Multifractal Thermal Characteristics of the Southwestern GIN Sea Upper Layer

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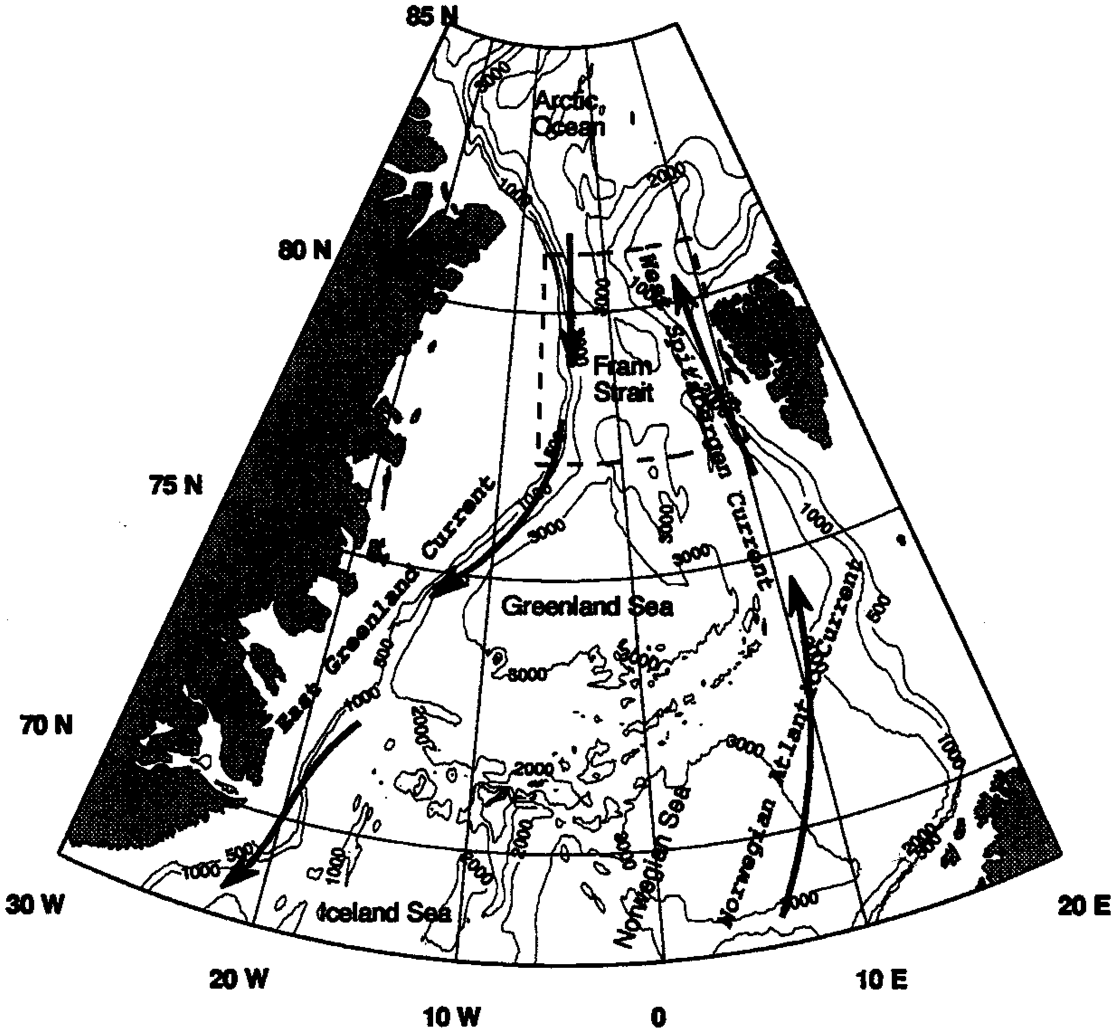
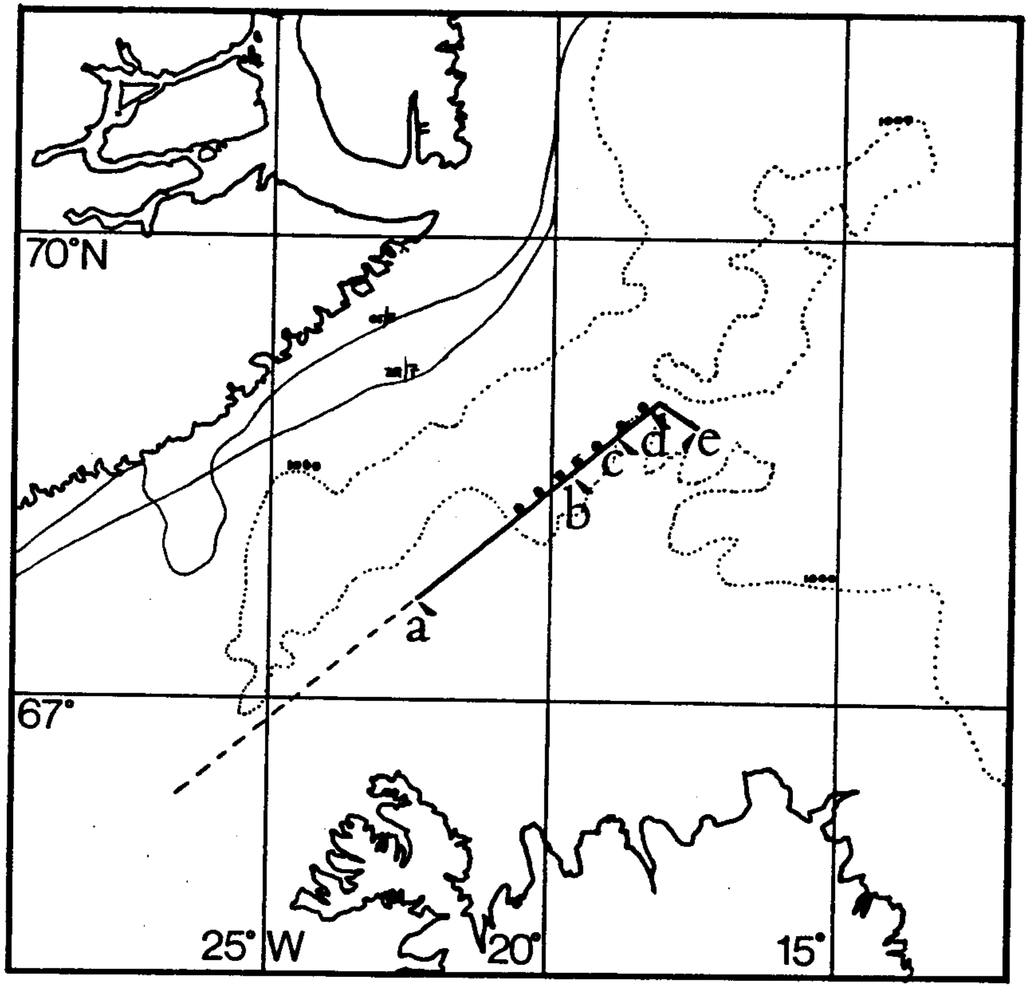
Reference

- Chu, P.C., 2004: Multifractal thermal characteristics of the southwestern GIN Sea upper layer, the journal "Chaos, Solitons and Fractals", 19 (2), 275-284.

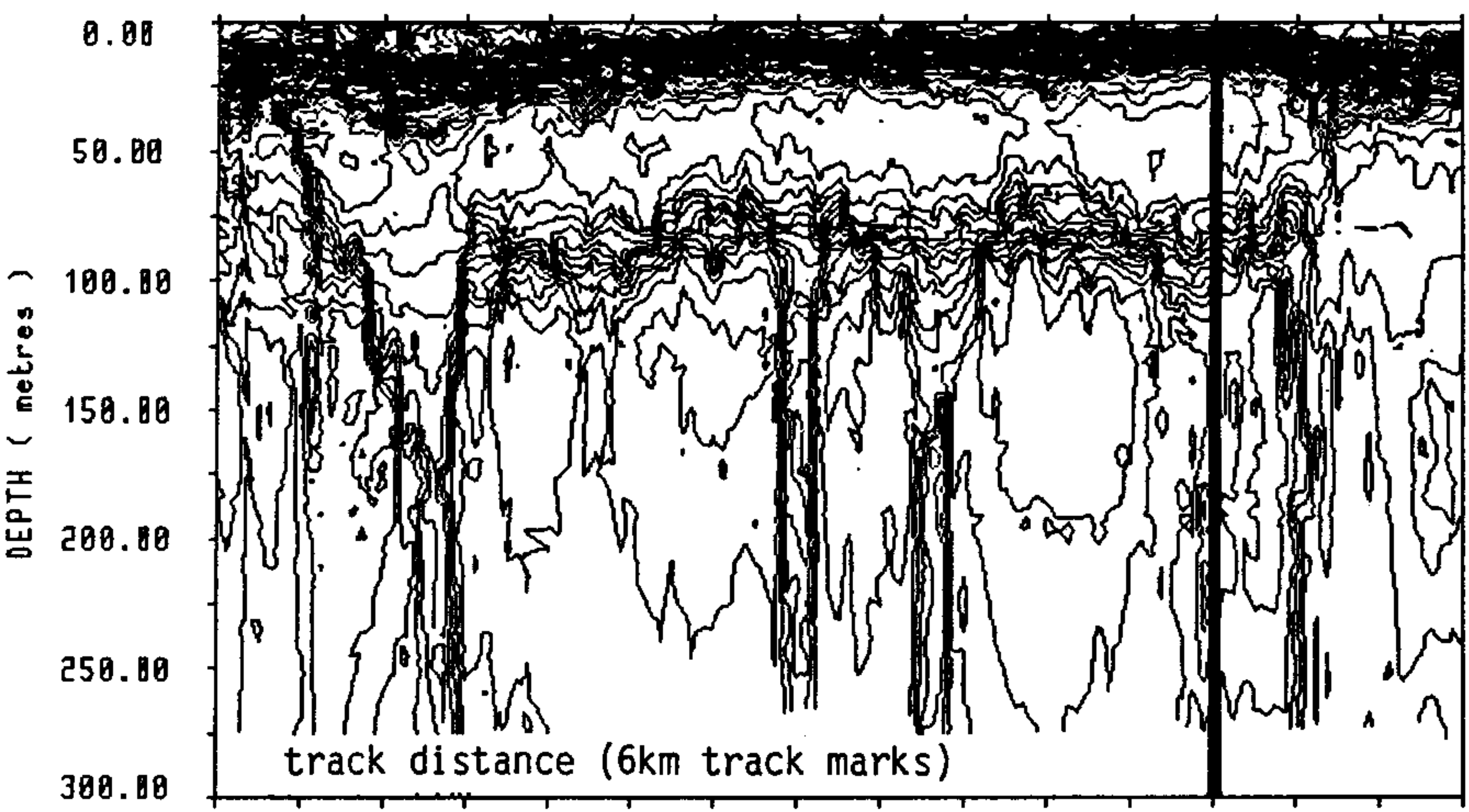
Greenland-Icelandic-Norwegian (GIN) Sea



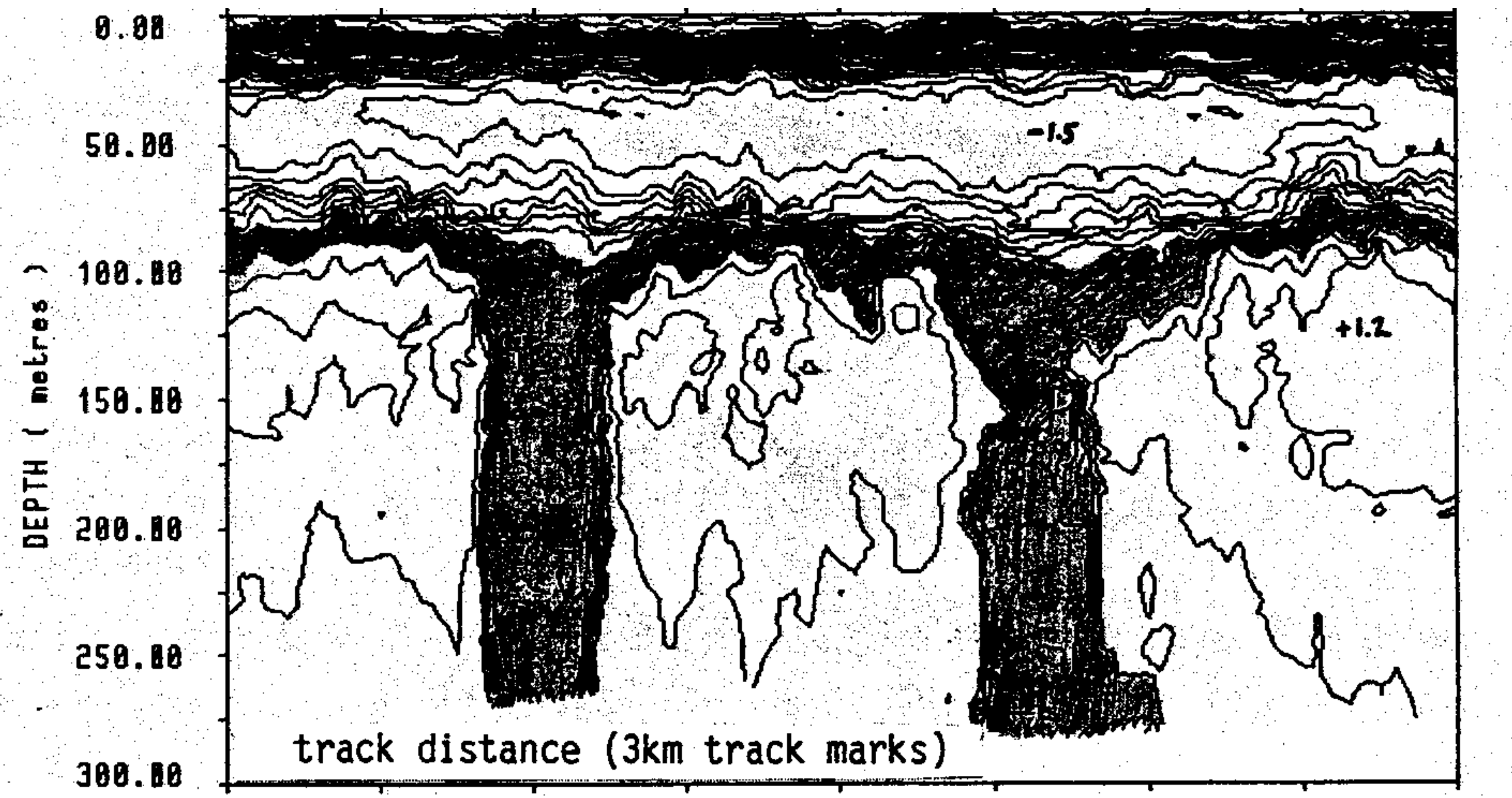
Thermister Chain



Temperature Field



Temperature Field (zoomed in)



Power Spectrum

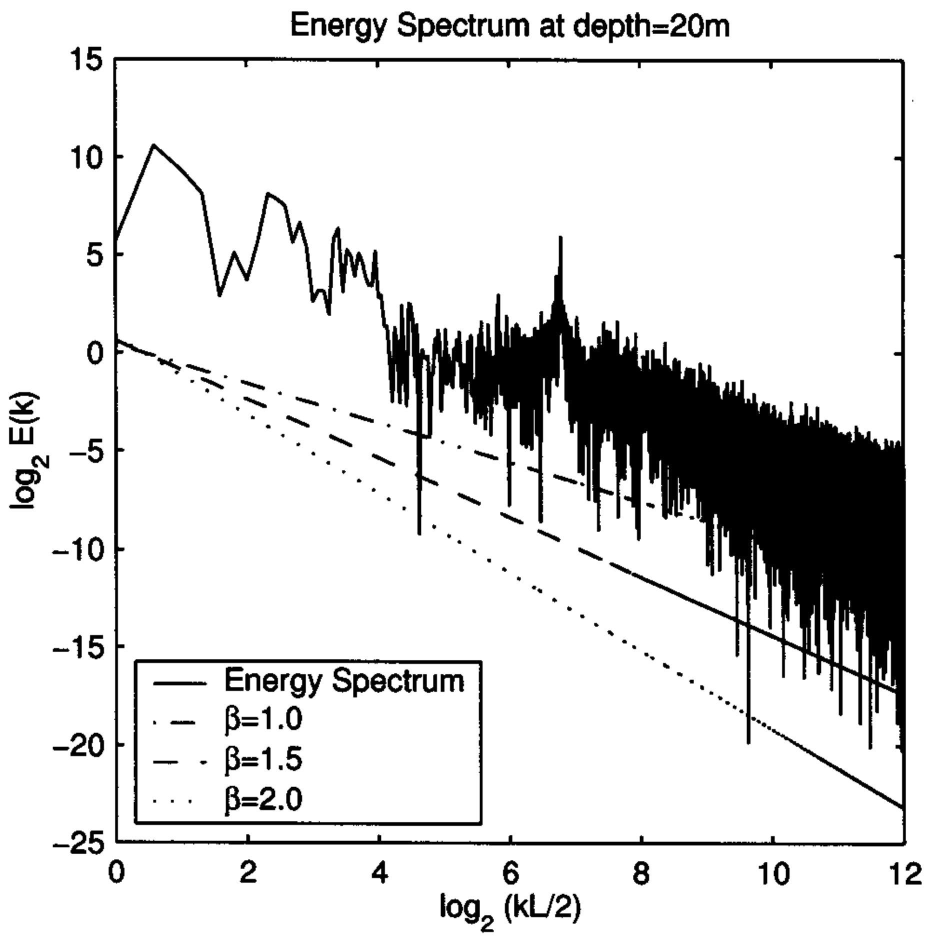
$$E(k) \propto k^{-\beta},$$

$$1 < \beta < 2,$$

Spike at

$$Log_2(kL/2) \simeq 6.5,$$

Corresponding to Chimney Scale ~ 3 km



Structure Function (1)

$$|\Delta T(r;x)| = |T(x_{i+r}) - T(x_i)|, \quad i = 0,1,\ldots,\Lambda - r,$$

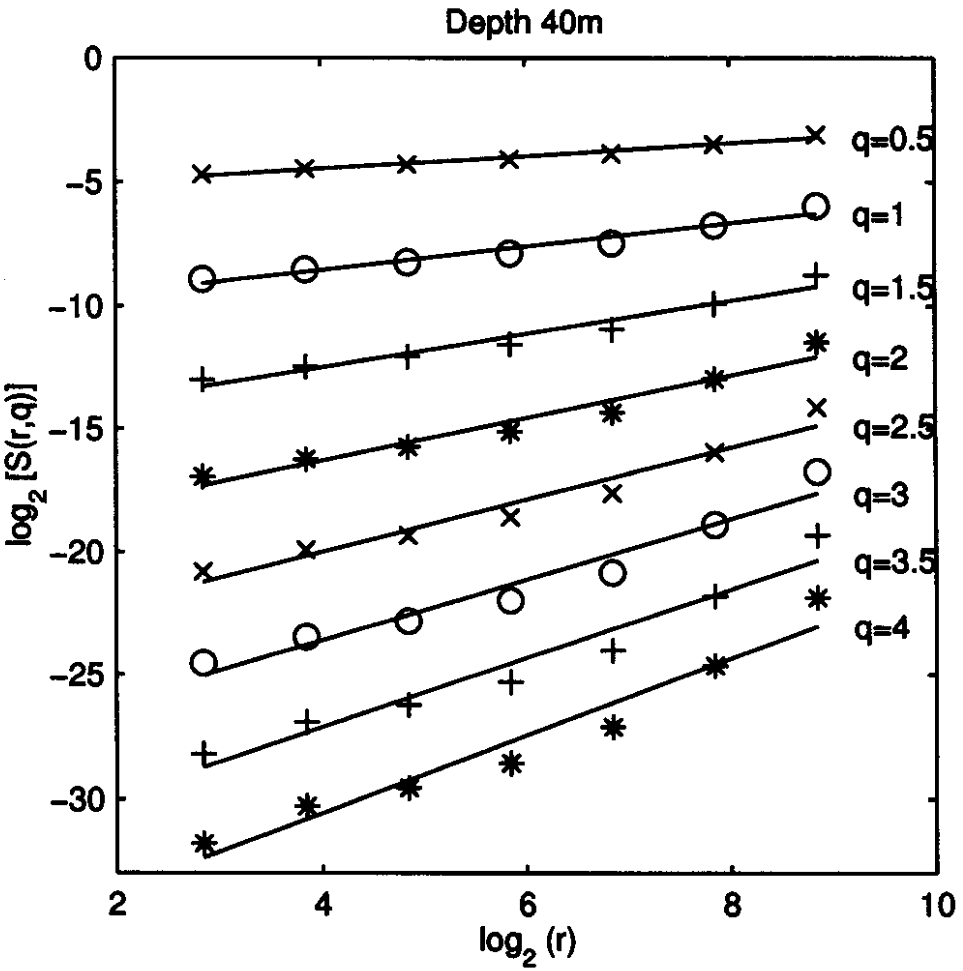
$$S(r,q) \equiv \left\langle |\Delta T(r;x)|^q \right\rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} |\Delta T(r;x)|^q .$$

Here, r is the lag, q is the order of the structure function

$$S(1,1) = \frac{1}{\Lambda - 1} \sum_{i=0}^{\Lambda - 1} |T(x_{i+1}) - T(x_i)|$$

S(1,1) is the mean gradient .

Structure Function (Power Law)



$$S(r,q) \propto r^{\zeta(q)},$$

$$\zeta(q) = H(q)q,$$

Dimension of GIN Sea Temperature Field

$$D_{g(T)} = 2 - H_1 .$$

Stationary

$$S(r,1) = const\ ,$$

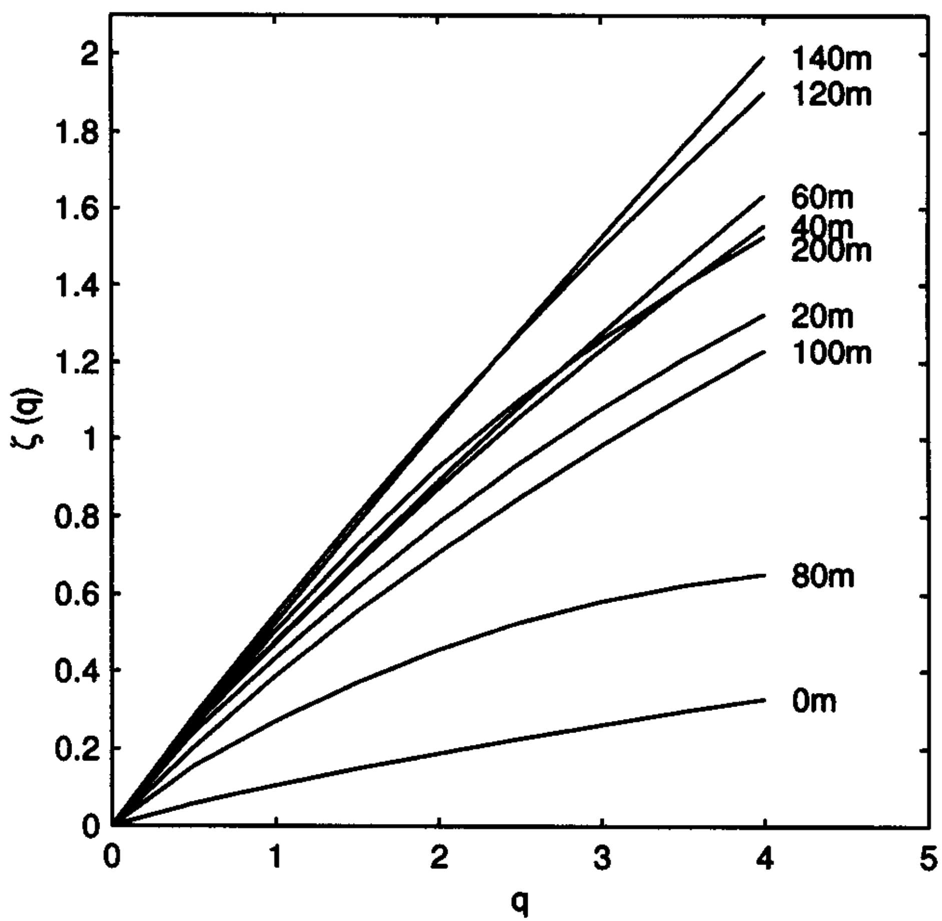
$$\zeta(1) = H(1) = 0\,.$$

$$D_{g(T)} = 2,$$

Power of the structure function S(r, 1) and the dimension of GIN Sea temperature field

Depth (m)	0	20	40	60	80	100	120	140	200
$\zeta(1) = H_1$	0.11	0.43	0.47	0.48	0.28	0.40	0.56	0.52	0.50
$D_{g(T)}$	1.89	1.57	1.53	1.52	1.78	1.60	1.44	1.48	1.50

Dependence of structure function's power ζ(q) on *q* and depth



Normalized Small Scale Absolute Gradient and Running Mean

$$\left\langle \left| \Delta T(1;x_i) \right| \right\rangle = \frac{1}{\Lambda} \sum_{i=0}^{\Lambda-1} \left| \Delta T(1;x_i) \right|,$$

$$\varepsilon(1;x_i) = \frac{\left| \Delta T(1;x_i) \right|}{\left\langle \left| \Delta T(1;x_i) \right| \right\rangle},$$

$$\varepsilon(r;x_i) = \frac{1}{r} \sum_{j=i}^{i+r-1} \varepsilon(1;x_j), \qquad i = 0,1,\ldots,\Lambda - r .$$

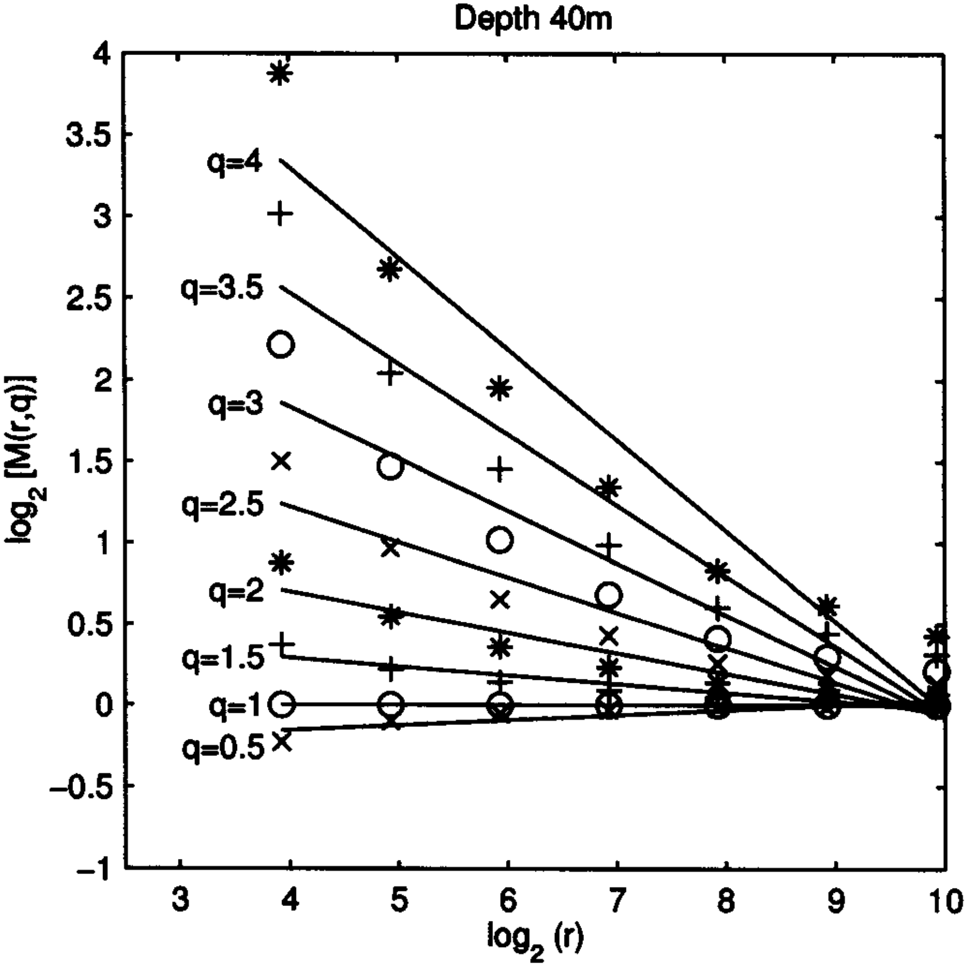
Singular Measure

$$M(r,q) \equiv \left\langle \varepsilon(r;x_i)^q \right\rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} [\varepsilon(r;x_i)]^q ,$$

$$M(r,1) \equiv \left\langle \varepsilon(r;x_i) \right\rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} [\varepsilon(r;x_i)]$$

$$= \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} \left[\frac{1}{r} \sum_{j=i}^{i+r-1} \varepsilon(1;x_j) \right] = 1.$$

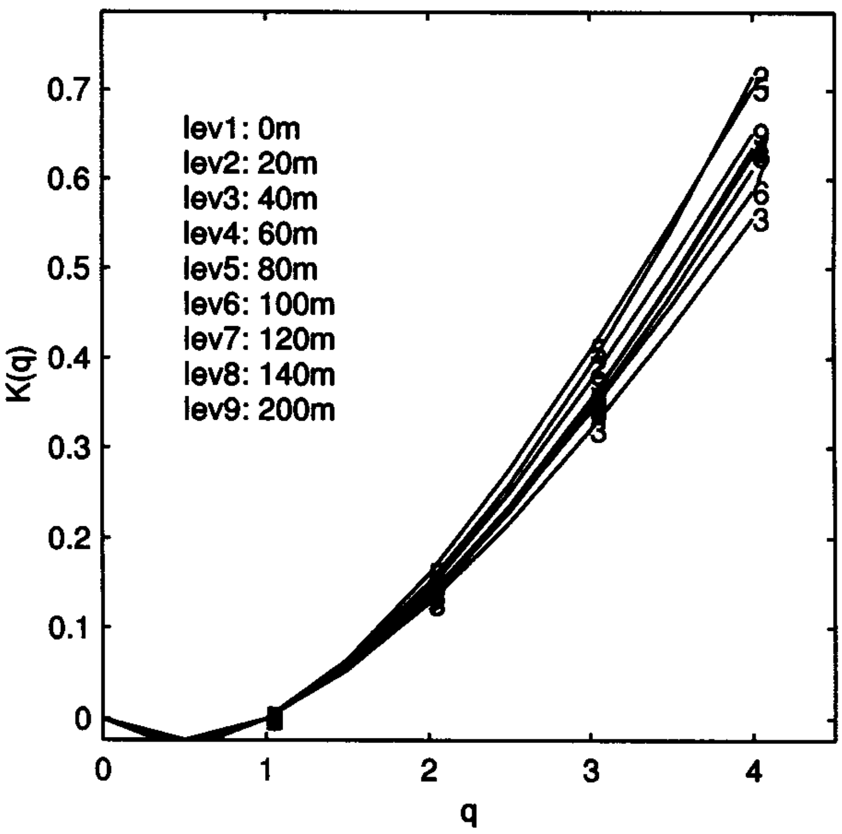
Singular Measure (Power Law)



Power of the Singular Measure

$$M(r,q) \propto r^{-K(q)}, \quad q \geq 0,$$

$$K(0) = K(1) = 0.$$



Characteristics of the Power of the Singular Measure

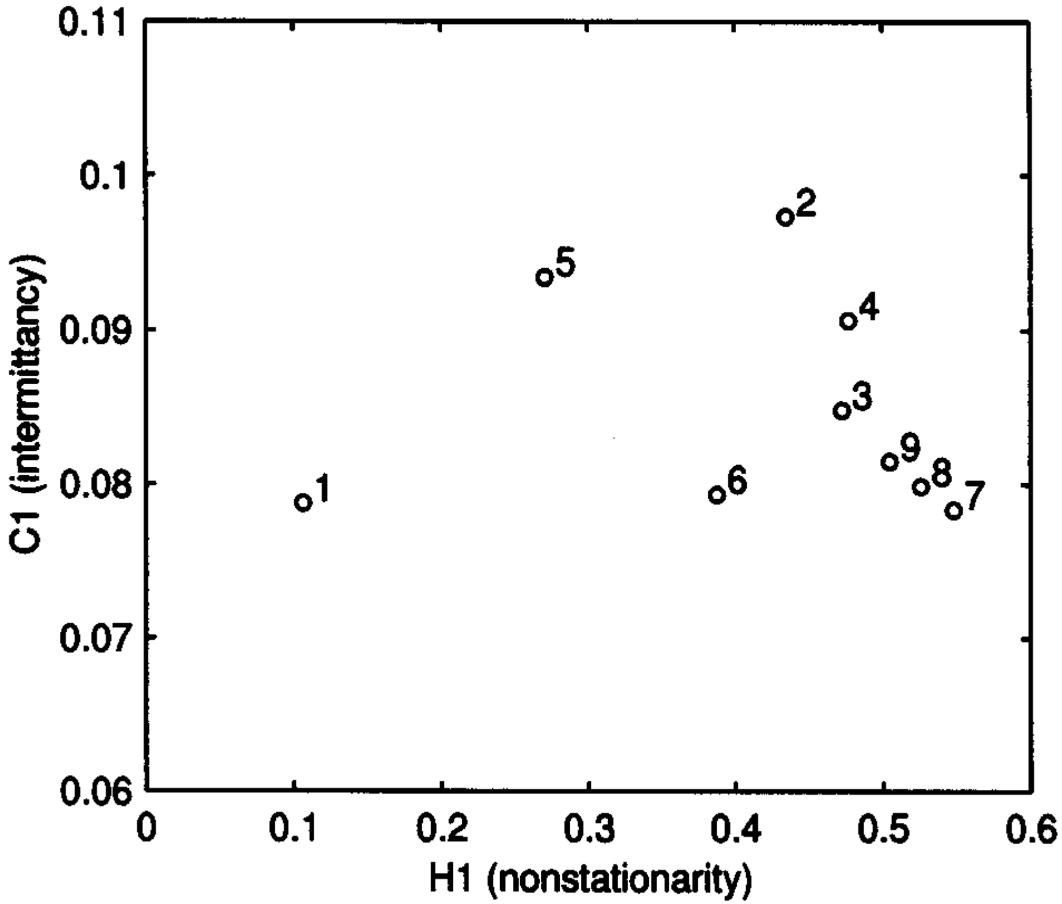
$$\frac{d^2K(q)}{dq^2} > 0,$$

$$K(q) < 0 \quad \text{only if} \quad 0 < q < 1$$

$$C(q) = \frac{K(q)}{q-1}.$$

$$C_1 \equiv C(1) = K'(1) \geq 0,$$

Mean Multifractal Plane (number representing level: 1 ~ 0 m, …, 9 ~ 200m)



Conclusions

- (1) Chimney width is aound 3 km.
- (2) GIN Sea sublayer is nonstationary.
- (3) The structure function has multifractal characteristics.