Variation of Marine Geoid Due to Ocean Circulation and Sea Level Change

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Thank you very much for coming to the last talk on the last day!

Outline

- 1. Marine Geoid before and after GRACE
- 2. Marine Geoid Anomaly due to Oceanic Motion and Sea Level Change
- 3. Governing Equation for Marine Geoid Anomaly
- 4. Temporally Averaged Global Marine Geoid Anomaly
- 5. Conclusions

1. Marine Geoid before and after GRACE

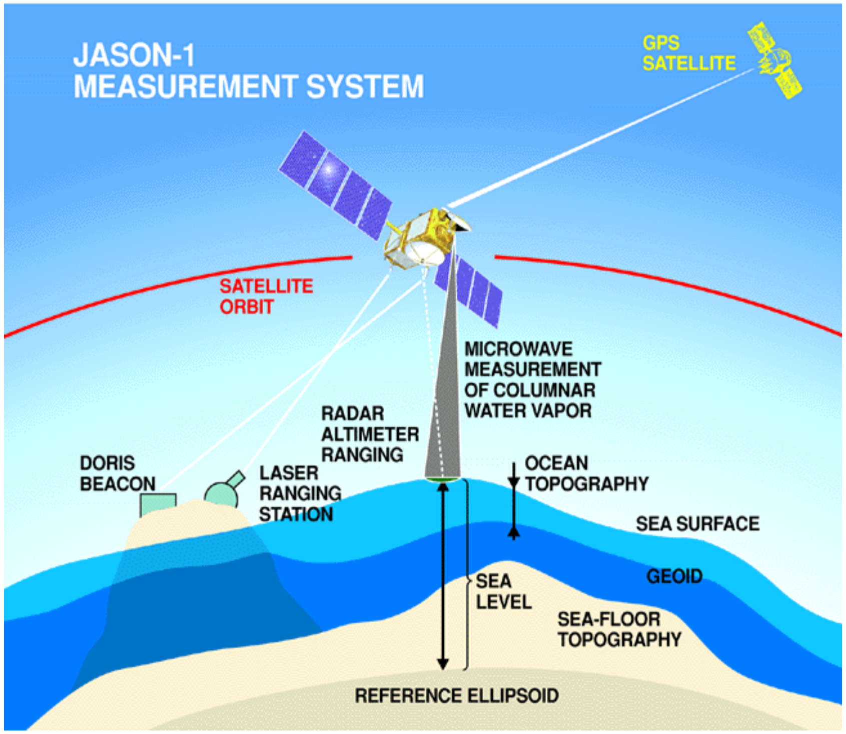
Classical Marine Geoid (N_0) before GRACE

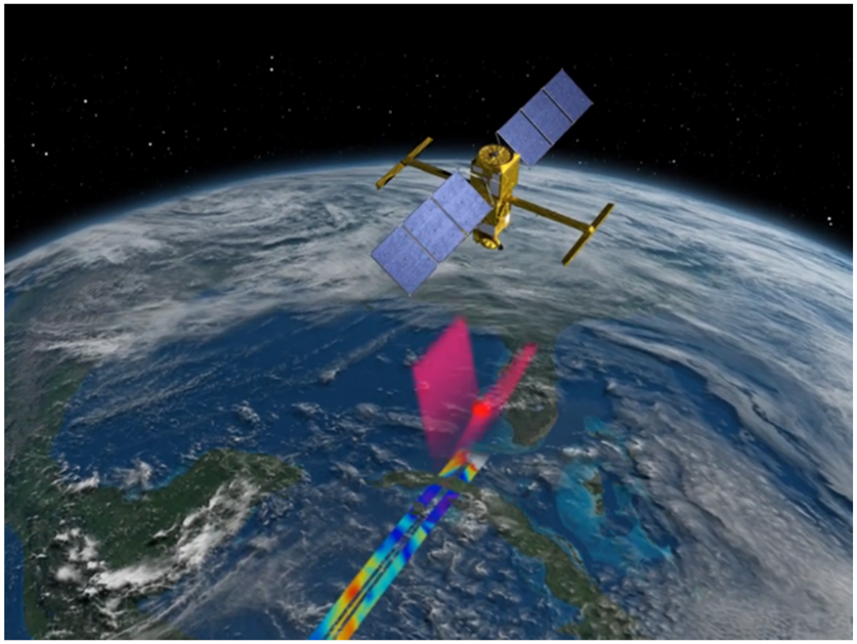
- An equipotential surface (N_0) which would coincide with the average level of the oceans (implying sea level not change) if the water were at rest.

Sea Surface Height (Topography) (η)

JASON-1 → Dec 7, 2001
JASON-2 → JUN 20, 2008
JASON-3 → Jan 17, 2016

Surface Water and Ocean Topography (SWOT) – Launch 2020 time Frame





https://swot.jpl.nasa.gov/mission/

https://en.wikipedia.org/wiki/Ocean_surface_topography#/media/File:Jason-1_measurement_system.gif

Fu and Ubelmann (2014)

Classical Marine Geoid and Dynamic Ocean Topography (DOT)

$$N_0 = \eta - D$$

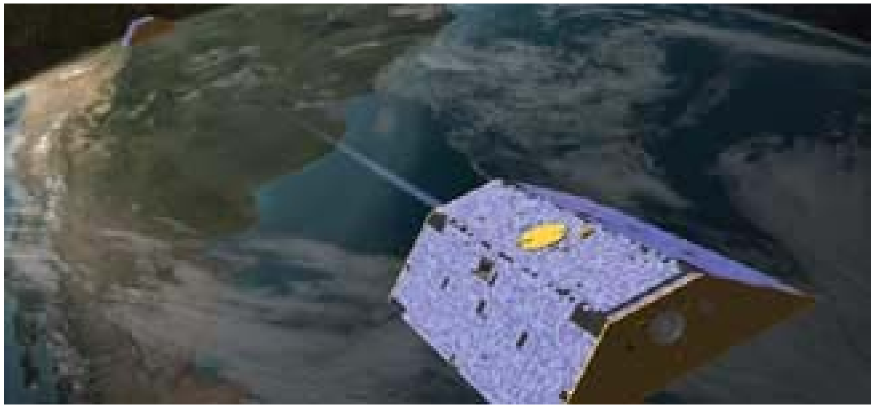
Surface Geostrophic Currents $[u_g(0),\ v_g(0)] \Rightarrow D$

$$u_g(0) = -\frac{g_*}{f}\frac{\partial D}{\partial y}, \qquad v_g(0) = \frac{g_*}{f}\frac{\partial D}{\partial x}$$

g_* → Globally Averaged Gravity

f → Coriolis Parameter

Marine Geoid (*N*) from GRACE



- Gravity field measured from GRACE →

with the ocean in ceaseless motion and changing sea level

Tapley et al. (2004)

Observed by GRACE
GRACE 2002-2017
GRACE-FO 2018

https://www.jpl.nasa.gov/missions/gravity-recovery-and-climate-experiment-follow-on-grace-fo/

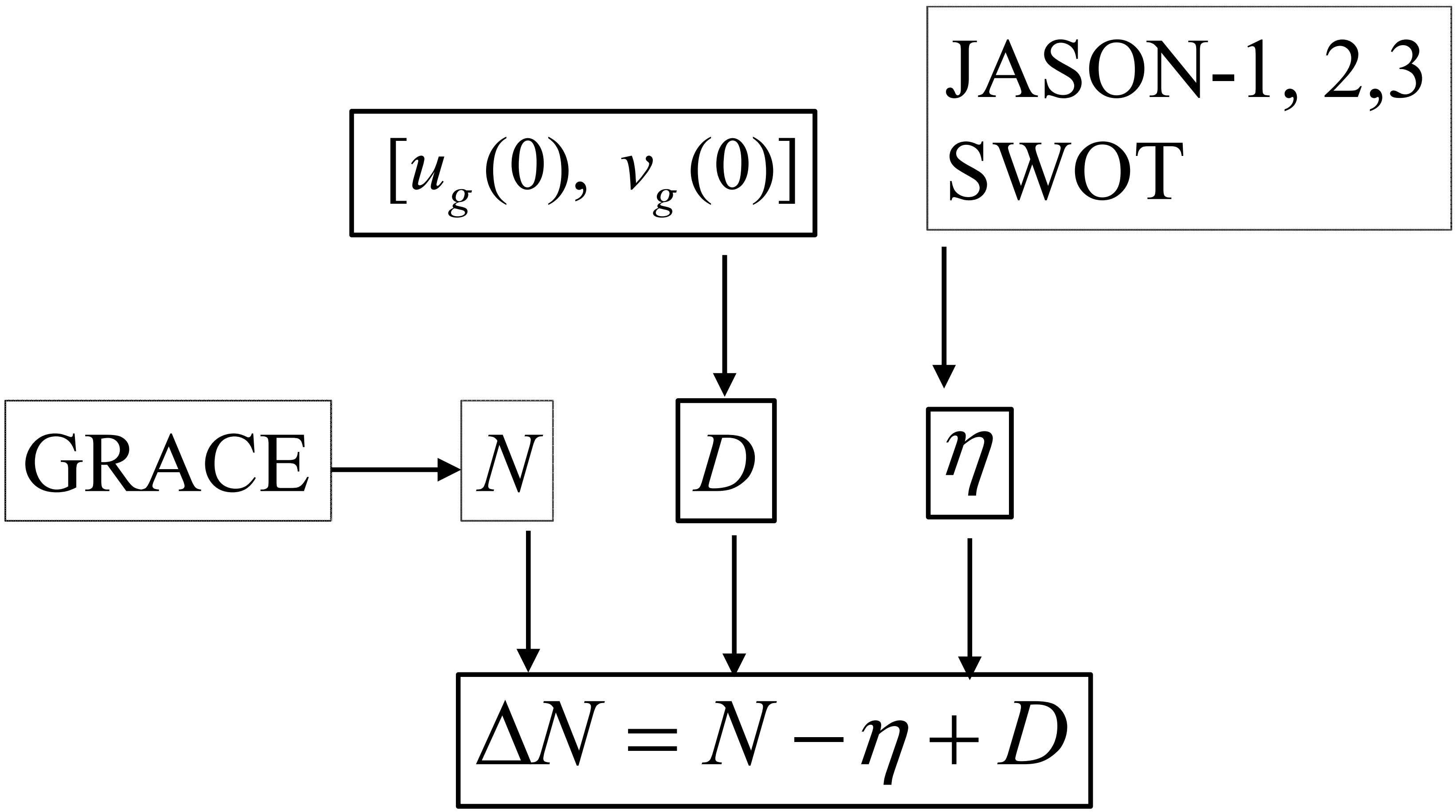
2. Marine Geoid Anomaly Due to Oceanic Motion and Sea Level Change

- N ← GRACE (Oceans in Ceaseless Motion and Changing Sea Level)
- N_0 ← Classical Marine Geoid (Oceans at Rest)

$$N \neq N_0$$

$$\Delta N \equiv N - N_0 = D - (\eta - N)$$

= Marine Geoid Anomaly due to Motion in Ocean and Sea Level Change



How to obtain D?

3. Governing Equation for Marine Geoid Anomaly

Theoretical Base

- (1) 3D geostrophic velocity is determined from 3D (T, S) fields and 2D DOT (D) using the thermal wind relation.
- (2) For large scale motion, geostrophic balance has a minimum energy state in an energy conserved basin (Vallis 1992).
- (3) Euler-Lagrangian equation of the global ocean mechanical energy leads to the governing equation

Thermal Wind Relation

$$u_g(z) = u_g(0) + u_{BC}(z), \quad v_g(z) = v_g(0) + v_{BC}(z)$$

$$u_g(0) = -\frac{g_*}{f}\frac{\partial D}{\partial y}, \qquad v_g(0) = \frac{g_*}{f}\frac{\partial D}{\partial x}$$

$$u_{BC}(z) = -\frac{g_*}{f\rho_0}\int_z^0 \frac{\partial \rho}{\partial y} dz', \quad v_{BC}(z) = \frac{g_*}{f\rho_0}\int_z^0 \frac{\partial \rho}{\partial x} dz'$$

$\rho = \rho(T, S, p)$ (Equation of State)

Minimum Energy State

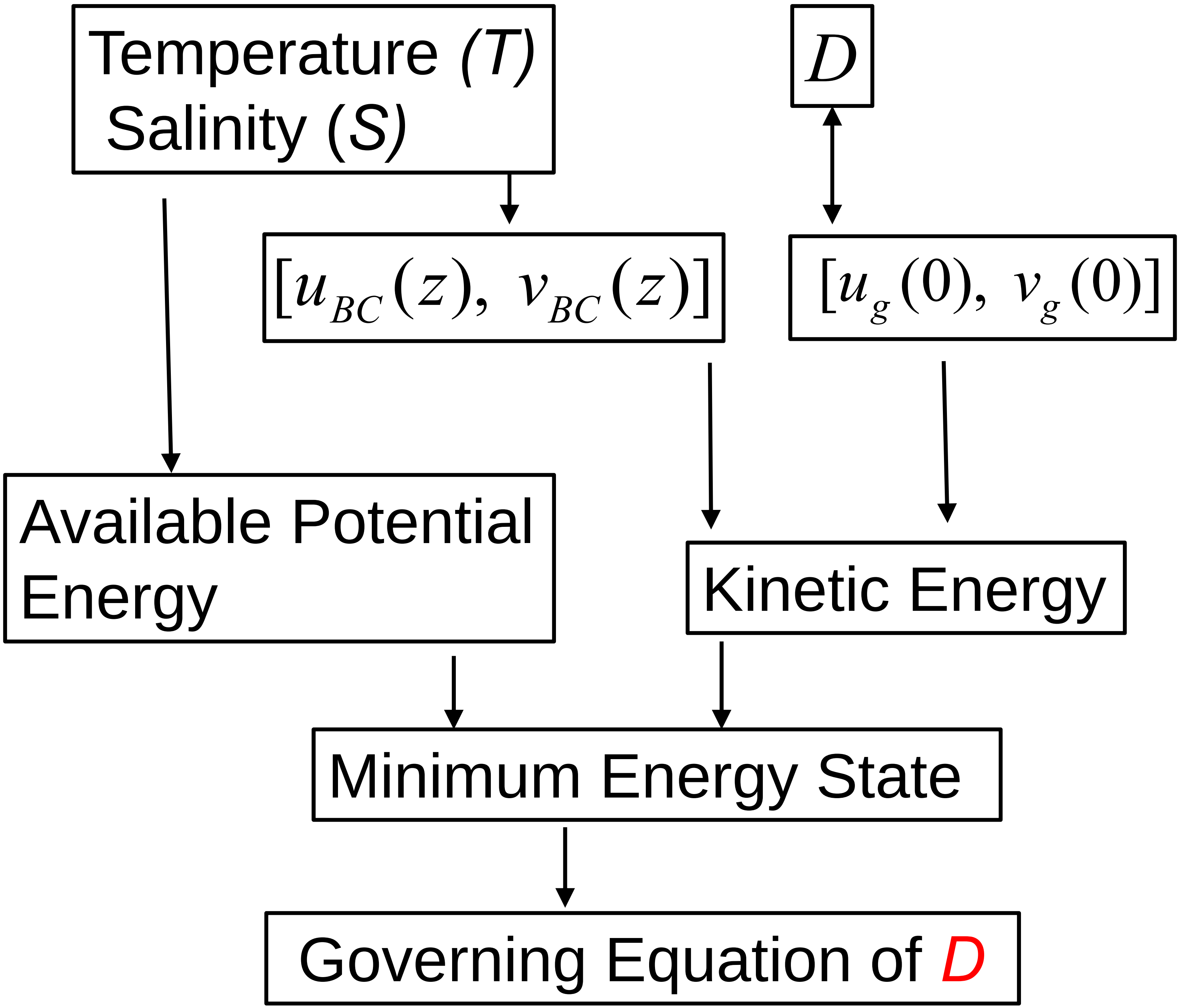
Kinetic Energy

Available Potential Energy

$$E = \iiint_V \left[\frac{1}{2}(u_g^2 + v_g^2) + \frac{g_*^2 \rho^2}{2\rho_0^2 n^2} \right] dxdydz$$

$$E(D_x, D_y, \rho) = \frac{g_*^2}{2} \iiint_V \left[(-D_y + \frac{f u_{BC}}{g_*})^2 / f^2 + (D_x + \frac{f v_{BC}}{g_*})^2 / f^2 + \frac{\rho^2}{\rho_0^2 n^2} \right] dxdydz$$

$$n^2 \equiv -\frac{g_*}{\rho_0} \frac{\partial \overline{\rho}}{\partial z}$$ (mean stratification)


Temperature (T)
Salinity (S)
D
$[u_{BC}(z),\ v_{BC}(z)]$
$[u_g(0),\ v_g(0)]$
Available Potential Energy
Kinetic Energy
Minimum Energy State
Governing Equation of D

Euler-Lagrangian Equation

Minimization of $E(D_x, D_y, \rho) \Rightarrow$

$$\boxed{H\left[\nabla^2 D + r^{(x)}\frac{\partial D}{\partial x} + r^{(y)}\frac{\partial D}{\partial y} - 2(\beta / f)\frac{\partial D}{\partial y}\right] = -F} \qquad \text{(A)}$$

$$F \equiv \left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right), \quad \nabla \equiv \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}, \quad r^{(x)} \equiv \frac{1}{H}\frac{\partial H}{\partial x}, \quad r^{(y)} \equiv \frac{1}{H}\frac{\partial H}{\partial y}$$

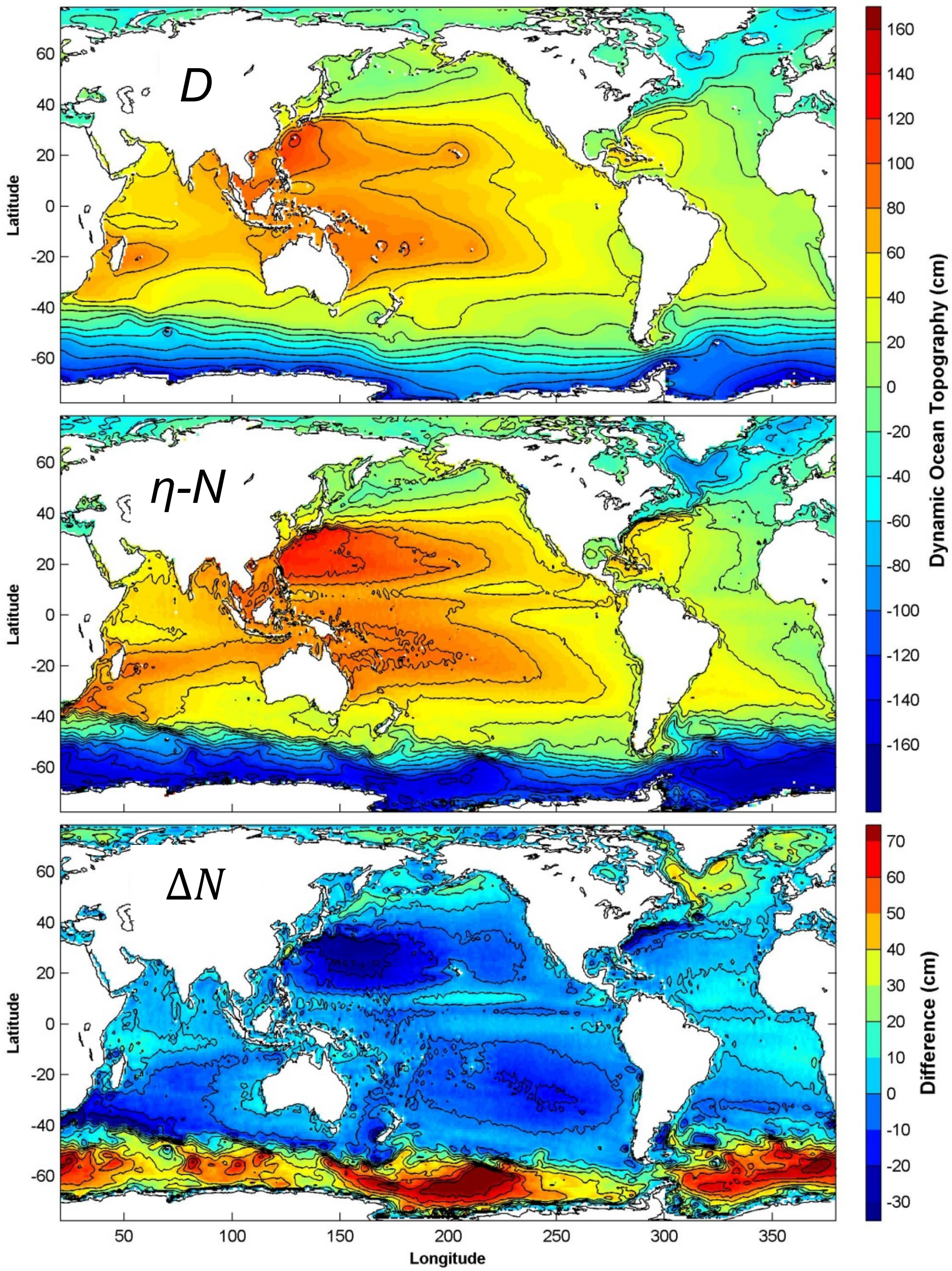
At ocean rigid boundary: $N_0 = N$ → $D = \eta - N$

H → Ocean Bottom Topography

4. Temporally Averaged Global Marine Geoid Anomaly

Ocean Data

- The climatological annual mean (T, S) data are obtained from the world ocean from the NOAA National Centers for Environmental Information (NCEI) World Ocean Atlas 2013 version 2 (WOA) at the website: http://www.nodc.noaa.gov/OC5/woa13/woa13data.html.
- The ocean depth data $H_{i,j}$ is downloaded from the NECI 5-Minute Gridded Global Relief Data Collection (ETOPO5) at the website: https://www.ngdc.noaa.gov/mgg/fliers/93mgg01.html



Numerical solution of the governing elliptic equation (A) of D with the rigid boundary values of

$$D = (\eta - N)$$

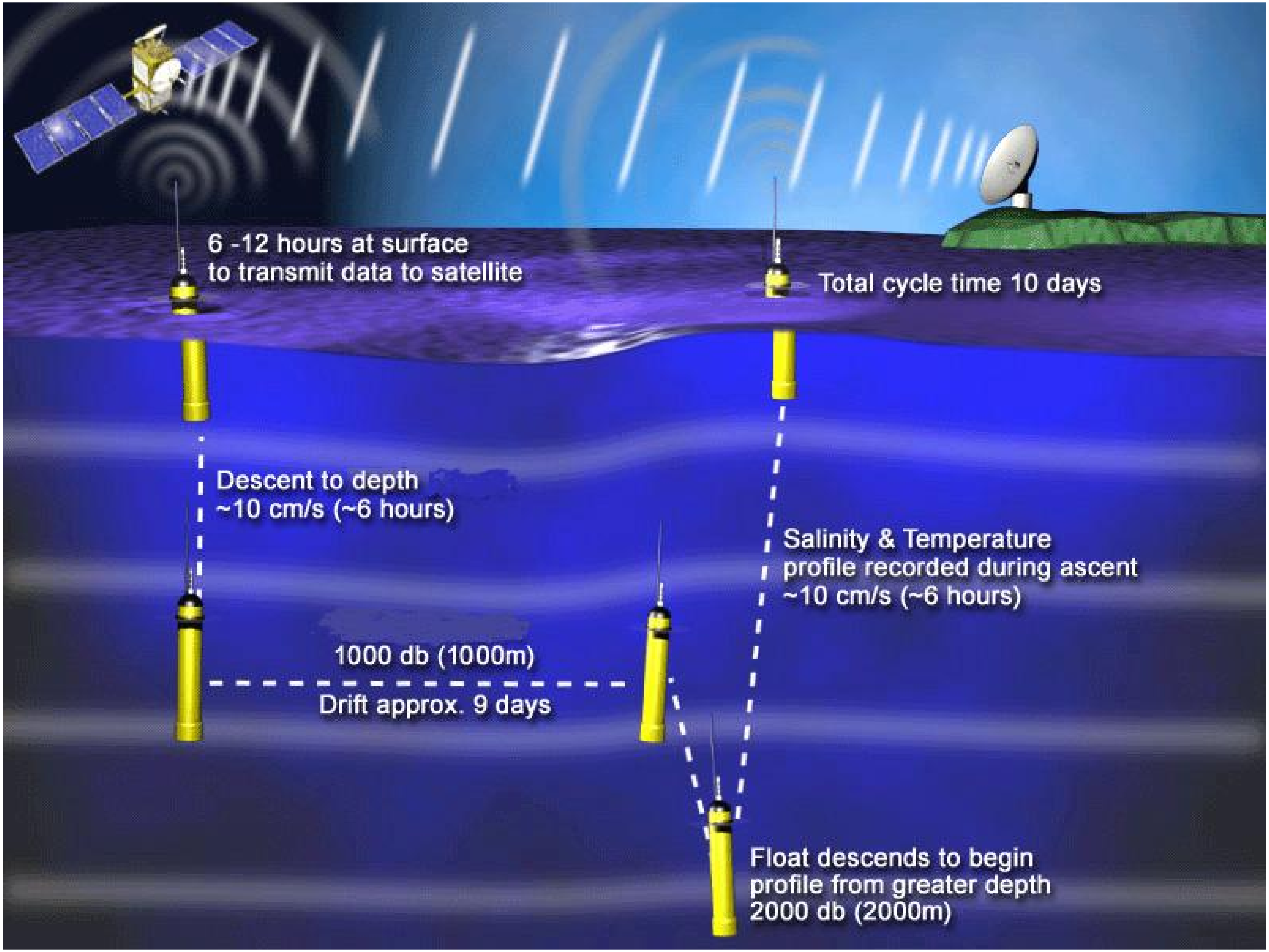
The mean (1993-2006) ($\eta - N$) "Dynamic Ocean Topography" downloaded from the NASA/JPL website: https://grace.jpl.nasa.gov/data/get-data/dynamic-ocean-typography

$$\Delta N = N - \eta + D$$

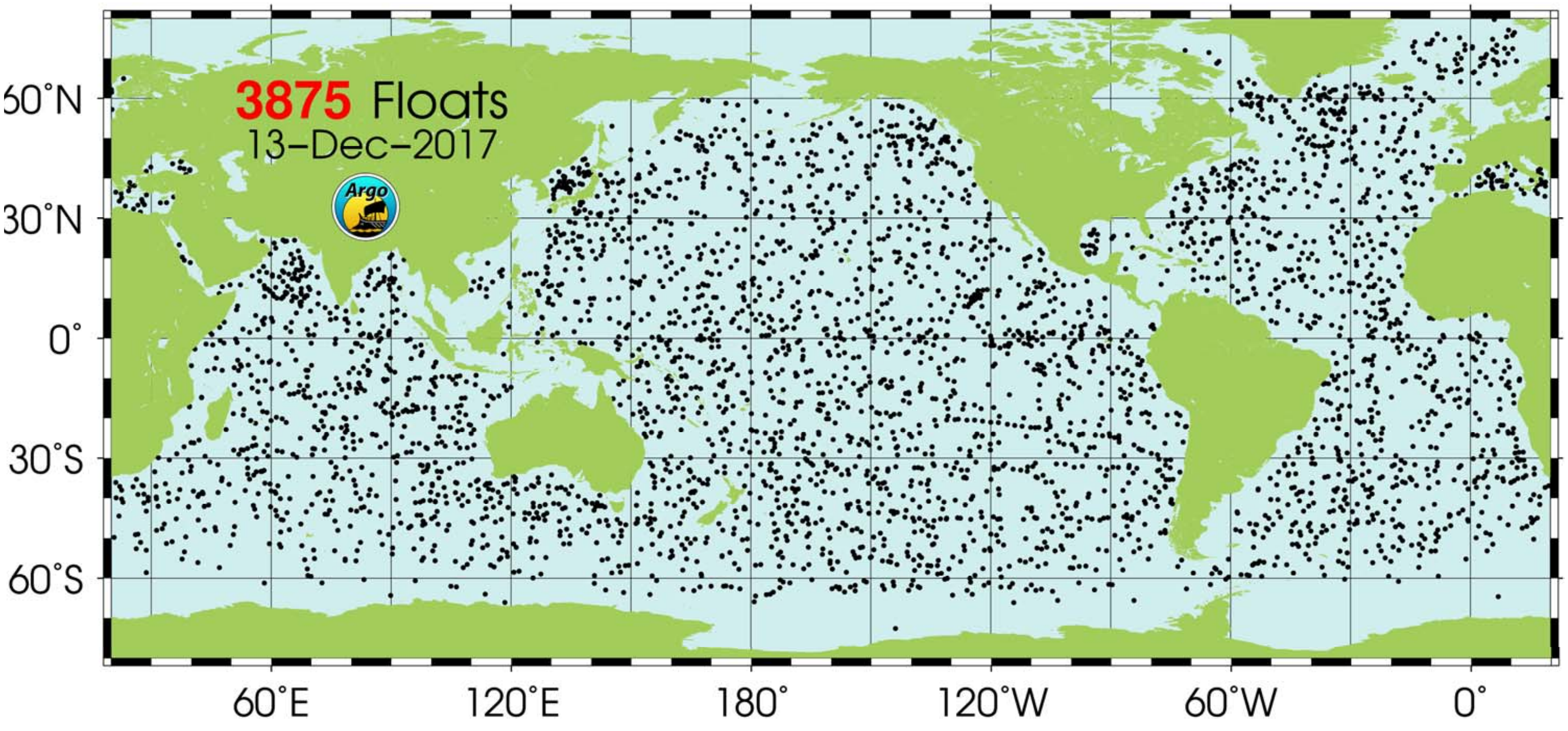
$$70 \text{ cm} > \Delta N > -30\text{cm}$$

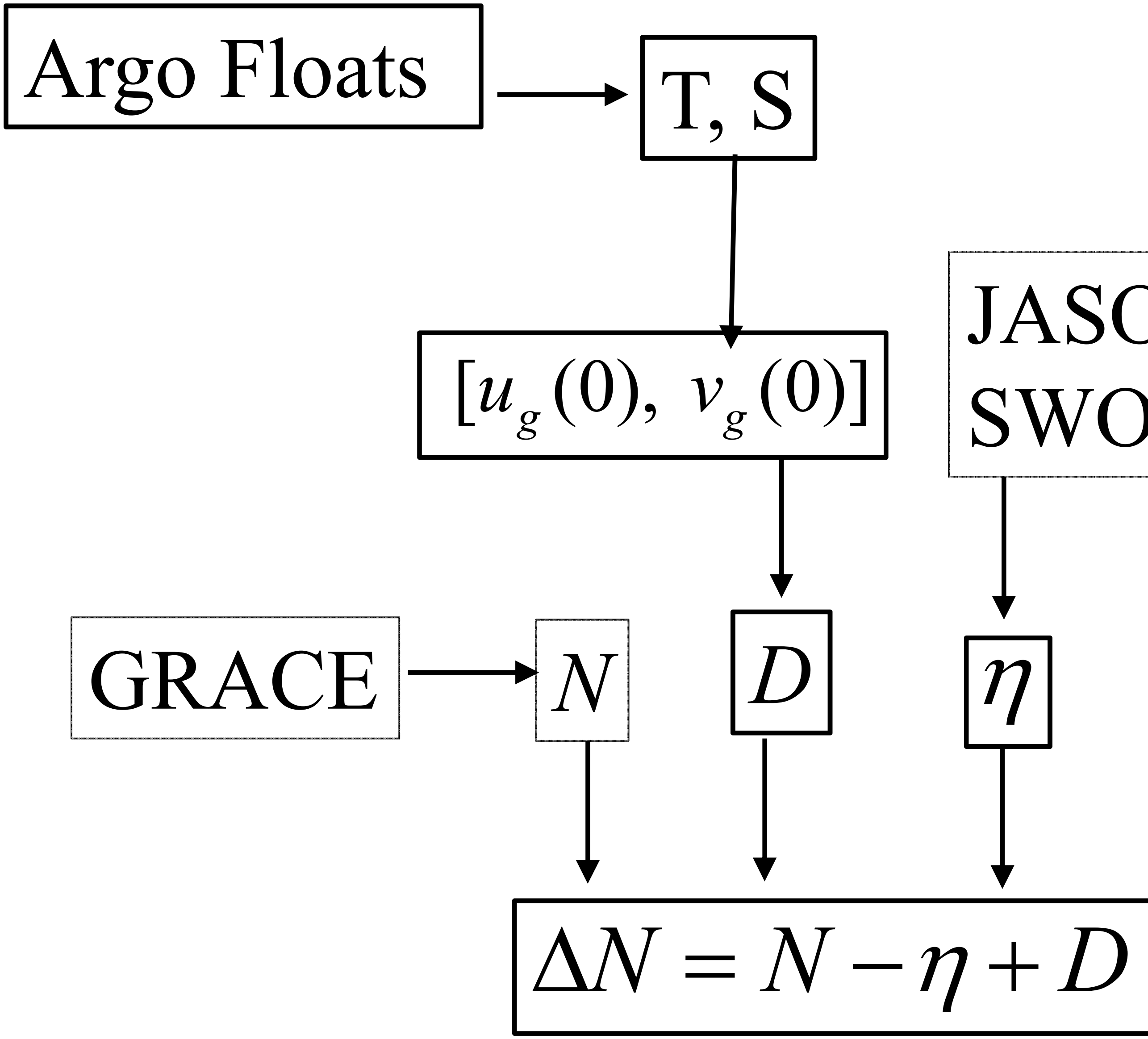
5. Conclusions

- Marine geoid anomaly due to oceanic motion and sea level change is not negligible.
- A new elliptic equation was derived for the marine geoid anomaly.
- Combined space and underwater remote sensing may be important for future marine geodesy.


6 -12 hours at surface
to transmit data to satellite
Total cycle time 10 days
Descent to depth
~10 cm/s (~6 hours)
Salinity & Temperature
profile recorded during ascent
~10 cm/s (~6 hours)
1000 db (1000m)
Drift approx. 9 days
Float descends to begin
profile from greater depth
2000 db (2000m)

Global Argo Floats → (T,S) Profiles





Argo Floats
T, S
$[u_g(0),\ v_g(0)]$
JASON-1, 2,3
SWOT
GRACE
N
D
η
$\Delta N = N - \eta + D$