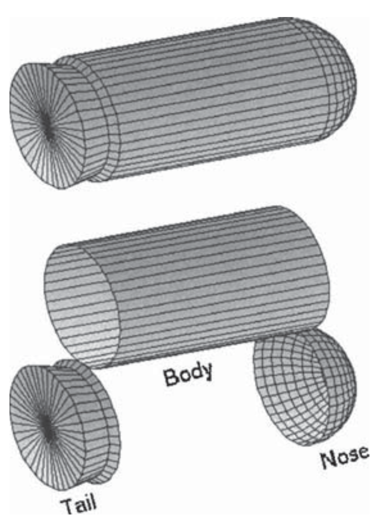


Fig. 3 Mine with nose, tail, and cylindrical body

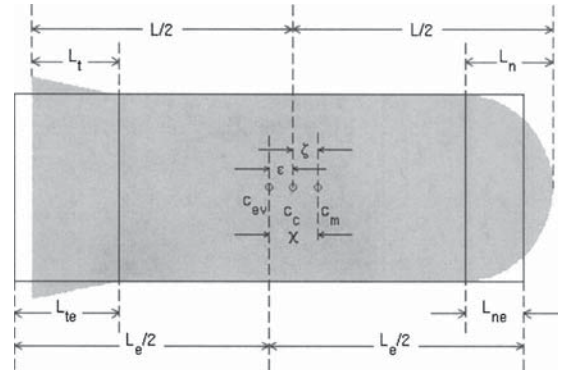


Fig. 4 Location of c_v, c_{ev}, and c_m. Here, ε is the distance between c_v and c_m; χ is the distance between c_{ev} and c_m.

4.2 Three-Dimensional Modeling. Three-dimensional model (called IMPACT35) consists of the three momentum equations for the mine's COM position

$$\frac{d^2}{dt^2}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \frac{\mathbf{F}_{nh} + \mathbf{F}_h}{\rho\Pi}, \tag{10}$$

which is written in the E-coordinate system. Here, Π is the mine's volume; ρ is the mine's density; $\rho\Pi = m$ is the cylinder mass; $\mathbf{F}_{nh}$ and $\mathbf{F}_h$ are integrated (over the whole volume) nonhydrodynamic and hydrodynamic forces. The moment of momentum equation for the mine's orientation (ψ_1, ψ_2, ψ_3) in the M-coordinate system is written by (see [3,5])

$$\mathbf{J} \dot{\rightarrow} \frac{d^2}{dt^2}\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \mathbf{M}_{nh} + \mathbf{M}_h, \tag{11}$$

where $\mathbf{J}$ is the moment of gyration tensor; and $\mathbf{M}_{nh}$ and $\mathbf{M}_h$ are the nonhydrodynamic and hydrodynamic torques. The hydrodynamic (drag and lift) forces and torques are computed using the F coordinate [3]. The sediment part is the same as that depicted in [5].

5 Mine Drop Experiments

Data from two mine drop experiments are used to verify the value added of the three-dimensional model. Exp-1 was designed to collect data on the mine's motion in the water column for various combinations of the mine's parameters. Exp-2 was designed to collect synchronized data on sediment parameters (shear strength and density) and the mine's burial depth and orientation.

5.1 Exp-1. Exp-1 was conducted at the pond (water depth: 7.92 m) of the Naval Surface Warfare Center Caderock Division, West Bethesda, Maryland in September 2001 using six model mines with a radius of 0.084 m, two lengths (1.01 m, 0.505 m), and an adjustable internal weight to change the mine's COM position (i.e., χ value) [6]. The mine shapes are fabricated from aluminum pipe with a urethane covered aluminum front plate (Fig. 5).

The controlled parameters for each drop were the L/R ratio, χ value, initial velocity ($\mathbf{V}_{\text{in}}$), and drop angle. The E-coordinate system is chosen with the origin at the corner of the swimming pool with the two sides as x and y axes and the vertical z axis. The initial injection of cylinders was in the (x,z) plane. The blunt nosed mines are released into the water from three orientations

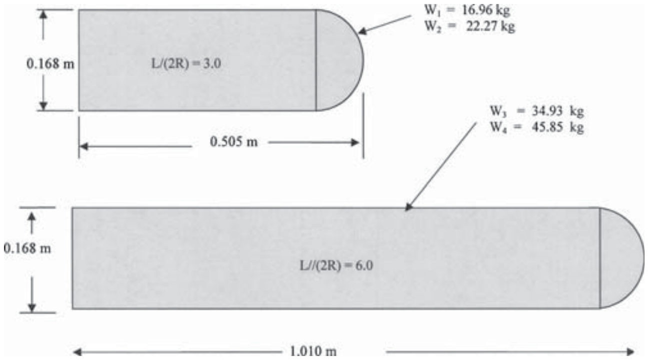


Fig. 5 Six model mines used in Carderock test pond with two different lengths

(horizontal, vertical, and 45° nose down) with a total of 42 drops. The observational data are $x(t)$, $z(t)$, and $\psi_2(t)$ in the water column. For detailed information, please contact Dr. Philip Valent at the Naval Research Laboratory Stennis Space Center (pvalent@nrlssc.navy.mil).

5.2 Exp-2. Exp-2 was conducted on the R/V John Martin on 23 May 2000 [7]. The purpose of this experiment is to collect mine burial, sediment density, and shear strength data simultaneously. The barrel with a density ratio of 1.8 was released horizontally while touching the surface with near zero velocity. The barrel was to be released 17 times. The diver would snap the quick-release shackle on the barrel and then dive down to measure the burial depth. The average depth of the water was 13 m. Before each drop, the gravity core is collected. The number of total mine drops (gravity cores) is 17. An analysis of the gravity cores was conducted at the USGS Laboratories in Menlo Park, California.

Table 1 Physical parameters of the six mines used in Exp-1

Mine	Mass (kg)	ρ (10^3 kg m^{-3})	L (m)	J_1 (kg m^2)	J_2 (J_3) (kg m^2)	χ (m)
1	16.96	1.60	0.505	0.0647	0.356	0
2	22.27	2.10	0.505	0.0806	0.477	0
3	34.93	1.60	1.010	0.1362	2.900	0
4	45.85	2.10	1.010	0.1696	3.820	0
5	45.85	2.10	1.010	0.1693	3.940	0.0045
6	45.85	2.10	1.010	0.1692	4.570	-0.077

6 Value-Added of 3D Model

6.1 Trajectory in Water Column. Improvement from IMPACT28 to IMPACT35 in predicting the cylinders' trajectory and orientation in the water column is verified using the Exp-1 data. The physical parameters of the six mines are presented in Table 1. Here, we only list three cases for illustration.

6.1.1 Near Horizontal Release. Model mine #6 is released to the water with $\psi_2=-14°$ (near horizontal, see Fig. 2). The initial conditions are given by

$$x_0 = y_0 = z_0 = 0, \quad u_0 = v_0 = w_0 = 0,$$
$$\psi_{10} = 0, \quad \psi_{20} = -14°, \quad \psi_{30} = 0, \quad \omega_{10} = \omega_{20} = \omega_{30} = 0. \tag{12}$$

Substitution of the model parameters and the initial conditions (12) into IMPACT28 and IMPACT35 leads to the prediction of the mine's translation and orientation that are compared with the data collected during Exp-1 at time steps (Fig. 6). The new 3D model (IMPACT35) simulated trajectory agrees well with the observed trajectory. Both show the same pattern and the same travel time (1.91 s) for the cylinder passing through the water column. However, the existing 2D model (IMPACT28) has less capability to predict the cylinder's movement in the water column.

6.1.2 Near 45° Release. Model mine #6 is released to the water with $\psi_2=42.2°$ (near 45°, see Fig. 2). The initial conditions are given by

$$x_0 = y_0 = z_0 = 0, \quad u_0 = v_0 = w_0 = 0,$$
$$\psi_{10} = 0, \quad \psi_{20} = 42.2°, \quad \psi_{30} = 0, \quad \omega_{10} = \omega_{20} = \omega_{30} = 0. \tag{13}$$

Substitution of the model parameters (79) and the initial conditions (81) into IMPACT28 and IMPACT35 leads to the prediction of the mine's translation and orientation that are compared with the data collected during Exp-1 at time steps (Fig. 7). The new 3D model (IMPACT35) simulated trajectory and travel time agree

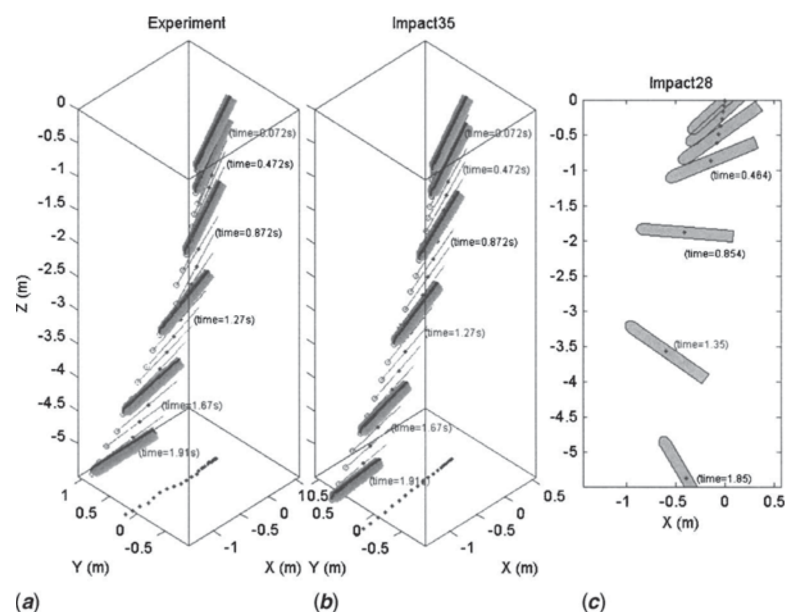


Fig. 6 Movement of Mine #6 (L=1.01 m, ρ=2.1×10^3 kg m^{-3}) with χ=−0.0077 m and ψ_2=−14.0° obtained from (*a*) experiment, (*b*) 3D IMPACT35 model, and (*c*) 2D Impact28 model

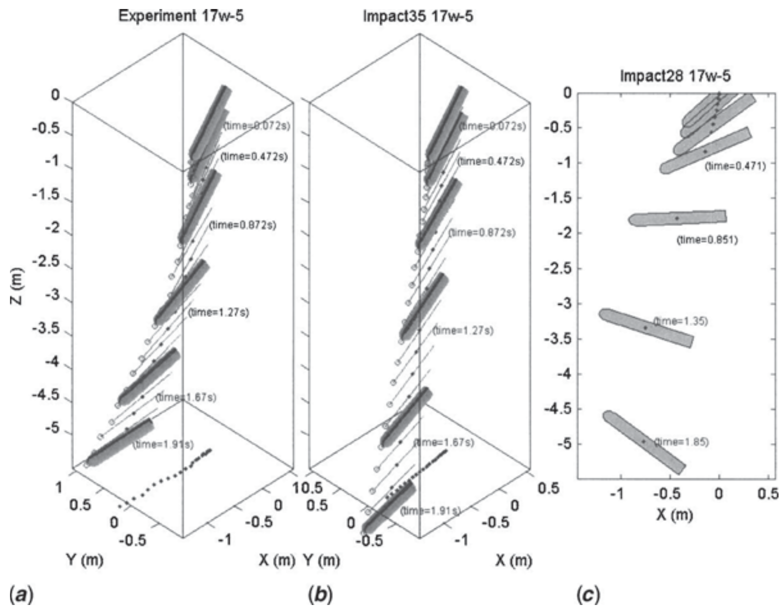


Fig. 7 Movement of Mine #5 (L=1.01 m, ρ=2.1×10^3 kg m^{-3}) with χ =0.0045 m and ψ_2=42.2° obtained from the (*a*) experiment, (*b*) 3D IMPACT35 model, and (*c*) 2D Impact28 model

well with the observed trajectory. However, the existing 2D model (IMPACT28) has less capability to predict the cylinder's movement in the water column.

6.1.3 Near Vertical Release. Model mine #2 is released to the water with ψ_2=87° (near vertical, see Fig. 2). The initial conditions are given by

$$x_0 = y_0 = z_0 = 0, \quad u_0 = v_0 = w_0 = 0,$$
$$\psi_{10} = 0, \quad \psi_{20} = 87\,^\circ, \quad \psi_{30} = 0, \quad \omega_{10} = \omega_{20} = \omega_{30} = 0. \tag{14}$$

The predicted cylinder's translation and orientation are compared with the data collected during Exp-1 at time steps (Fig. 8). The 3D model (IMPACT35) simulated trajectory agrees well with the observed trajectory. Both show the same straight pattern and the same travel time (1.83 s) for the cylinder passing through the water column. However, the existing 2D model (IMPACT28) does not predict the travel time well.

6.2 Burial Depth. After running the two models (IMPACT35 and IMPACT28) for each gravity core regime $[\rho_s(z), S(z)]$, the burial depths were compared with measured burial depth data (Fig. 9). As evident, IMPACT35 improves the prediction capability. The existing 2D model (IMPACT28) overpredicts the actual burial depth by an order of magnitude, on average. However, the

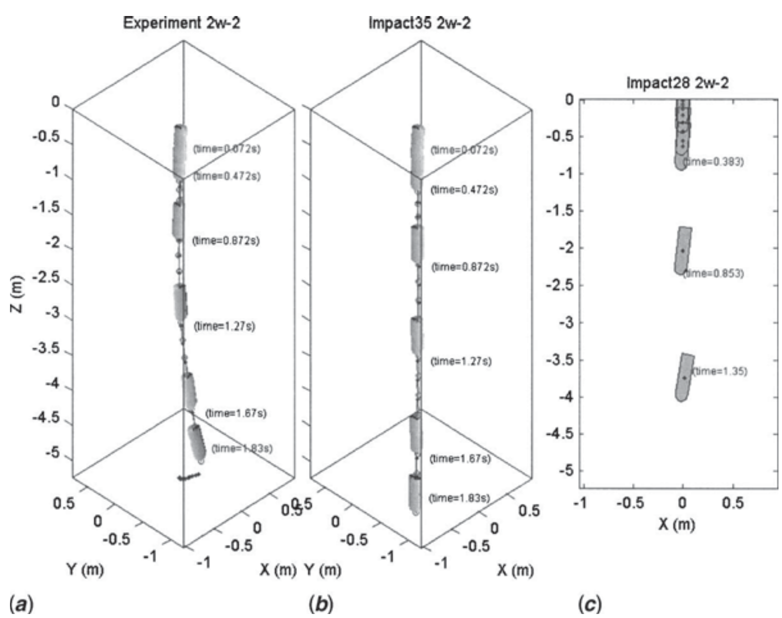
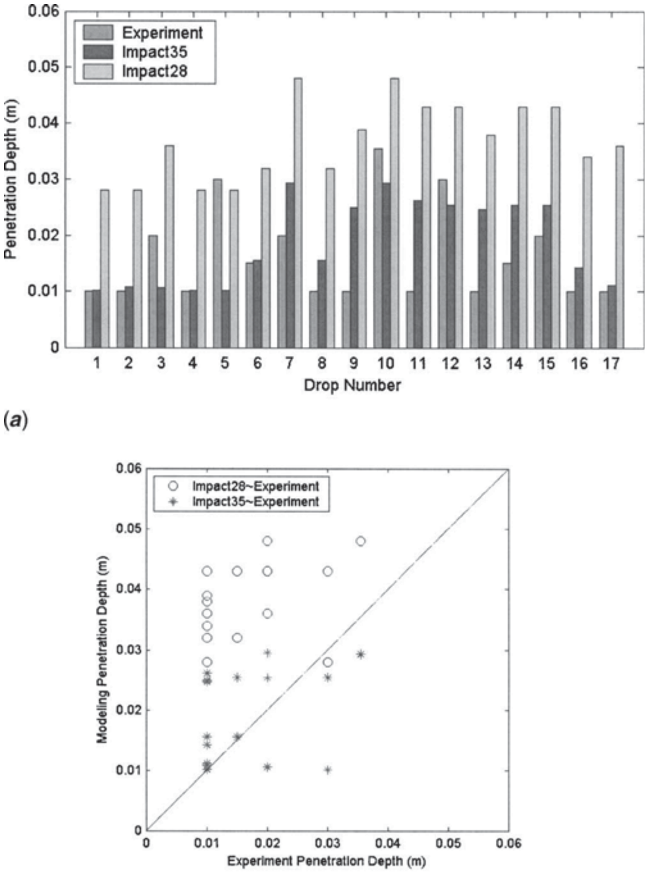


Fig. 8 Movement of cylinder #2 (L=0.505 m, ρ=2.1×10^3 kg m^{-3}) with χ=0 and ψ_2=87.0° obtained from the (*a*) experiment, (*b*) 3D IMPACT35 model, and (*c*) 2D Impact28 model



(b)

Fig. 9 A comparison among observed (Ext-2 data) and predicted burial depths

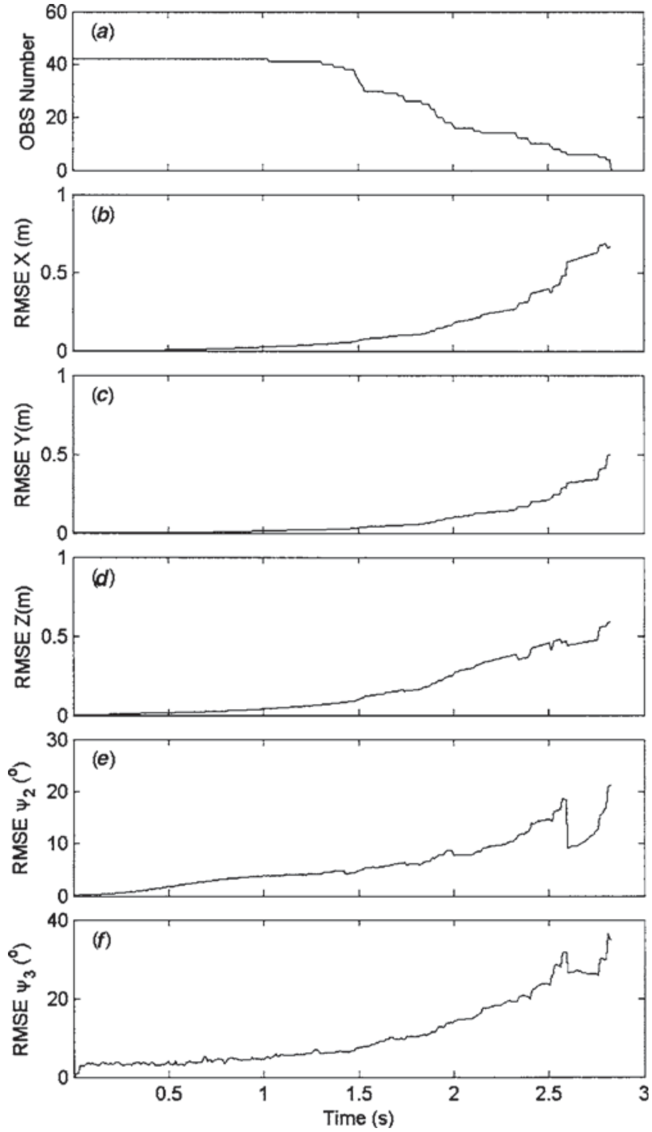


Fig. 10 Dependence of (*a*) observational data number, and of root-mean-square errors of the prediction for (*b*) *x*, (*c*) *y*, (*d*) *z*, (*e*) ψ_2, (*f*) ψ_3 on the mines' falling time.

3D model (IMPACT35) predicts the burial depth reasonably well without evident overprediction. Since the gravity cores were taken for approximately two to three meters from the impact location, several cores were taken for each drop. This allowed an average to be calculated in order to yield more accurate data for each drop.

7 Sensitivity and Weakness of IMPACT35

IMPACT35 has two major model parameters: (1) distance (χ) between COM and COV, and (2) L/R ratio. In the two mine drop experiments, COM almost coincides with COV. Model sensitivity is tested with respect to the aspect ratio and initial release velocity using the observational data. A comparison among Figs. 6–8 shows that the model has better predictability for a large L/R ratio. When the L/R ratio is reduced, the prediction error increases.

Let ζ represent any of the five parameters $(x, y, z, \psi_2, \psi_3)$, and let (ζ_p, ζ_o) be the predicted and observed values. The difference between the two

$$\Delta\varsigma(i,t) = \varsigma_p(i,t) - \varsigma_o(i,t), \tag{15}$$

is defined as the model error. Here, the index (i) is the case number. The root-mean-square error (RMSE) for IMPACT35 is defined by

$$\mathrm{RMSE}(t) = \sqrt{\frac{1}{N(t)}\sum_i [\Delta\varsigma(i,t)]^2}, \tag{16}$$

where $N(t)$ is the total number of observational data at the falling time t. Figure 10(*a*) shows that $N(t)$ is around 40 as $t < 1.5$ s and reduces quickly with time as $t > 1.5$ s. For $t > 2.5$ s, the observational data points are less than 10. RMSEs of the (x, y, z, ψ_2) prediction are very low when $t < 1.5$ s and increases drastically when $t > 1.5$ s. The large values of RMSE as $t > 1.5$ s may also be related to less observational data (Fig. 10).

Although the number of observational data may affect RMSE (Fig. 10), a tendency of RMSE growing with time does exist. For example, a RMSE of z increases from 0 at $t = 0$ to 0.1 m at $t = 1.5$ s. If we set

$$\mathrm{RMSE}_z \leq 0.4 \text{ m}, \tag{17}$$

as the tolerance level, IMPACT35 has capability within 6 s. In Exp-1, it takes around 3 s for the mines falling in the pond (water depth: 7.92 m) of the Naval Surface Warfare Center Caderock Division. Therefore, at a water column depth of about 16 m, IMPACT35 becomes unable to make reliable predictions.

Another weakness of the current version of IMPACT35 is only for near-cylindrical mines. The hypothesis used in this study is that the mine can be parametrized into a pseudocylinder. The model neglects the effect of the mine shape and only addresses only the effect of the L/R ratio, χ, and density. The effect of mine shape is a significant issue if the model is used operationally, because the most popular mines such as Rockan and Manta are not near cylindrical. The most important issue is to determine the hydrodynamic (drag and lift) force and torque for noncylindrical mines. There is no existing formula for calculating the drag and lift forces and torques for noncylindrical objects. The conformal projection may be used to transfer the noncylindrical mine into an "equivalent" cylindrical mine. In the model development, the nonlinear instability and model sensitivity should be studied. Within the correct physics of the model there is a possibility of chaotic behavior. The chaotic features will be handled by the instability and predictability analyses.

To overcome such a weakness, test data are crucial to include the mine shape. Since it is not likely to conduct full-size mine experiments with varying mine shapes, controlled reduced-size mine impact experiments are needed. The reduced size model mines with various shapes (Rockan, Manta, Korean, Bowen mines) should be conducted similar to the past experiments for the cylindrical mines.

8 Conclusions

(1) Pseudo-cylinder parametrization is presented and included into the recently developed 3D model (IMPACT35) to predict the translation and orientation of falling mine with near cylindrical shape through air, water, and sediment. After the pseudocylinder parametrization, the drag and lift forces and torques can be easily calculated using the existing formulas to calculate the drag and lift coefficients.

(2) A model-data comparison shows that IMPACT35 improves the prediction capability drastically versus the 2D model (IMPACT28) with an order of error reduction in the mine burial depth, more accurate cylinder track (depth and orientation) prediction, and more accurate travel time of the cylinder through air–water–sediment.

(3) The root-mean-square error of IMPACT35 grows with time. If the error in predicting vertical position of COM is required less than 0.4 m, at a water column depth of about 16 m, IMPACT35 becomes unable to make reliable predictions.

(4) Pseudocylinder parametrization is valid only for near-cylindrical mines. The effect of the mine shape (rather than near-cylindrical) is a significant issue if IMPACT35 is used operationally, because the most popular mines such as Rockan and Manta are not near-cylindrical. To overcome the weakness, test data are crucial to include the mine shape. Reduced size mines with various shapes (Rockan, Manta, Korean, Bowen mines) should be conducted similar to the past experiments for the cylindrical or near-cylindrical mines.

Acknowledgments

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