NAVAL POSTGRADUATE SCHOOL

USE OF TRUE GRAVITY IN METEOROLOGY AND OCEANOGRAPHY (METOC)

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Inaugural Department of Defense 6.1 Basic Research Conference (Hybrid) September 6-9, 2022

Background

- (1) Gravity = Newton's Gravitational Acceleration
- + Centrifugal Acceleration
- (2) Untrue gravity, i.e., effective gravity ($\mathbf{g}_{eff}$), is used in METOC.
- (3) True gravity ($\mathbf{g}_{true}$) is used in Geodesy.
- (4) Gravity Disturbance Vector (GD),

$$\delta\mathbf{g} = \mathbf{g}_{true} - \mathbf{g}_{eff}$$

- (5) $\delta\mathbf{g}$ is the most important variable in Geodesy.
- (6) $\delta\mathbf{g}$ has never been considered in METOC.

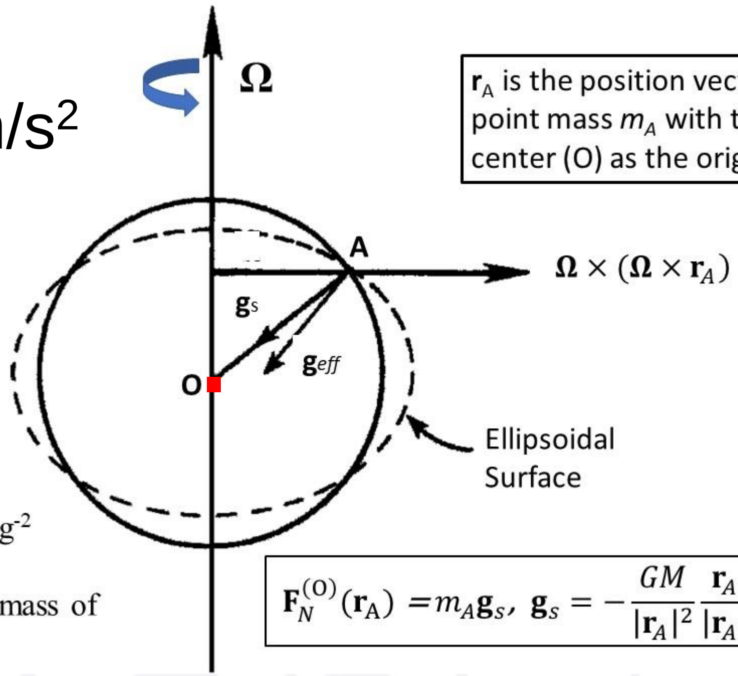
Gravity in METOC is UNTRUE

In Meteorology and Oceanography,

the whole solid Earth is treated as one point mass with the entire Earth mass to be located at the Earth centre O

One Point Mass at Earth Center O

$g_0 = 9.81$ m/s^2



$\mathbf{r}_A$ is the position vector of the point mass m_A with the Earth's center (O) as the origin

$G = 6.67408 \times 10^{-11}$ Nm^{2}kg^{-2}

$M = 5.98 \times 10^{24}$ kg is the mass of the Earth

$$\mathbf{F}_N^{(O)}(\mathbf{r}_A) = m_A \mathbf{g}_s, \quad \mathbf{g}_s = -\frac{GM}{|\mathbf{r}_A|^2}\frac{\mathbf{r}_A}{|\mathbf{r}_A|} = -g_0 \frac{\mathbf{r}_A}{|\mathbf{r}_A|}$$

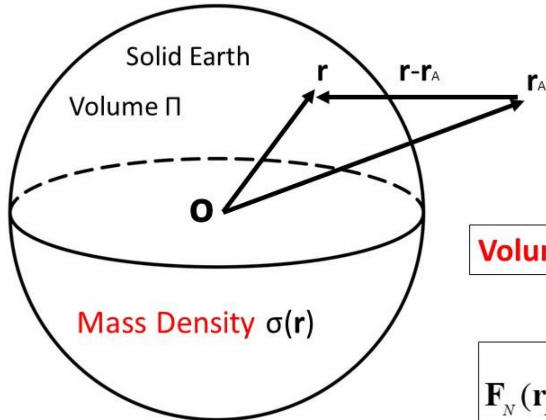
UNTRUE!!

Effective Gravity $\boldsymbol{g}_{eff}$

$$\mathbf{g}_{eff} = -g_0 \frac{\mathbf{r}_A}{|r_A|} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_A)$$

True Gravity and Gravity Disturbance Vector $\delta\mathbf{g}$

In Geodesy, the Newton's gravitation is the volume integration over all the point masses inside the solid Earth to the point mass in atmosphere and oceans.



Newton's Gravitational Force of the Solid Earth on a Point Mass m_A at $\mathbf{r}_A$ in atmosphere

↓

Volume Integration over the Solid Earth

↓

$$\mathbf{F}_N(\mathbf{r}_A) = Gm_A \iiint_\Pi \frac{\sigma(\mathbf{r})}{|\mathbf{r}-\mathbf{r}_A|^3}(\mathbf{r}-\mathbf{r}_A)d\Pi$$

$$\mathbf{g}_{true} \equiv \frac{\mathbf{F}_N(\mathbf{r}_A)}{m_A} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}_A)$$
$$= G\iiint_\Pi \frac{\sigma(\mathbf{r})}{|\mathbf{r}-\mathbf{r}_A|^3}(\mathbf{r}-\mathbf{r}_A)d\Pi + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}_A)$$

Gravity Disturbance Vector

$$\delta\mathbf{g} = \mathbf{g}_{true} - \mathbf{g}_{eff} = \nabla_3 T$$
$$\approx g_0 \nabla N(\lambda,\varphi)$$

T: Disturbing Gravity Potential
N: Geoid, λ:longitude, φ: latitude

Question

Is $\delta \mathbf{g}$ negligible in METOC?

Objective

Identify the importance of δg in METOC through comparison to other forces such as Coriolis force, pressure gradient force, wind stress (for oceans) using 4 publicly available datasets:

(a) International Center for Global Earth Models (ICGEM) static gravity field model EIGEN-6C4 (http://icgem.gfz-potsdam.de/home) for $N(\lambda, \varphi)$

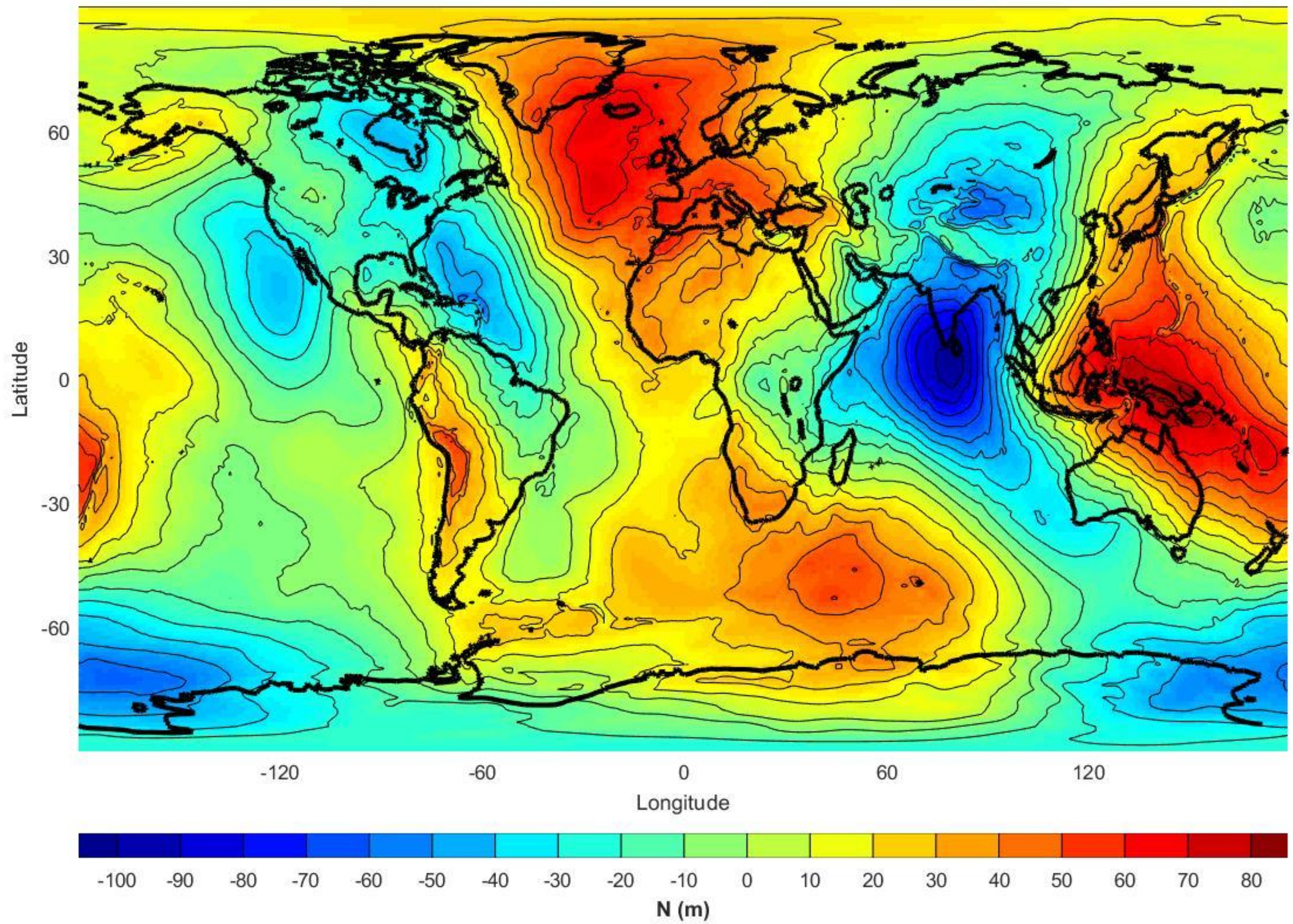
(b) NCEP/NCAR Reanalysed Monthly long-term mean (effective) geopotential height (Z), wind velocity (u, v), and temperature (T_a) at 12 pressure levels 1,000, 925, 850, 700, 600, 500, 400, 300, 250, 200, 150, and 100 hPa (https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.derived.pressue.html)

(c) Climatological annual mean temperature and salinity from the NOAA/NCEI WOA18 for the sea water density (ρ) data (from https://www.ncei.noaa.gov/access/world-ocean-atlas-2018/)

(d) Climatological annual mean surface wind stress (τ_λ, τ_φ) from the Atlas of Surface Marine Data 1994 (SMD94) (from https://iridl.ldeo.columbia.edu/SOURCES/.DASILVA/.)

Geoid Undulation *N* from EIGEN-6C4

$N(\lambda, \varphi)$



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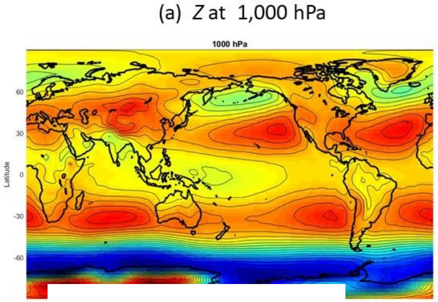
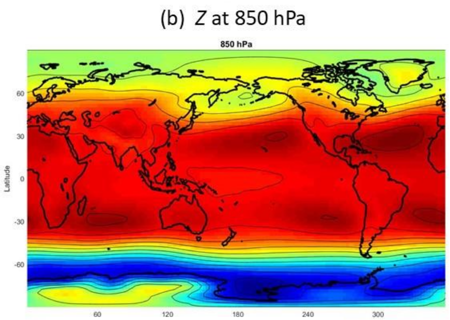
NCEP/NCAR reanalyzed global long-term atmospheric annual mean (Z, T_a, u, v) data

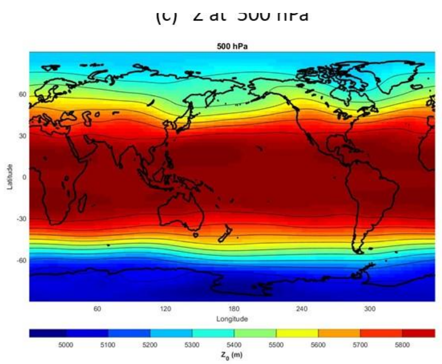
Z at 1000 hPa



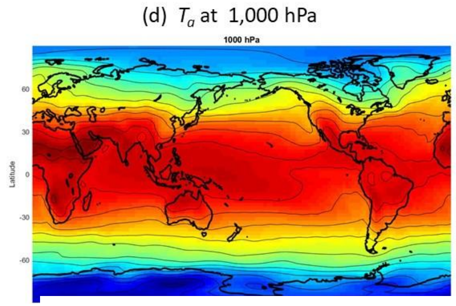
Z at 850 hPa



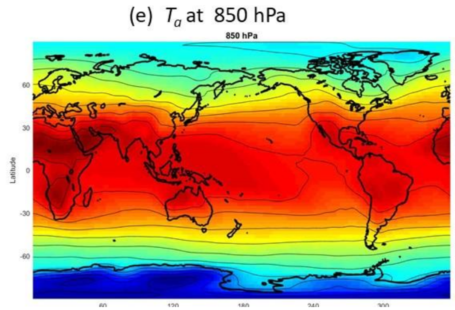
Z at 500 hPa



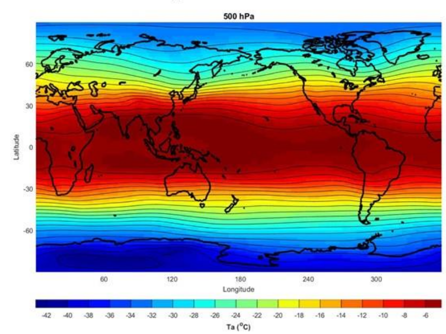
T*a* (^{o}C) at 1000 hPa



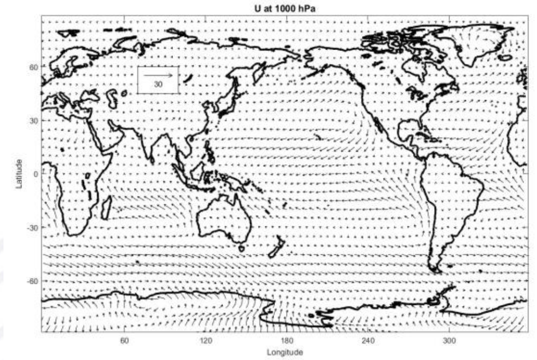
T*a* (^{o}C) at 850 hPa



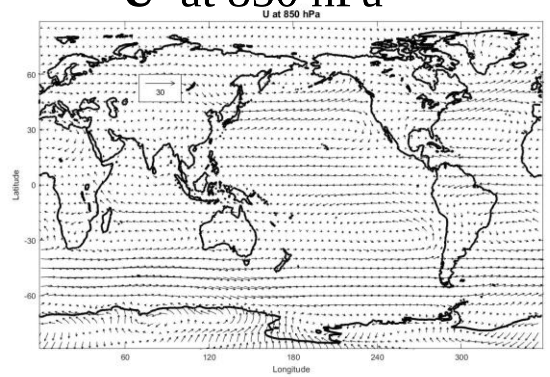
T*a* (^{o}C) at 500 hPa



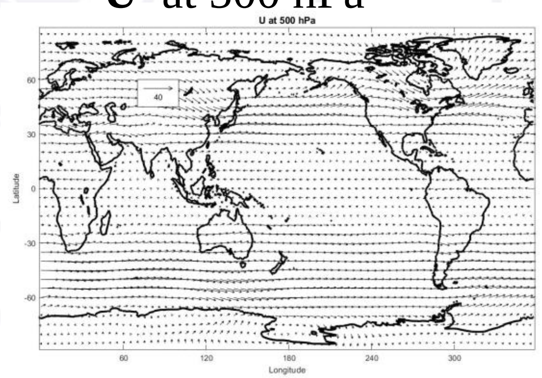
U at 1000 hPa



U at 850 hPa



U at 500 hPa



Meteorology – *B* and *C* Numbers

Horizontal Momentum Equation

$$\frac{D\mathbf{U}}{Dt} + f\mathbf{k}\times\mathbf{U} = -g_0\nabla Z + \delta\mathbf{g} + \mathbf{F}$$

$$\frac{D\mathbf{U}}{Dt} + f\mathbf{k}\times\mathbf{U} = -g_0\nabla Z + g_0\nabla N + \mathbf{F}$$

F is the frictional force

$$B \equiv \frac{O(|\delta\mathbf{g}|)}{O(|\text{ Pressure Gradient Force}|)} = \frac{\mathrm{O(|\nabla N|)}}{\mathrm{O[|\nabla Z|]}} = \frac{\mathrm{mean}(|\nabla N|)}{\mathrm{mean}[|\nabla Z|]}$$

$$C \equiv \frac{O(|\delta\mathbf{g}|)}{O(|\text{ Coriolis Force}|)} = \frac{g_0 O(|\nabla N|)}{O(|f\mathbf{U}|)} = g_0\frac{\mathrm{mean}(|\nabla N|)}{\mathrm{mean}(|f\mathbf{U}|)}$$

D-Number

Geostrophic Vorticity

$$\zeta = \frac{1}{f}\nabla^2\Phi = \zeta_{eff} + \zeta_{gd}, \quad \zeta_{eff} = \frac{1}{f}\nabla^2\Phi_{eff} = \frac{g_0}{f}\nabla^2 Z, \quad \zeta_{gd} = -\frac{g_0}{f}\nabla^2 N$$

Ekman Pumping Velocity

$$w_{Ekman} = \frac{\zeta}{2\gamma} = \frac{1}{2\gamma}(\zeta_{eff} + \zeta_{gd}), \quad \gamma \equiv \left|\frac{f}{2K}\right|^{1/2}\left(\frac{f}{|f|}\right)$$

D-Number

$$D \equiv \frac{O\left(\left|\zeta_{gd}\right|\right)}{O\left(\left|\zeta_{eff}\right|\right)} = \frac{O\left[\left|\nabla^2 N\right|\right]}{O\left[\left|\nabla^2 Z\right|\right]}$$

Q Vector (q_1, q_2) and Omega Equation

$$\sigma\nabla^2\omega + f^2\frac{\partial^2\omega}{\partial p^2} = -2\nabla\bullet\mathbf{Q} - \frac{\kappa}{p}\nabla^2 J$$

$$\nabla\bullet\mathbf{Q}^{eff} = \frac{R_a g_0}{pf}\nabla\bullet\mathbf{q}^{eff}, \quad \nabla\bullet\mathbf{Q}^{gd} = \frac{R_a g_0}{pf}\nabla\bullet\mathbf{q}^{gd}$$

Non-Dimensional (E_1, E_2, E_3) Numbers

$$E_1 \equiv \frac{O\left(\left|q_1^{gd}\right|\right)}{O\left(\left|q_1^{eff}\right|\right)} = \text{mean}\left(\left|J\left(\frac{\partial N}{\partial x}, T_a\right)\right|\right) / \text{mean}\left(\left|J\left(\frac{\partial Z}{\partial x}, T_a\right)\right|\right)$$

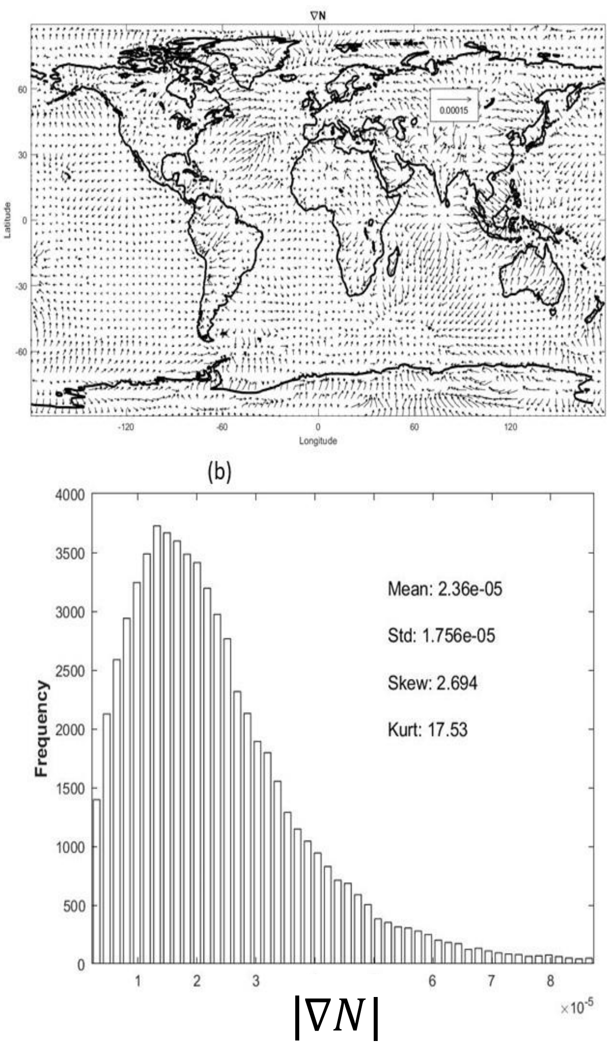
$$E_2 \equiv \frac{O\left(\left|q_2^{gd}\right|\right)}{O\left(\left|q_2^{eff}\right|\right)} = \text{mean}\left(\left|J\left(\frac{\partial N}{\partial y}, T_a\right)\right|\right) / \text{mean}\left(\left|J\left(\frac{\partial Z}{\partial y}, T_a\right)\right|\right)$$

$$E_3 \equiv \frac{O\left(\left|\nabla\bullet\mathbf{q}^{gd}\right|\right)}{O\left(\left|\nabla\bullet\mathbf{q}^{eff}\right|\right)}$$

Nondimensional *C* number

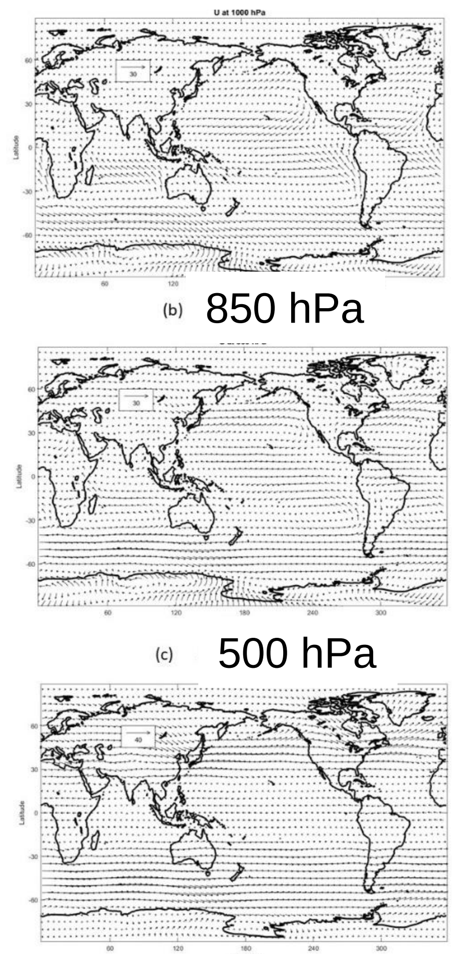
$$C = g_0 \frac{\mathrm{mean}(|\nabla N|)}{\mathrm{mean}(|f\mathbf{U}|)}$$

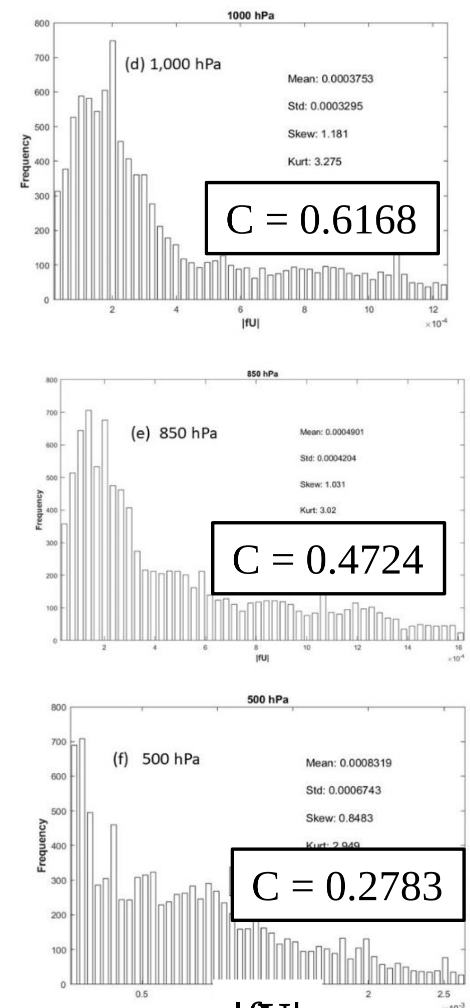
∇N



$|\nabla N|$

U





C = 0.6168

C = 0.4724

C = 0.2783

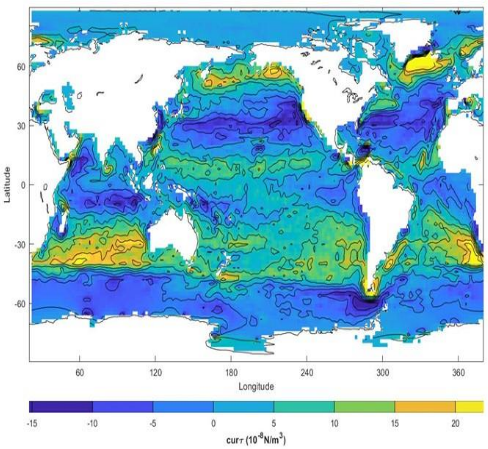
$|f\mathbf{U}|$

Non-Dimensional Numbers in Atmosphere

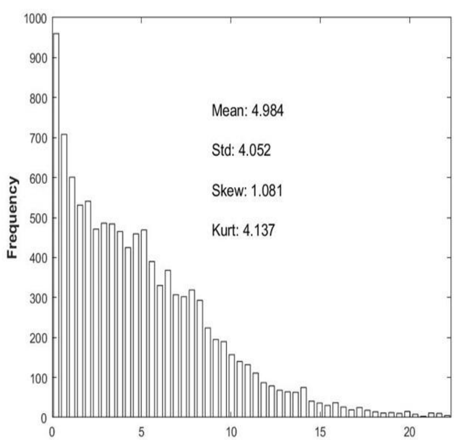
Pressure Level (hPa)	B Pressure Gradient Force	C Coriolis Force	D Geostrophic Vorticity	E_1 Q_1	E_2 Q_2	E_3 Omega Equation
1,000	0.4052	0.6168	0.6712	1.4268	0.5482	2.584
925	0.4151	0.5086	0.7539	1.6708	0.6205	3.366
850	0.4176	0.4724	0.8534	1.9850	0.7247	4.510
700	0.3836	0.3829	1.1078	2.8620	0.9097	6.911
600	0.3435	0.3327	1.3344	3.3175	1.0570	8.622
500	0.2849	0.2783	1.3381	3.3550	1.0077	8.797
400	0.2298	0.2241	1.2529	3.3121	0.9395	8.834
300	0.1861	0.1797	1.1169	3.0013	0.8091	7.495
250	0.1713	0.1645	1.0701	3.2204	0.7786	7.590
200	0.1630	0.1573	1.0564	3.1943	0.6829	7.667
150	0.1639	0.1611	1.0955	3.4567	0.7789	9.165
100	0.1869	0.1837	1.2109	4.3834	0.9965	11.713
Mean	0.2792	0.3052	1.0718	2.9313	0.8211	7.271

Wind-Driven Ocean Circulation
Combined Sverdrup-Stommel-Munk Dynamics

Wind Forcing

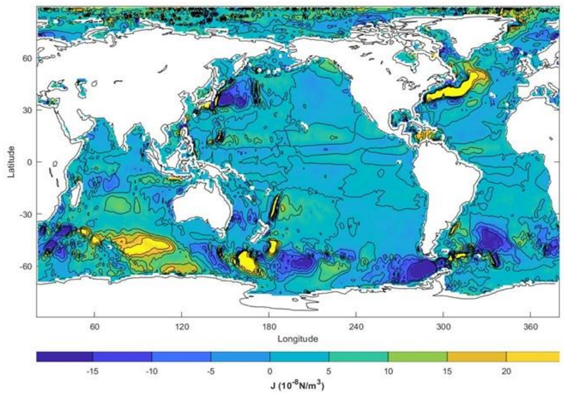


(b)



$|\text{curl}\ \boldsymbol{\tau}|$ (10^{-8} N/m^3)

Forcing due to $\delta\mathbf{g}$



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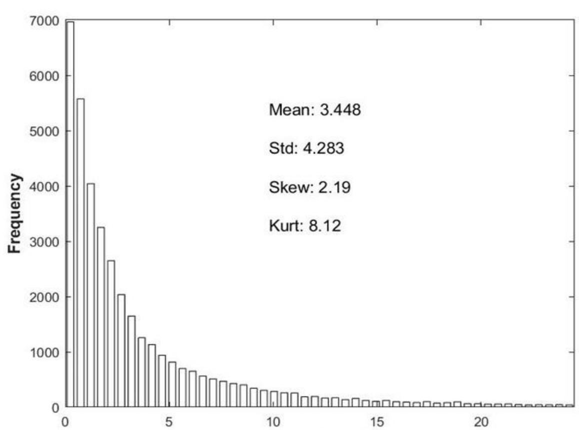

|J| (10^{-8} N/m^3)

$$-A\nabla^4\Psi + \gamma\nabla^2\Psi + \beta\frac{\partial\Psi}{\partial x} = \frac{1}{\rho_0}\left[\text{curl}\ \boldsymbol{\tau} + g_0\int_{-H}^{0}\mathbf{k}\bullet\left(\nabla\rho\times\nabla N\right)dz\right]$$

$$J = g_0\int_{-H}^{0}\mathbf{k}\bullet\left(\nabla\rho\times\nabla N\right)dz$$

$$F = \frac{\delta\mathbf{g}\text{-forcing}}{\text{wind-forcing}} = \frac{O\left[|J|\right]}{O\left[|\text{curl}\ \boldsymbol{\tau}|\right]} = \frac{3.448\times10^{-8}\ \text{N/m}^3}{4.984\times10^{-8}\ \text{N/m}^3} = 0.69$$

Results

- (1) Effective gravity $\mathbf{g}_{eff}$ used in meteorology and oceanography is untrue.

- (2) The effect of gravity disturbance vector $\delta\mathbf{g}$ in atmospheric and oceanic dynamics is important.

- (3) It is urgent to include $\delta\mathbf{g}$ in atmospheric and oceanic dynamics and numerical models.

References

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