Determination of mean dynamic topography (MDT) using the minimum energy state

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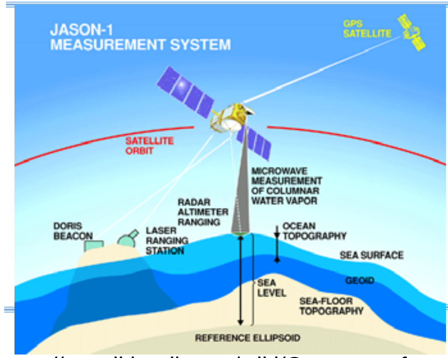
1. Introduction

Geostrophic balance represents the minimum energy state in linear Boussinesq primitive equations with conservation of potential vorticity. On the base of that, an elliptic equation is derived for the absolute dynamic topography (η) with H the water depth, g the gravitational acceleration, and coefficients (B, C) determined by temperature (T) and salinity (S).



https://en.wikipedia.org/wiki/Ocean_surface_topography#/media/File:Jason-1_measurement_system.gif

2. Geostrophic balance

$$u_g(z) = -\frac{g}{f}\frac{\partial \eta}{\partial y} + u_{BC}, \quad v_g(z) = \frac{g}{f}\frac{\partial \eta}{\partial x} + v_{BC}$$

$$u_{BC} = -\frac{g}{f\rho_0}\int_z^0 \frac{\partial \hat{\rho}}{\partial y} dz', \quad v_{BC} = \frac{g}{f\rho_0}\int_z^0 \frac{\partial \hat{\rho}}{\partial x} dz'$$

η → Dynamic ocean topography (DOT)

3. Minimum mechanical energy

$$\eta_x = \partial \eta / \partial x, \quad \eta_y = \partial \eta / \partial y$$

The total mechanical energy E

$$E(\eta, \eta_x, \eta_y) = \frac{g^2 \rho_0}{2} \iiint_\Omega \left[\begin{array}{l} (-\eta_y + \frac{f u_{BC}}{g})^2 / f^2 \\ +(\eta_x + \frac{f v_{BC}}{g})^2 / f^2 \\ +\frac{\rho^2}{\rho_0^2 N^2} + \frac{\eta^2}{gH} \end{array} \right] d\Omega$$

reaches the minimum state as the velocity takes he geostrophic velocity (u_g, v_g) subject to a given potential vorticity (Gill, 1982)

4. Elliptic equation for η

$$E(\eta, \eta_x, \eta_y) = \iint_R L(\eta, \eta_x, \eta_y) dxdy \to \min$$

$$\frac{\partial L}{\partial \eta} - \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \eta_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial L}{\partial \eta_y}\right) = 0$$

$$\boxed{-f^2 \nabla \bullet \left(\frac{H}{f^2}\nabla \eta\right) + \frac{f^2}{g}\eta = \left(\frac{\partial C}{\partial x} - \frac{\partial B}{\partial y}\right)}$$

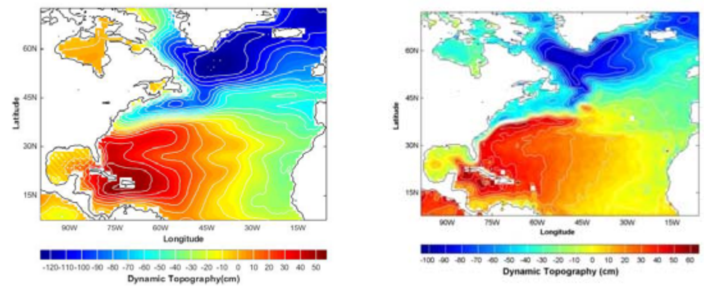
$$\nabla \equiv \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}$$

$$\alpha_1 \mathbf{n} \bullet \nabla \eta + \alpha_2 \eta = 0, \quad \text{at boundary } \Gamma$$

$$B(x,y) \equiv \frac{f}{g}\int_{-H}^{0} u_{BC} dz = -\frac{1}{\rho_0}\int_{-H}^{0}\int_{z}^{0} \frac{\partial \hat{\rho}}{\partial y} dz' dz$$

$$C(x,y) \equiv \frac{f}{g}\int_{-H}^{0} v_{BC} dz = \frac{1}{\rho_0}\int_{-H}^{0}\int_{z}^{0} \frac{\partial \hat{\rho}}{\partial x} dz' dz$$

5. Mean dynamic topography



Climatological mean dynamic topography η (cm) from (left panel) the solution of the elliptic equation with coefficients (B, C) calculated from the WOA annual mean (T, S) data and H from the NOAA ETOPO5, and (right panel) the difference between the time-averaged SSH and the geoid from the NASA/JPL website: http://gracetellus.jpl.nasa.gov/data/dot/.

6. Summary

On the theoretical base of minimum energy state of the geostrophic velocity with given potential vorticity, an elliptic partial differential equation (η-equation) for the dynamic topography (η) is derived with the coefficients depending on temperature (T), salinity (S), and water depth (H). The η-equation provides a link between the mean sea surface height (SSH) and the geoid.

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