Hilbert-Huang Transform to Estimate Turbulent Diffusion Coefficient from a Single Lagrangian Drifter Trajectory

Peter C. Chu
Naval Postgraduate School
Monterey, California, USA

Outline

- 1. Introduction
- 2. Hilbert-Huang Transform (HHT)

    Empirical Mode Decomposition (EMD)
- 3. Separation of Deterministic and Stochastic Components
- 4. Determination of (K_x, K_y)
- 5. Conclusions

1. Introduction

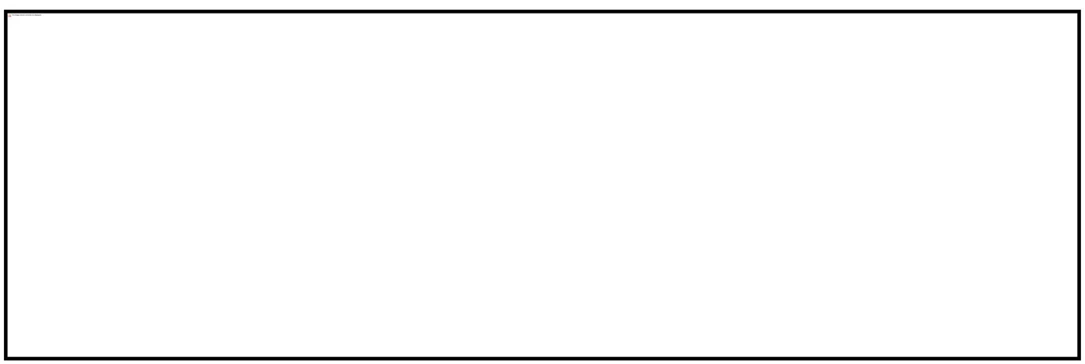
Eddy Diffusivity
Multiple Lagragian Drifters

$$[x_n(m), y_n(m)], \quad [u_n(m), v_n(m)]$$
$$n = 1, 2, \ \dots \ , N; \quad m = 1, 2, \dots, M$$

N → Number of drifters
M → Number of time instances

$$K_x(m) = c\hat{\sigma}_u(m)\hat{\sigma}_x(m)$$
$$K_y(m) = c\hat{\sigma}_v(m)\hat{\sigma}_y(m)$$

Okubo and Ebbesmeyer
(1976 DSR)



Minimum Number of Drifters

$$\hat{\sigma}_x(m) = \sqrt{\frac{1}{(N-1)} \sum_{n=1}^{N} \left[x_n(m) - \overline{x}(m) \right]^2}$$

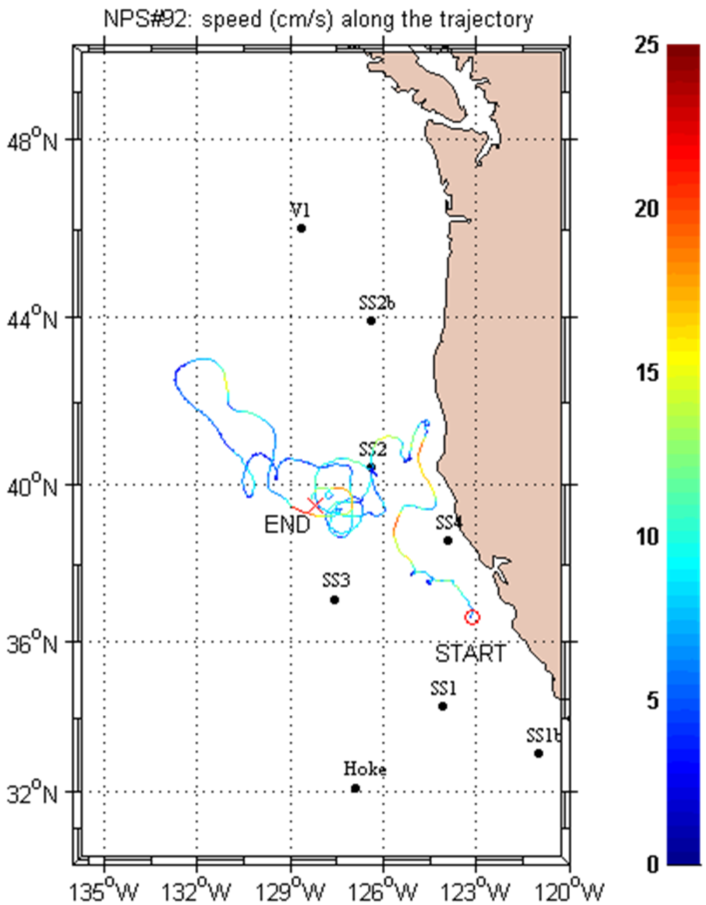
$$\hat{\sigma}_y(m) = \sqrt{\frac{1}{(N-1)} \sum_{n=1}^{N} \left[y_n(m) - \overline{y}(m) \right]^2}$$

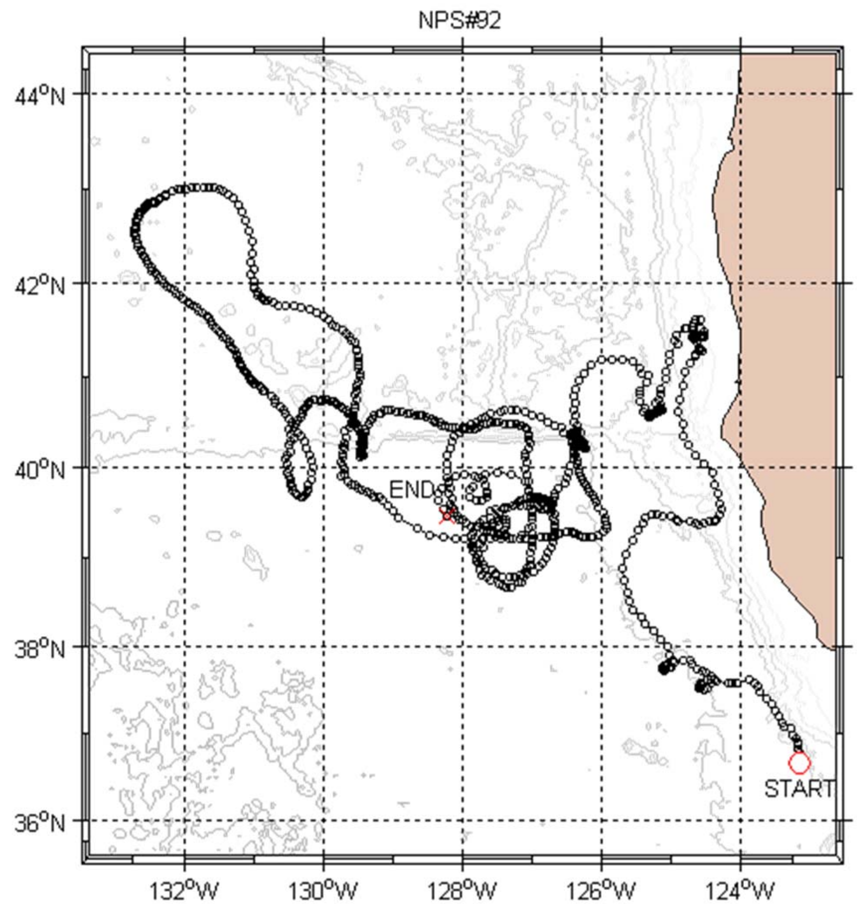
$$\overline{x}(m) = \frac{1}{N} \sum_{n=1}^{N} x_n(m), \;\; \overline{y}(m) = \frac{1}{N} \sum_{n=1}^{N} y_n(m)$$

N > 6 (Okubo and Ebbesmeyer, 1976) → Statistical meaningful

$(\hat{\sigma}_x, \hat{\sigma}_x, \hat{\sigma}_u, \hat{\sigma}_v)$

What Happen if N is much smaller than 6?





NPS RAFOS Subsurface Data: http://www.oc.nps.edu/npsRAFOS/

Can the turbulent eddy diffusivity be determined from a single Lagrangian drifter?

2. Hilbert-Huang Transform (HHT)

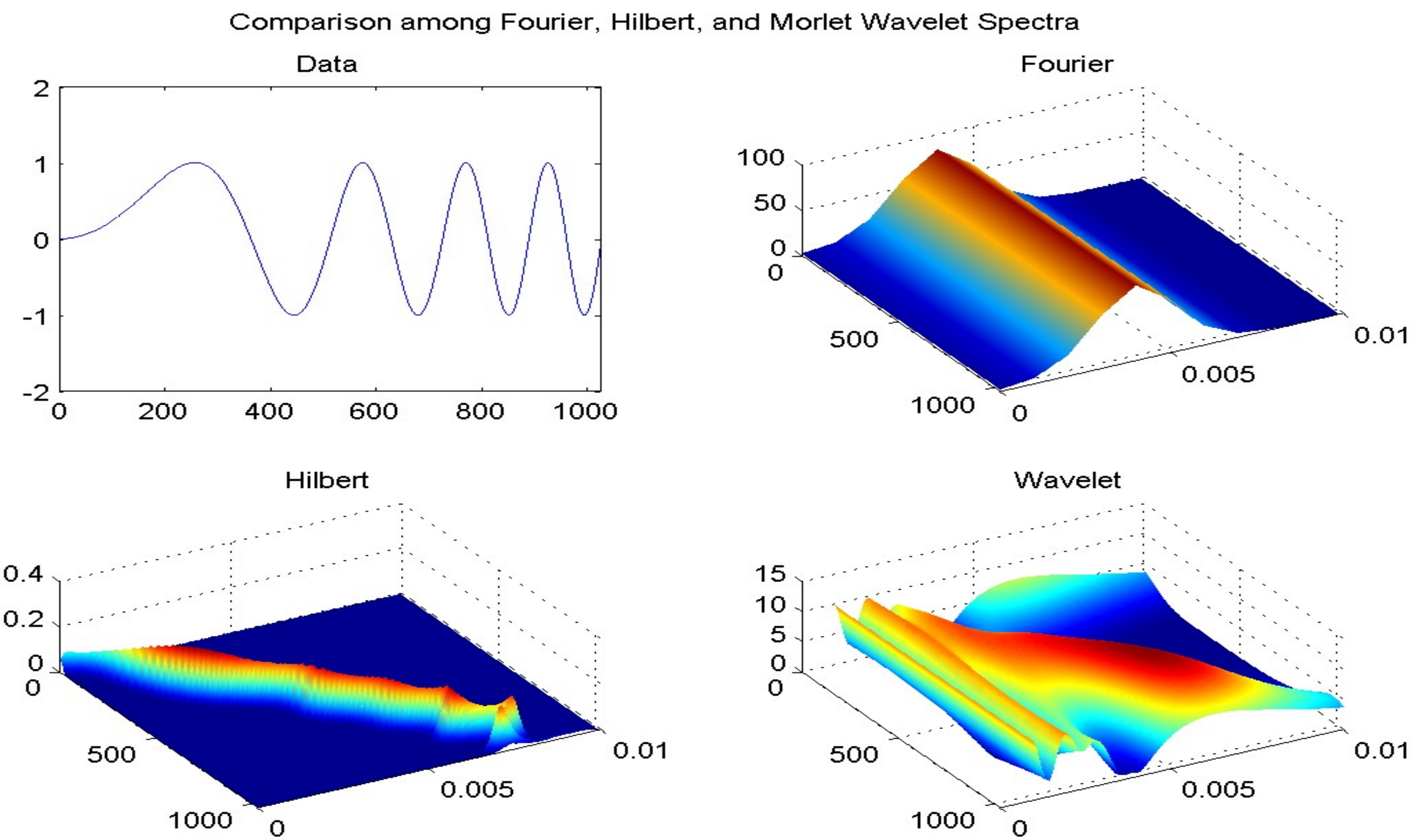
FFT→

$$x(t) = \Re \sum_j a_j e^{i\omega_j t}$$

HHT →

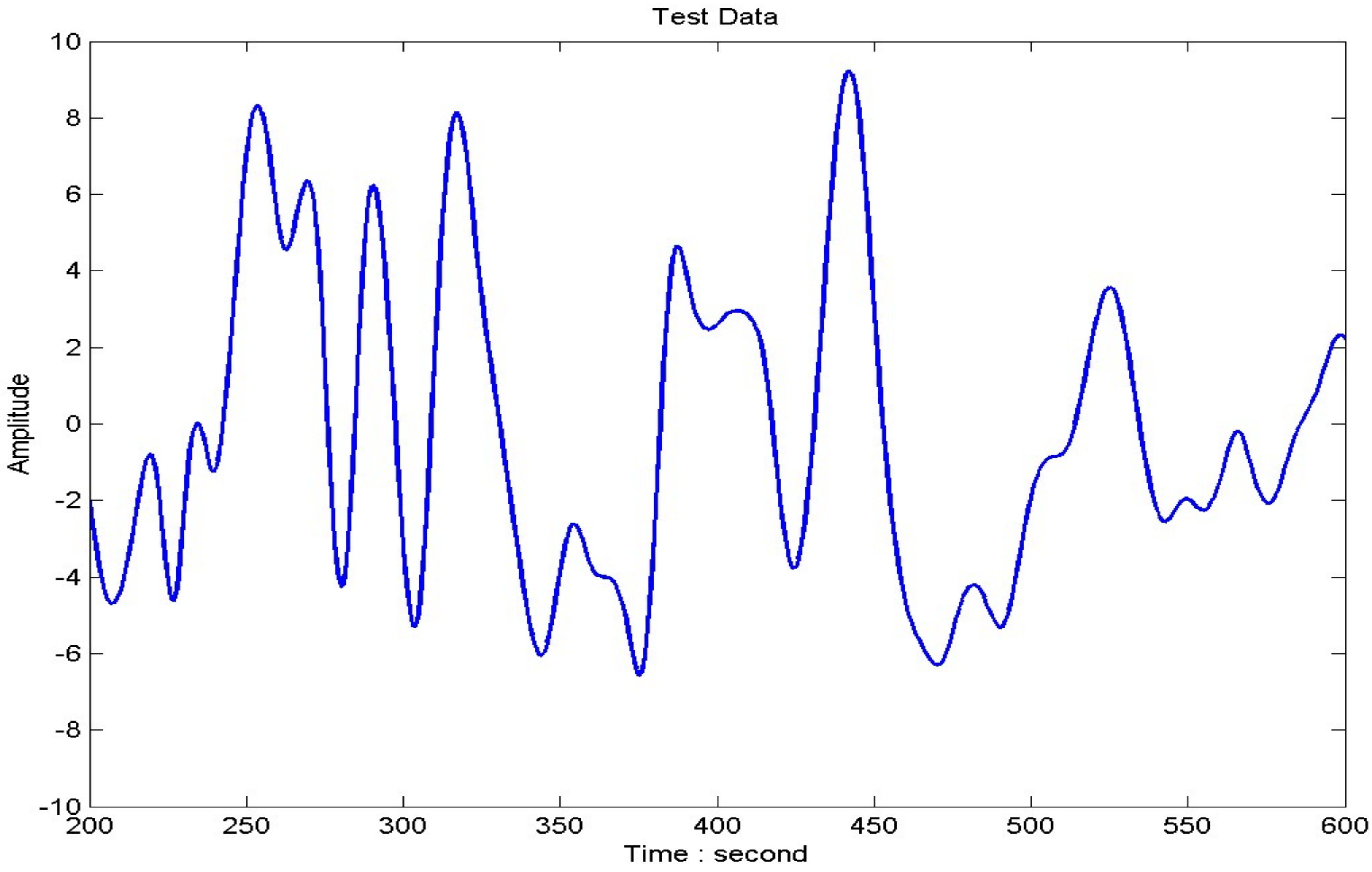
$$x(t) = \Re \sum_j a_j(t)\, e^{i\int_t \omega_j(\tau)\, d\tau}$$

Comparisons:
Fourier, Hilbert & Wavelet



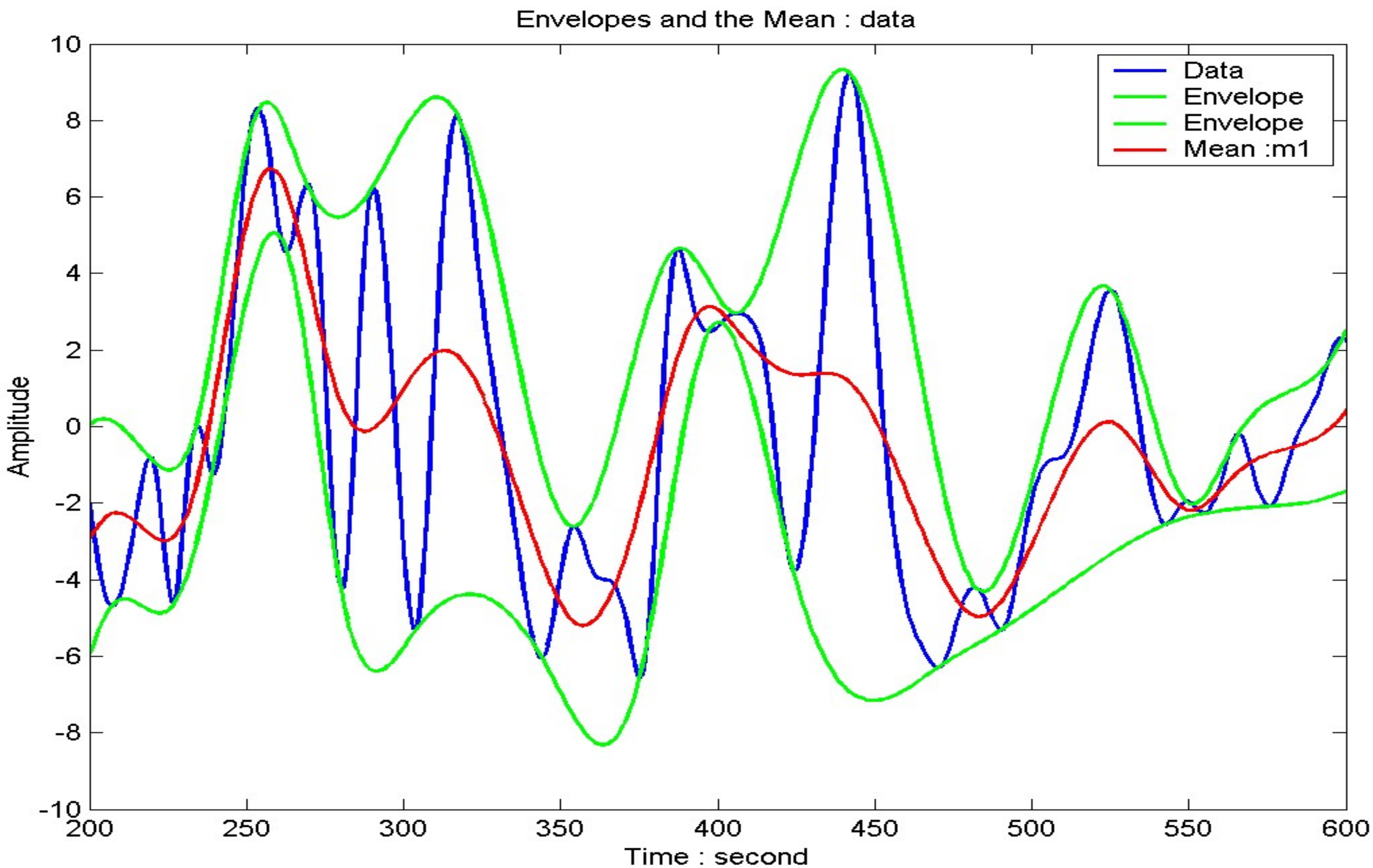
Empirical Mode Decomposition:

Methodology : Test Data



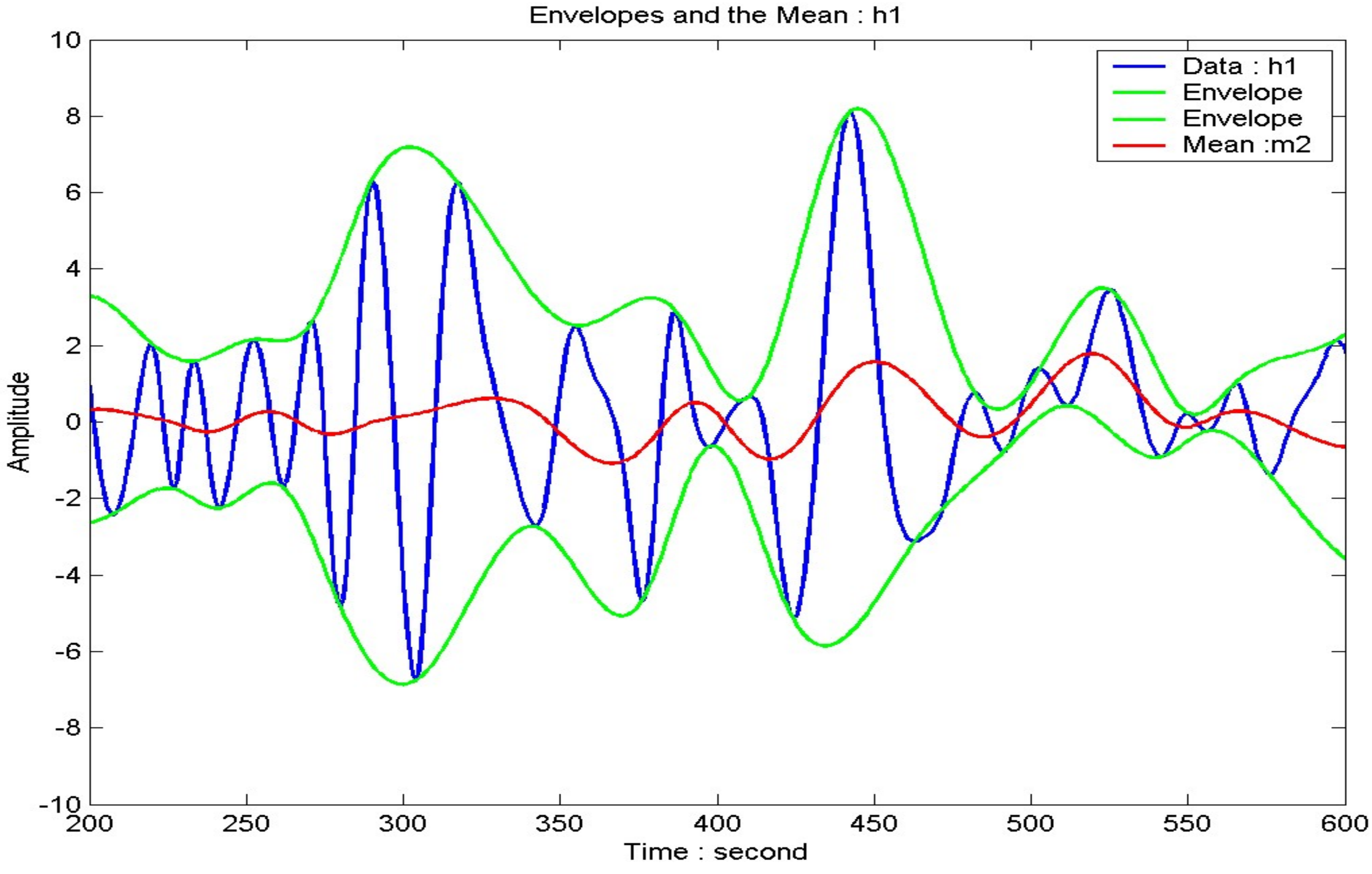
Empirical Mode Decomposition:

Methodology : data and m1



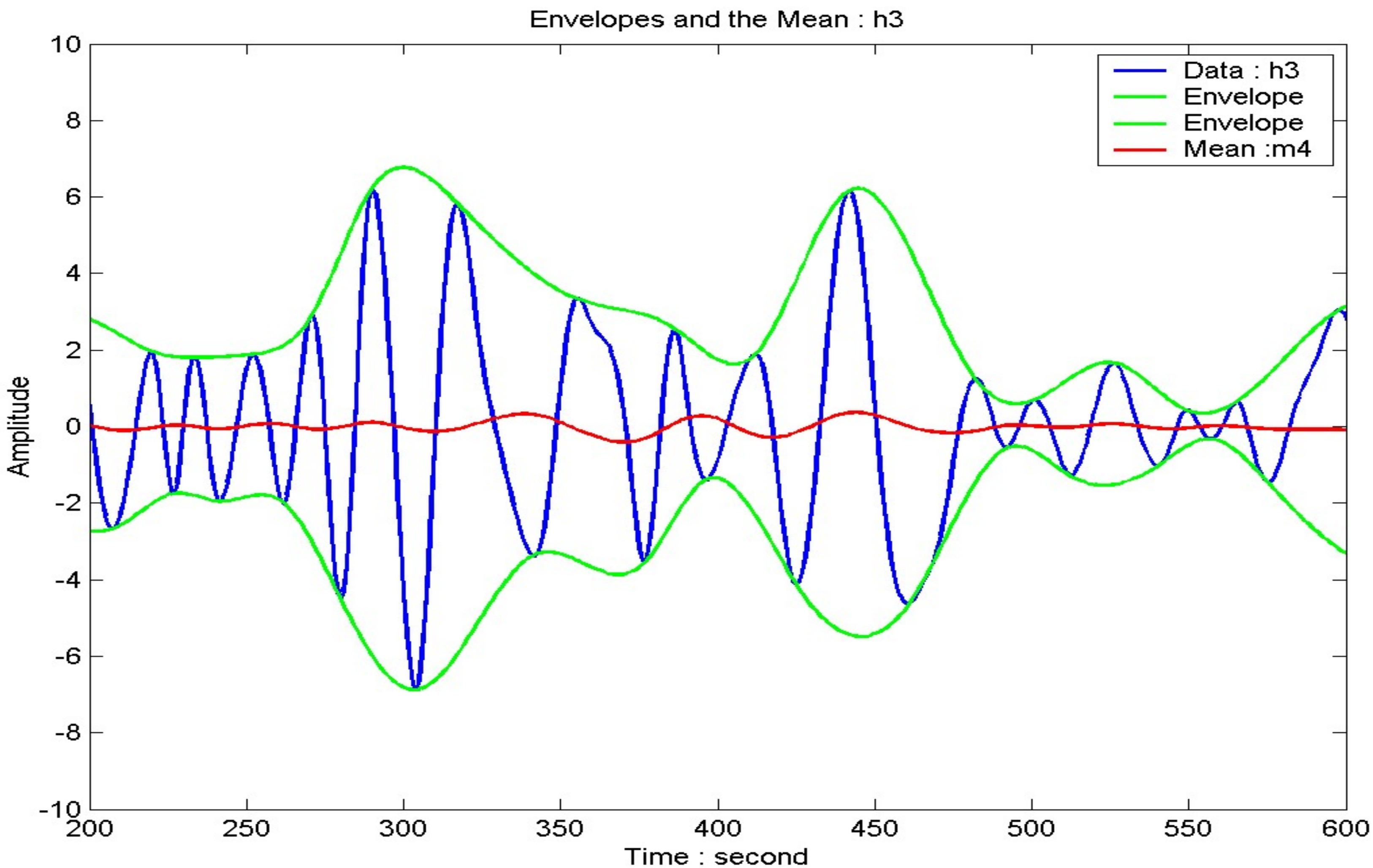
Empirical Mode Decomposition:

Methodology : h1 & m2



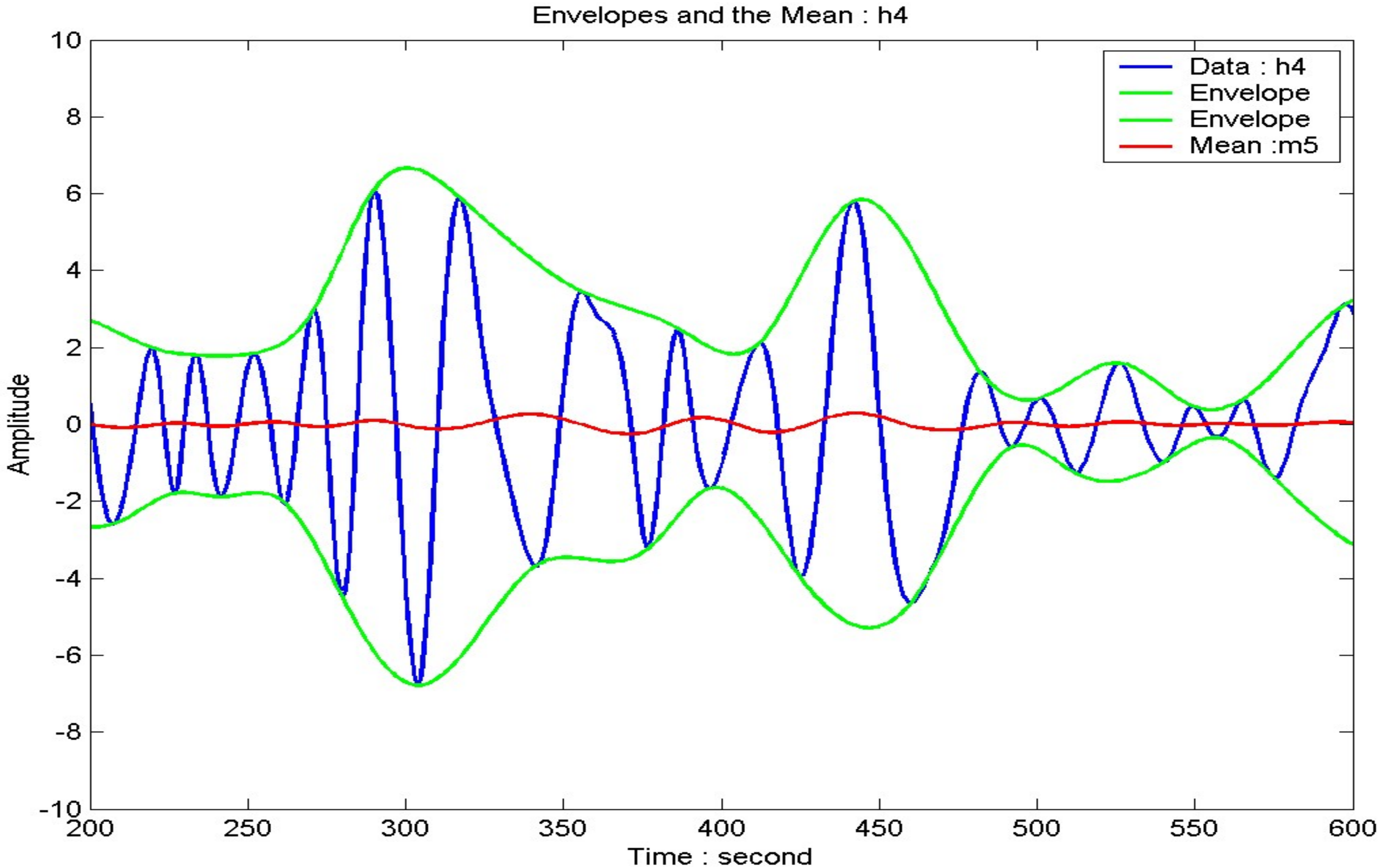
Empirical Mode Decomposition:

Methodology : h3 & m4



Empirical Mode Decomposition:

Methodology : h4 & m5



Empirical Mode Decomposition

Sifting : to get one IMF component

$$x(t) - m_1(t) = x_1(t),$$

$$m_1(t) - m_2(t) = x_2(t),$$

$$\cdots\cdots$$

$$m_{k-1}(t) - m_k(t) = x_k(t).$$

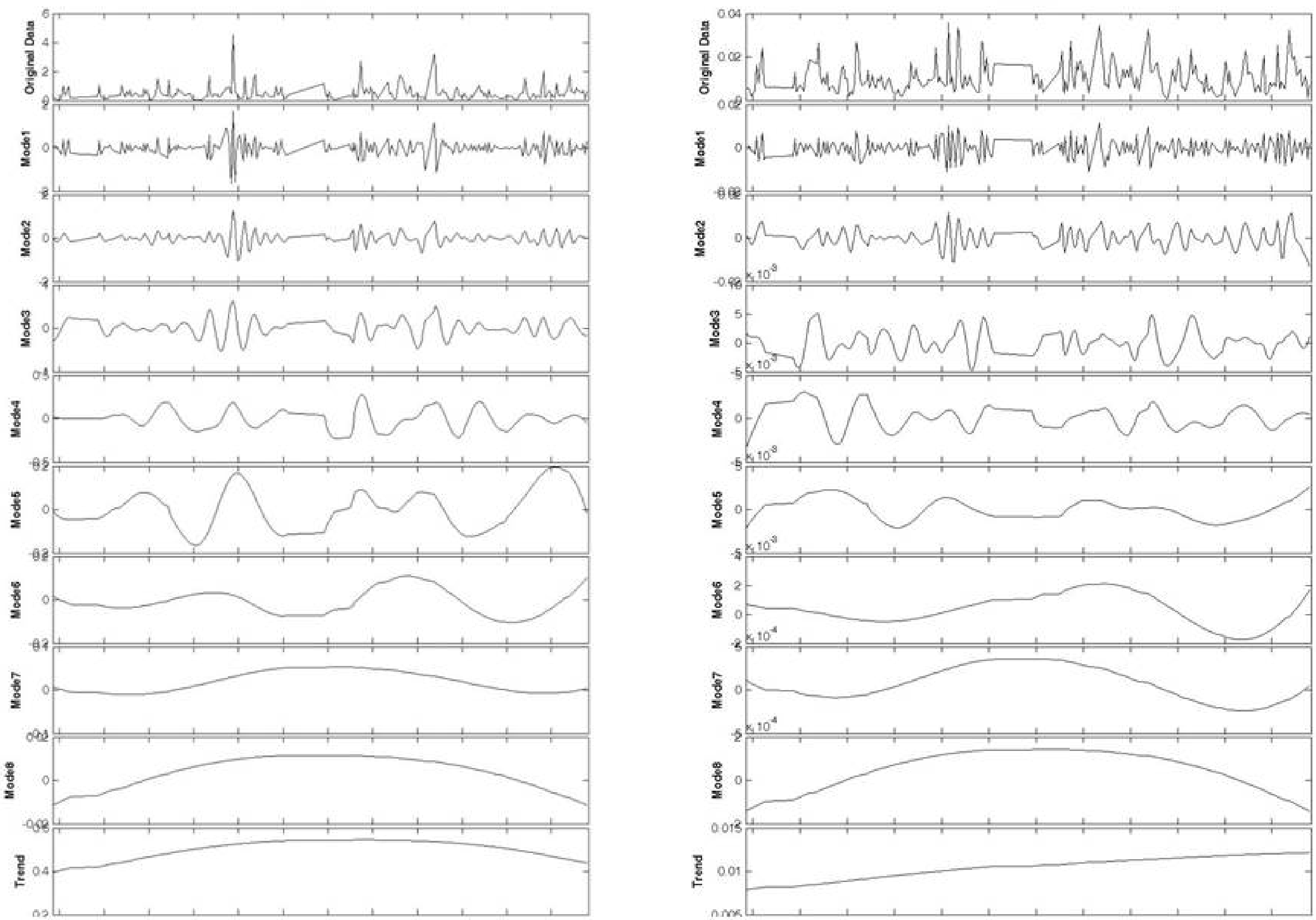
$$\Rightarrow \quad x(t) = m_K(t) + \sum_{k=1}^{K} x_k(t).$$

$x_k(t)$ → Intrinsic Mode Function (IMF)

$m_K(t)$ → Trend

3. Separation of Deterministic and Stochastic Components

EMD of a Single Drifter Trajectory [x(t), y(t)] → Multi-scales



Hilbert Transform on Each IMF → Hilbert-Huang Transform

$$z_k(t) = c_k(t) + \mathrm{i}\hat{c}_k(t), \qquad \mathrm{i} \equiv \sqrt{-1}$$

$$\hat{c}_k(t) = \frac{1}{\pi} CP \int_{-\infty}^{+\infty} \frac{c_k(s)}{t-s} ds,$$

CP is the Cauchy principal value

Instantaneous Amplitude and Frequency

$$z_k(t) = c_k(t) + \mathrm{i}\hat{c}_k(t) = a_k(t)\exp[\mathrm{i}\theta_k(t)],$$

$\omega_k(t) = d\theta_k(t)/dt$ → Instantaneous Frequency

$a_k(t)$ → Instantaneous Amplitude

Separation of Deterministic and Stochastic IMFs

- Rios & de Mello' Method (2016, Signal Processing, 118, 159-176)

- Phase Spectra

$$\theta_k(t) \rightarrow \begin{cases} \text{low frequency} \rightarrow \text{Deterministic} \\ \text{high frequency} \rightarrow \text{Stochastic} \end{cases}$$

4. Determination of (K_x, K_y)

The first 7 IMFs → high frequency

→ Stochastic

The last three IMFs → low frequency

→ Deterministic

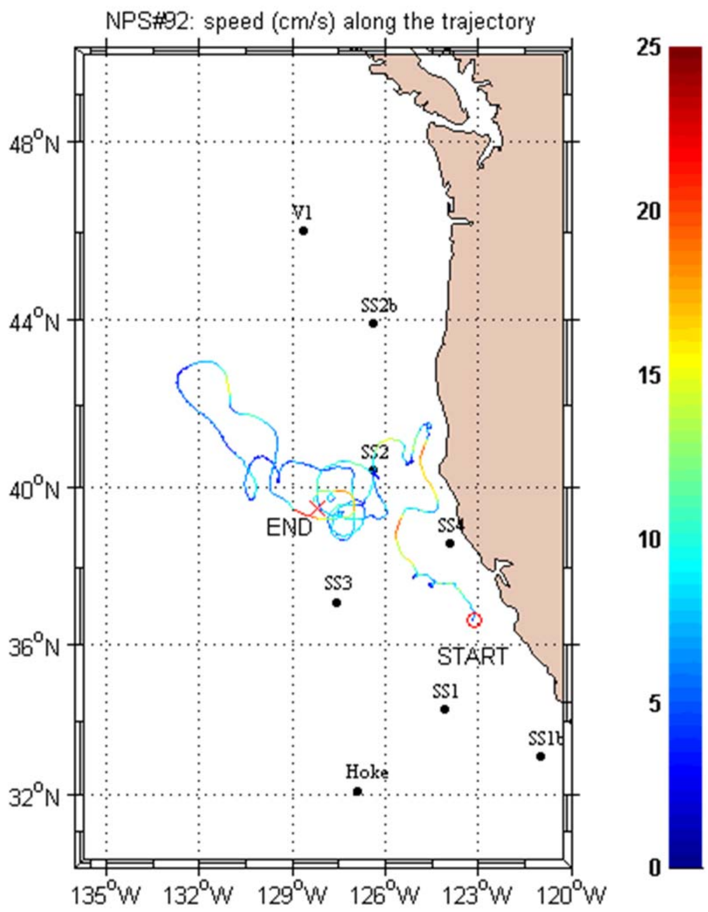
High-Frequency IMFs → Set of "Trajectories"

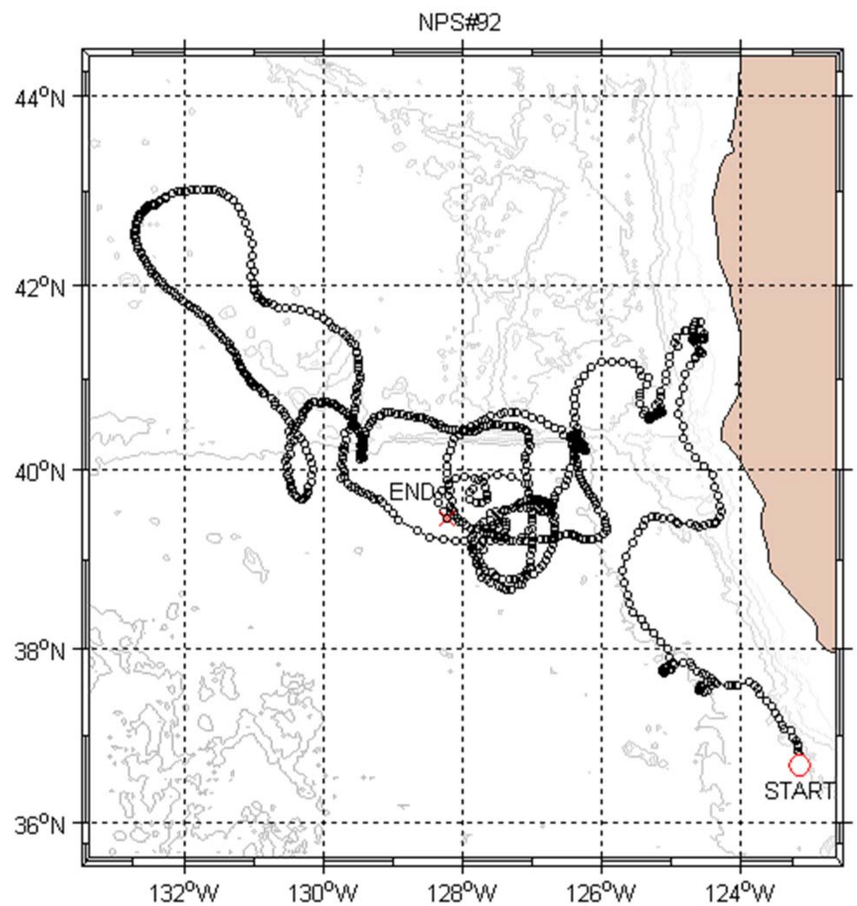
$$[x_n(m), y_n(m)], \quad [u_n(m), v_n(m)]$$
$$n = 1, 2, \ \ldots \ , 7; \quad m = 1, 2, \ldots, M$$

$$K_x(m) = c\hat{\sigma}_u(m)\hat{\sigma}_x(m)$$
$$K_y(m) = c\hat{\sigma}_v(m)\hat{\sigma}_y(m)$$

Temporally averaged (Kx, Ky) are the eddy diffusivities of the single Lagrangian drifter

$$K_x \simeq 1.27 \times 10^7 \ \mathrm{cm}^2/\mathrm{s}, \ K_y \simeq 0.86 \times 10^7 \ \mathrm{cm}^2/\mathrm{s}$$





NPS RAFOS Subsurface Data: http://www.oc.nps.edu/npsRAFOS/

4. Conclusions

- Turbulent eddy diffusivity can be identified from a single Lagragian drifter.
- EMD is a simple and effective method.
- Other methods such as Wavelet, S-transform may also be used.