Does satellite-determined dynamic ocean topography represent the surface absolute geostrophic currents?

Peter C. Chu
Naval Postgraduate School
Monterey, California, USA

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Outline

- 1. Marine Geoid
- 2. Two Types of Dynamic Ocean Topography (DOT)
- 3. Difference between Two Types of DOT
- 4. Conclusions

- Reference:

Chu, P.C., 2018: Two types of absolute dynamic ocean topography. *Ocean Science*, **14**, 947-957, https://doi.org/10.5194/os-14-947-2018.

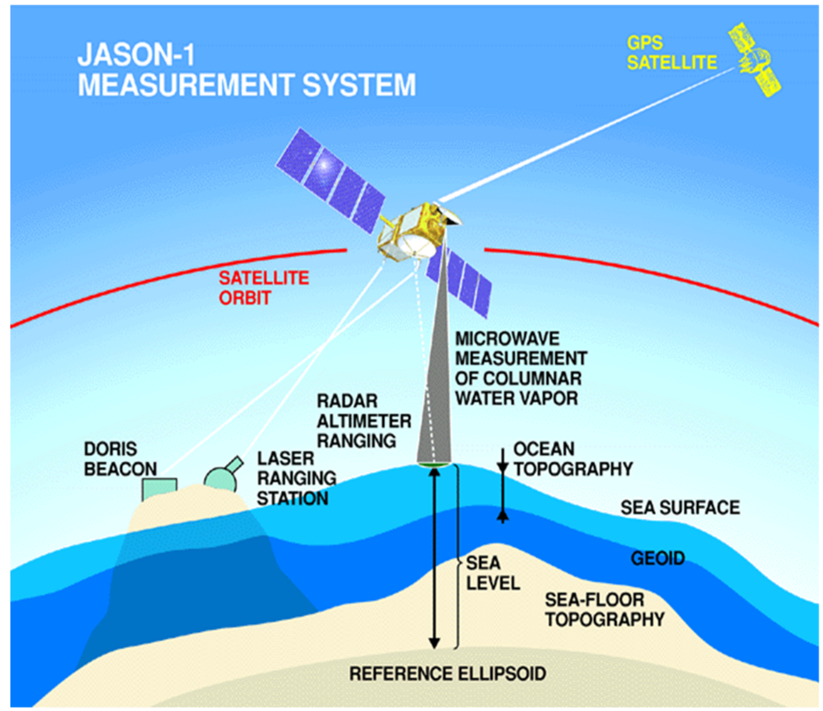
1. Marine Geoid

Three Surfaces

S – Sea surface height (SSH)

N – Marine geoid

D – Dynamic ocean topography



https://en.wikipedia.org/wiki/Ocean_surface_topography#/media/File:Jason-1_measurement_system.gif

Marine Geoid

- Definition

An equipotential surface of the Earth's gravity anomaly

Brun's Formula

$$\left(\frac{\partial^2 \hat{N}}{\partial x^2}+\frac{\partial^2 \hat{N}}{\partial y^2}\right)=\frac{1}{g}\frac{\partial(\Delta g)}{\partial z} \qquad (1)$$

g = 9.81 m/s^2, is the globally mean normal gravity;

$\Delta g(x, y, t)$ is gravity anomaly at $z = 0$

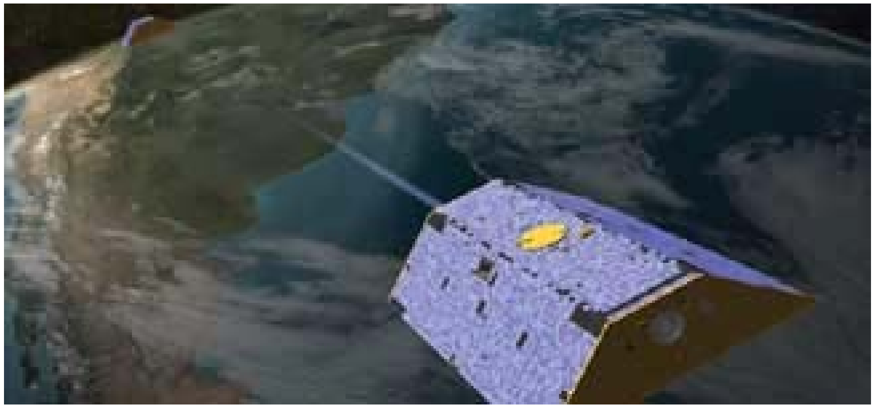
- Classical Approximation

The average level of the oceans (implying sea level not change) if the water were at rest → N

Up until now, nobody can approve that "N satisfies (1)"

$N_*(t)$ is the solution of (1) when Δg is determined through gravity anomaly observation by satellites.

N_* from Satellite Observations



Observed by GRACE

GRACE 2002-2017

GRACE-FO 2018

- Gravity field measured from GRACE →

with the ocean in ceaseless motion and changing sea level

Tapley et al. (2004)

https://www.jpl.nasa.gov/missions/gravity-recovery-and-climate-experiment-follow-on-grace-fo/

2. Two Types of Dynamic Ocean Topography (DOT)

Two Types of Dynamic Ocean Topography (DOT)

First Type DOT (D)

$$D = S - N$$

$$S = \text{SSH}$$

$$u_g(0) - u_g(N) = -\frac{g}{f}\frac{\partial D}{\partial y},$$

$$v_g(0) - v_g(N) = \frac{g}{f}\frac{\partial D}{\partial x}$$

$$u_g(N) = v_g(N) = 0$$

↓

$$u_g(0) = -\frac{g}{f}\frac{\partial D}{\partial y}, \quad v_g(0) = \frac{g}{f}\frac{\partial D}{\partial x}$$

Horizontal Gradient of D

↓

Absolute Surface Geostrophic Currents

Second Type DOT (D_*)

$$D_*(t) = S - N_*(t)$$

$$u_g(0) - u_g(N_*) = -\frac{g}{f}\frac{\partial D_*}{\partial y},$$

$$v_g(0) - v_g(N_*) = \frac{g}{f}\frac{\partial D_*}{\partial x}$$

$$u_g(N_*) \neq 0, \quad v_g(N_*) \neq 0$$

Horizontal Gradient of D_*

↓

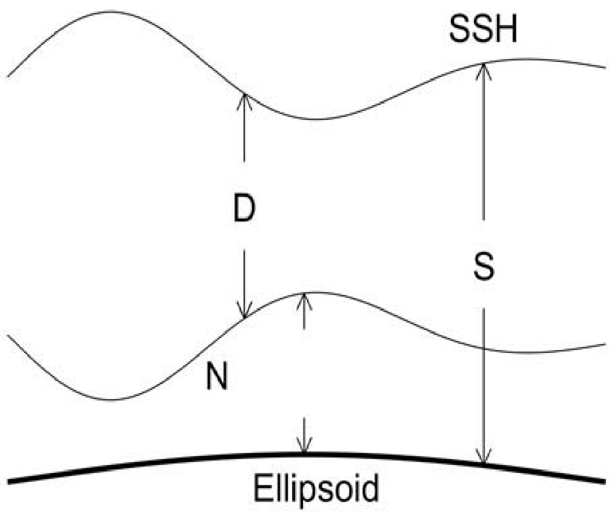
Relative Geostrophic Currents between the Surface and geoid (N_*)

↑

Not Surface Geostrophic Currents

Oceanography Implication

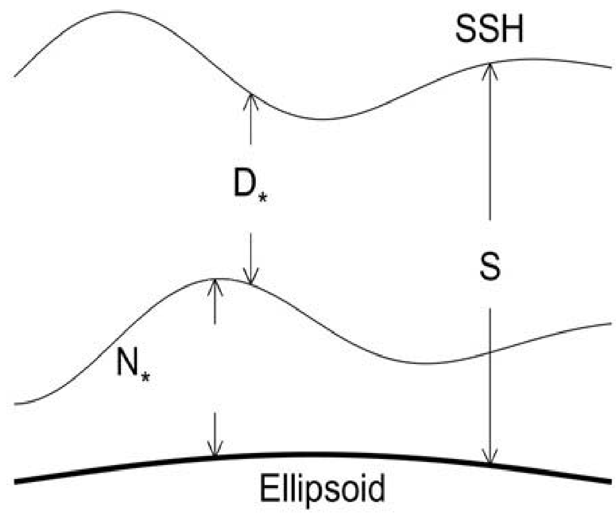
First Type



Water at rest on N

N is a level of no-motion

Second Type



Water in motion on N_*

Nobody can approve that N_* is a level of no-motion

Comparison Between Two Type DOTs

$$D_*(t) = S - N_*(t)$$

$(\bar{N}_*, \bar{D}_*)$ → Temporally Mean [N_*(t), D_*(t)]

$$\Delta N = \bar{N}_* - N, \quad \Delta D = \bar{D}_* - D = -\Delta N$$

$$\bar{D}_* \Leftrightarrow D$$

Continuation of geoid from land to oceans →

$$D|_\Gamma = \bar{D}_*|_\Gamma \qquad (2)$$

Γ is the coastline of the ocean basin.

Poisson Equation for D (but Not D*)

(Chu, 2018, OS) https://www.ocean-sci.net/14/947/2018/

$$H\left[\nabla^2 D + r^{(x)}\frac{\partial D}{\partial x} + r^{(y)}\frac{\partial D}{\partial y} - 2(\beta / f)\frac{\partial D}{\partial y}\right] = -F \qquad (3)$$

$$F \equiv \left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right), \quad \nabla \equiv \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}, \quad r^{(x)} \equiv \frac{1}{H}\frac{\partial H}{\partial x}, \quad r^{(y)} \equiv \frac{1}{H}\frac{\partial H}{\partial y}$$

$$X(x,y) = -\frac{1}{\rho_0}\int_{-H}^{0}\int_{z}^{0}\frac{\partial \hat{\rho}}{\partial y}dz'dz \qquad Y(x,y) = \frac{1}{\rho_0}\int_{-H}^{0}\int_{z}^{0}\frac{\partial \hat{\rho}}{\partial x}dz'dz$$

H → Ocean Bottom Topography

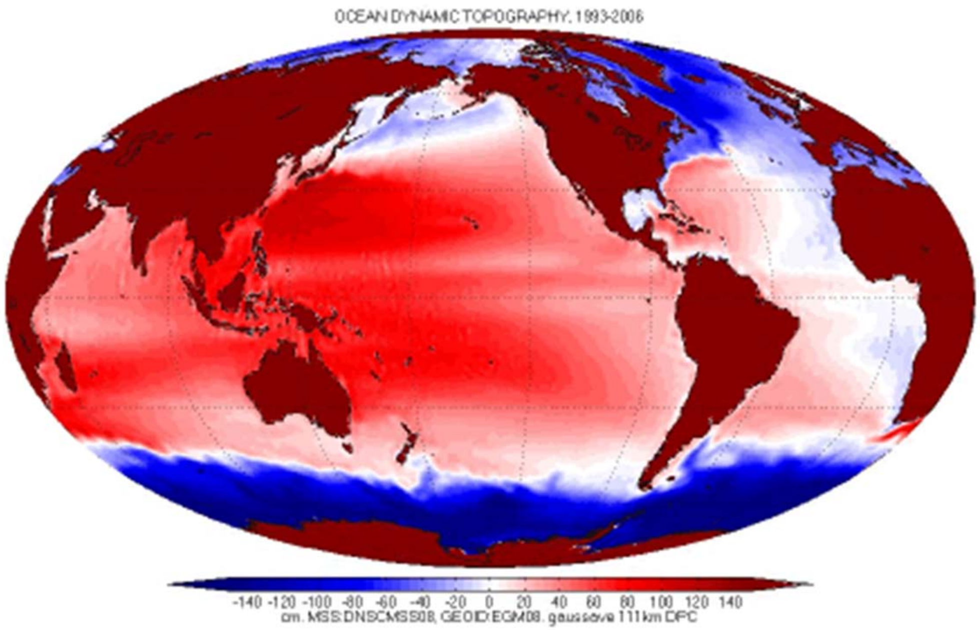
$\beta = df/dy$

Ocean Data → Forcing of (2)

- The climatological annual mean (T, S) data are obtained from the world ocean from the NOAA National Centers for Environmental Information (NCEI) World Ocean Atlas 2013 version 2 (WOA) at the website: http://www.nodc.noaa.gov/OC5/woa13/woa13data.html.
- The ocean depth data $H_{i,j}$ is downloaded from the NECI 5-Minute Gridded Global Relief Data Collection (ETOPO5) at the website: https://www.ngdc.noaa.gov/mgg/fliers/93mgg01.html

- Solving (3) with the boundary condition (2) → D

Climatological Mean $\bar{D}_*$



$$u_g(0) - u_g(\bar{N}_*) = -\frac{g}{f}\frac{\partial \bar{D}_*}{\partial y},$$

$$v_g(0) - v_g(\bar{N}_*) = \frac{g}{f}\frac{\partial \bar{D}_*}{\partial x}$$

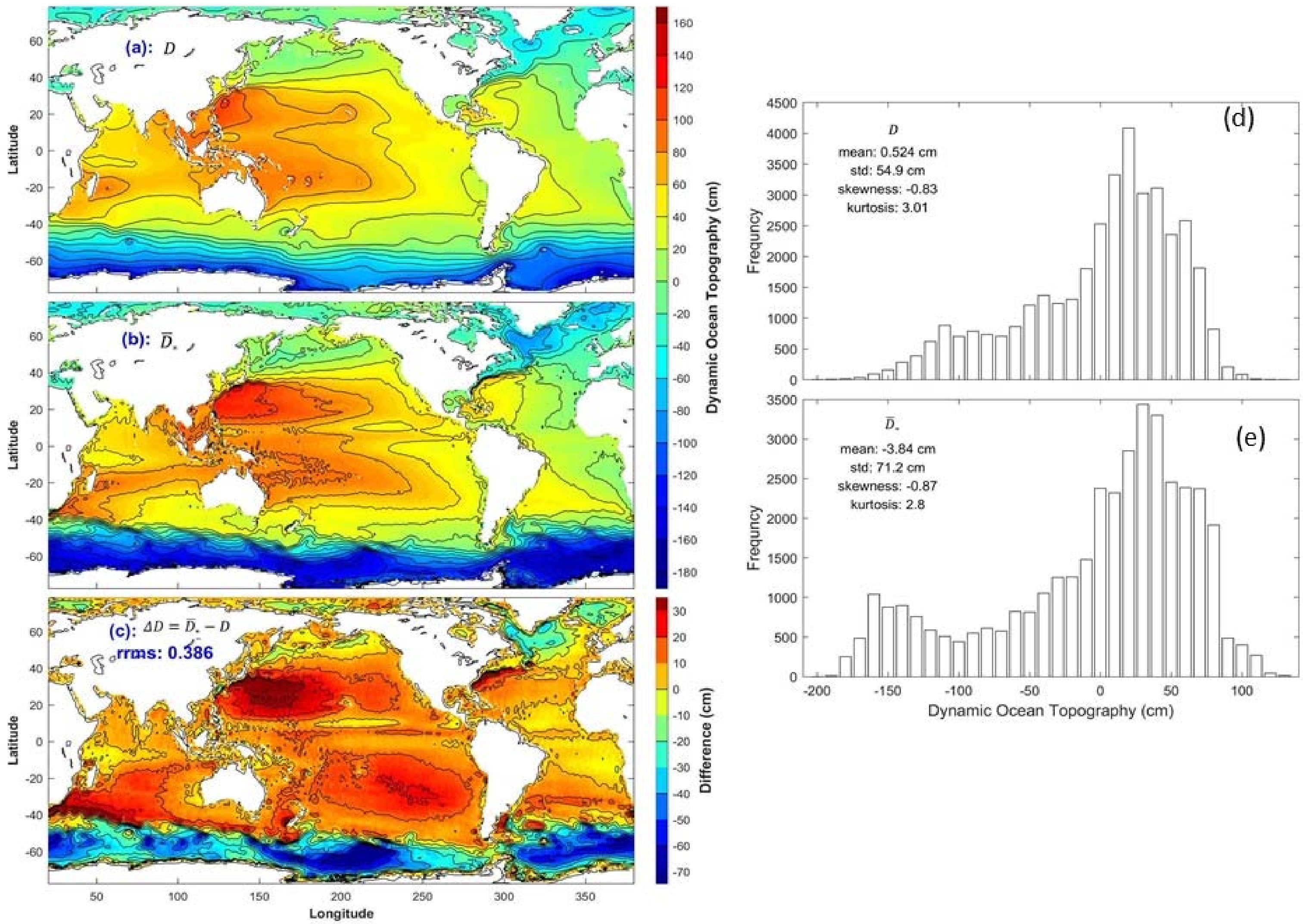
$$u_g(\bar{N}_*) \neq 0, \quad v_g(\bar{N}_*) \neq 0$$

https://grace.jpl.nasa.gov/data/get-data/dynamic-ocean-typography/

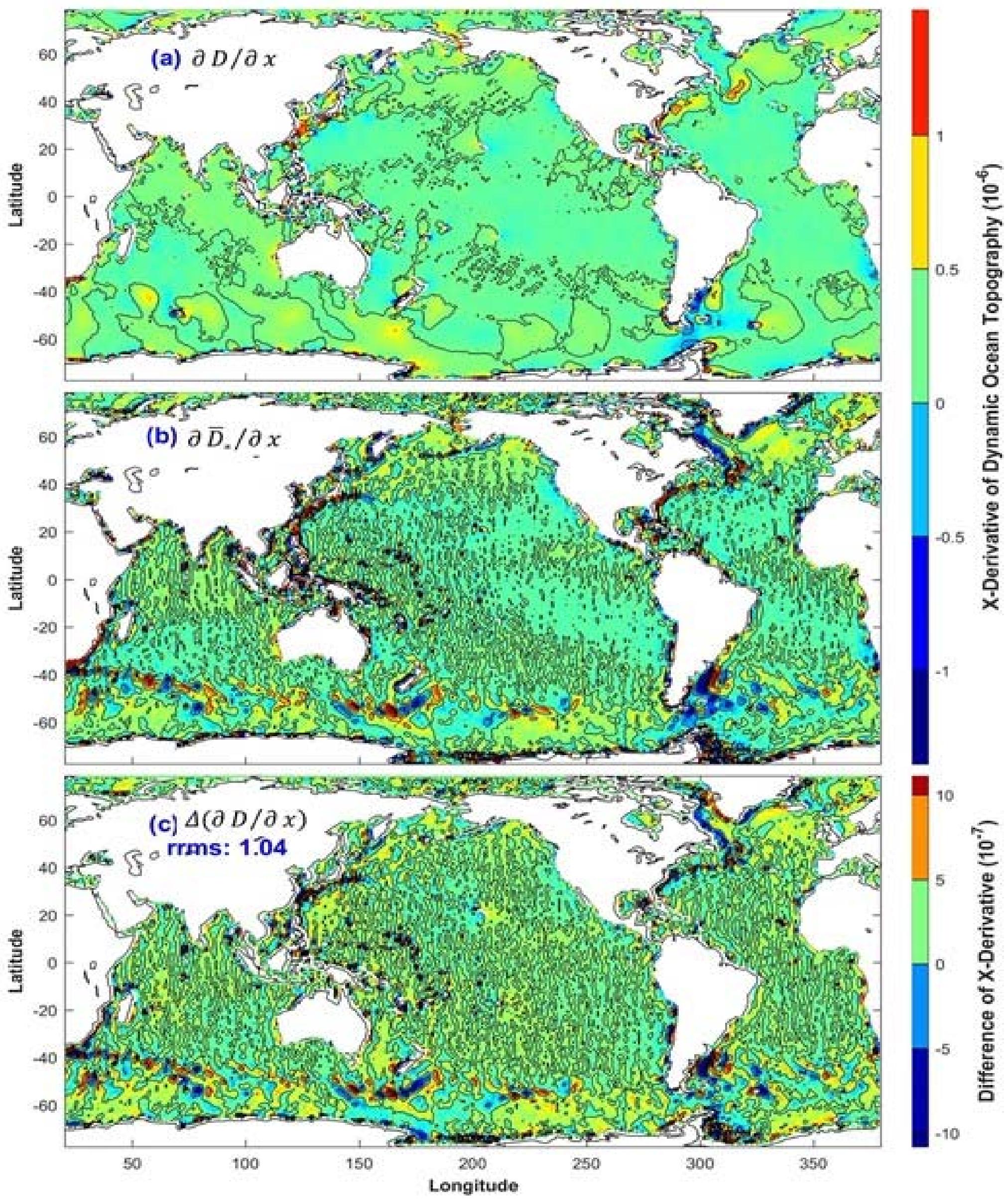
Horizontal gradient of $\bar{D}_*$ does not represent the surface absolute geostrophic velocity

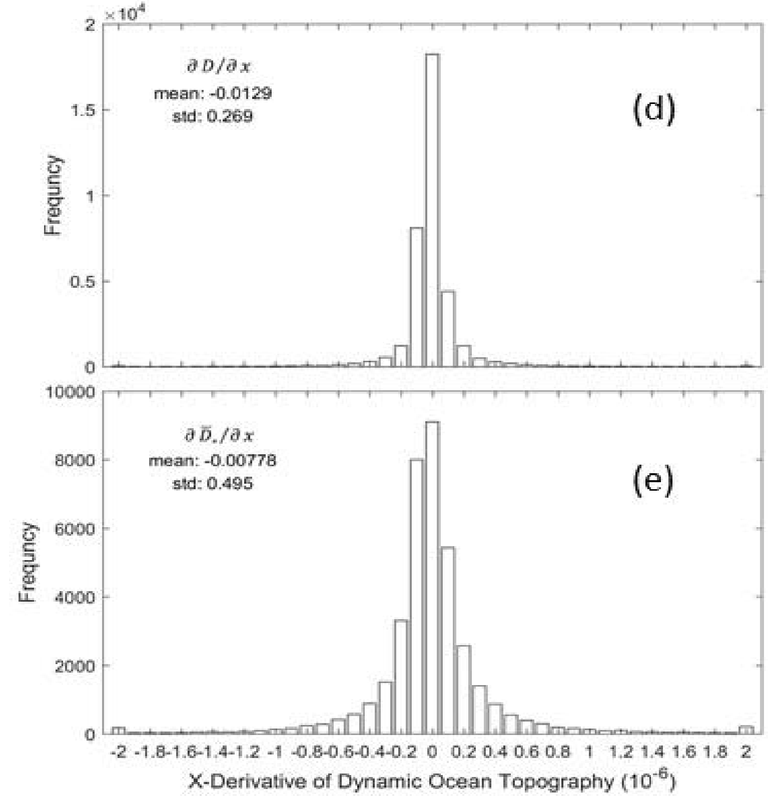
3. Difference between Two Types of DOT

Difference between $\bar{D}_*$ and D

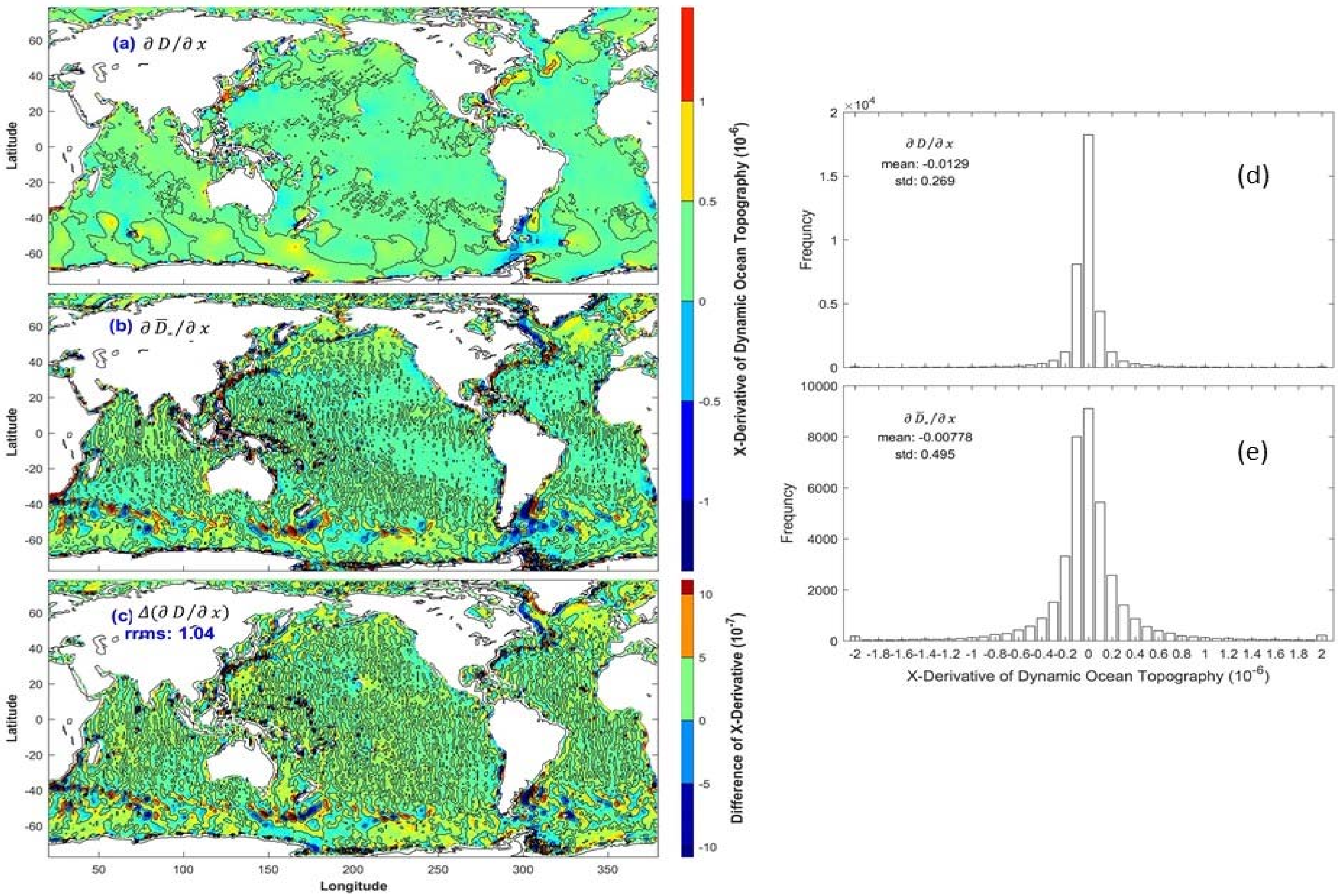


Difference between $\partial\overline{D}_*/\partial x$ and $\partial D/\partial x$





Difference between $\partial \overline{D}_* / \partial y$ and $\partial D / \partial y$



4. Conclusions

- Satellite-determined dynamic ocean topography (i.e. second type) does not represent the surface absolute geostrophic currents.

- Difference between the two types of DOT is evident with relative root-mean-square difference of 38.6%.