Near-Surface Ocean Currents

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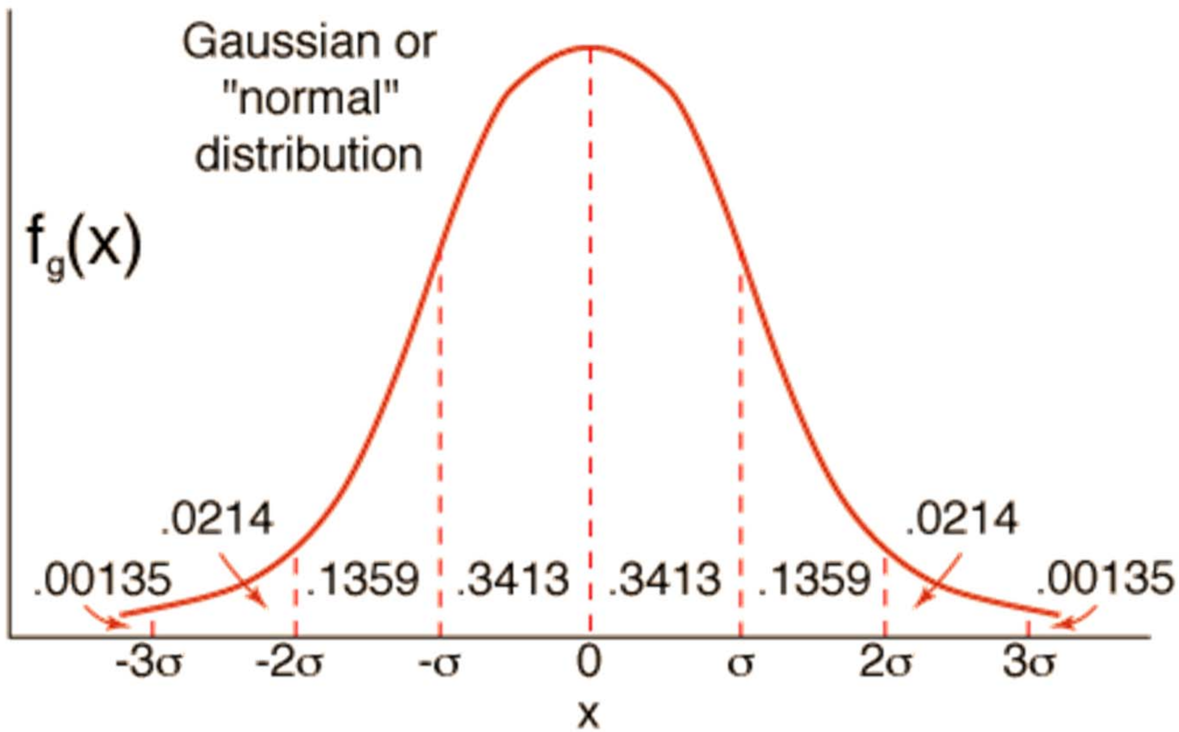
- Magnitude of Near Surface Currents

    10 cm/s

- What are the statistical features?

Dealing with Uncertainty

- Gaussian Distribution is often used



Gaussian Distribution

- Mean = μ
- Standard Deviation = σ
- Skewness = 0
- Kurtosis – 3 = 0

$$p(x) = \frac{1}{\sqrt{2\pi\,\sigma^2}} \cdot \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$$

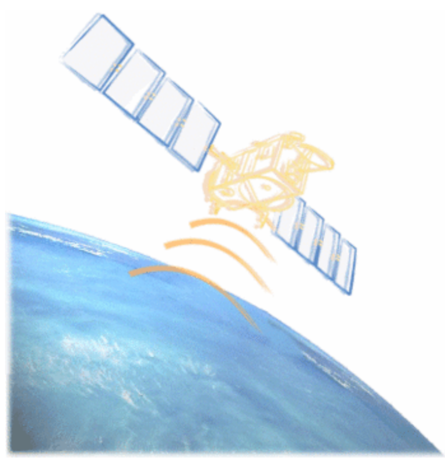
Gaussian Process

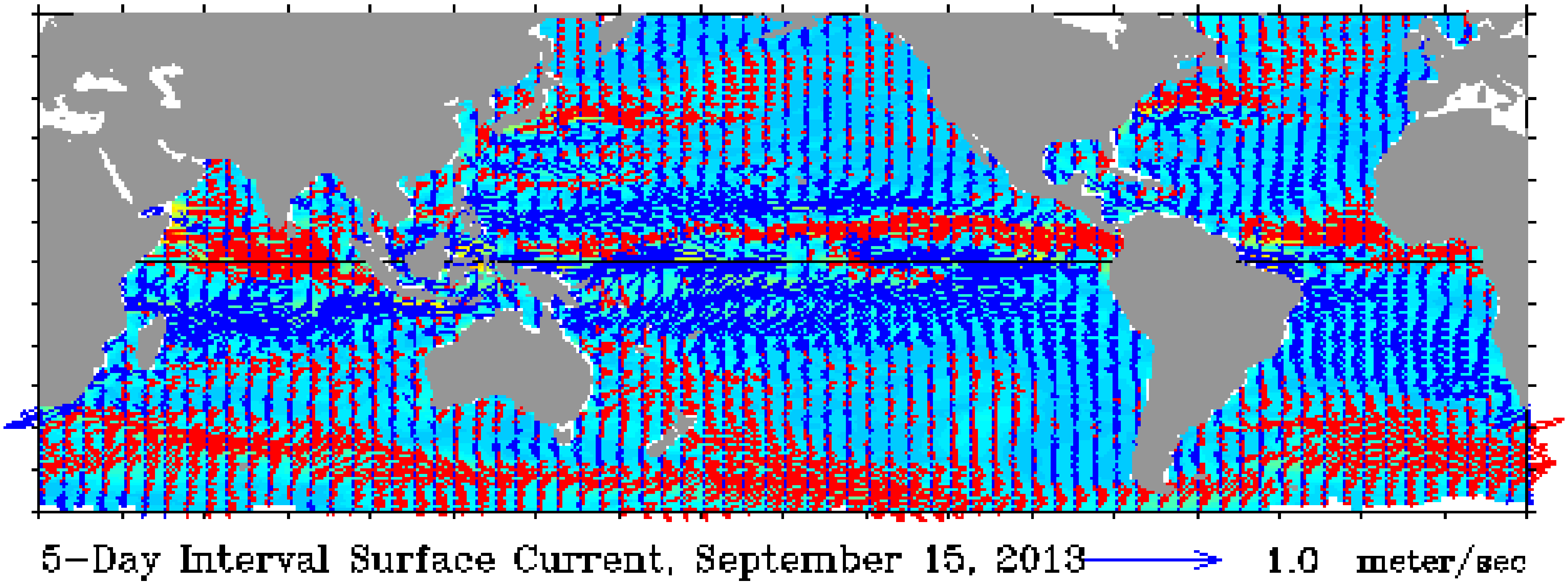
- Gaussian process is much easier to analyze, so is useful to produce a first approximation to reality. A frequent mistake is to confuse first approximation with the truth.

Satellite Altimeters
(JASON-1, GFO, ENVISAT)
Scatterometer (QSCAT)

→

Ocean
Surface Velocity





Ocean Surface currents data available for whole world' oceans at www.oscar.noa

Data continuously available every 5 days

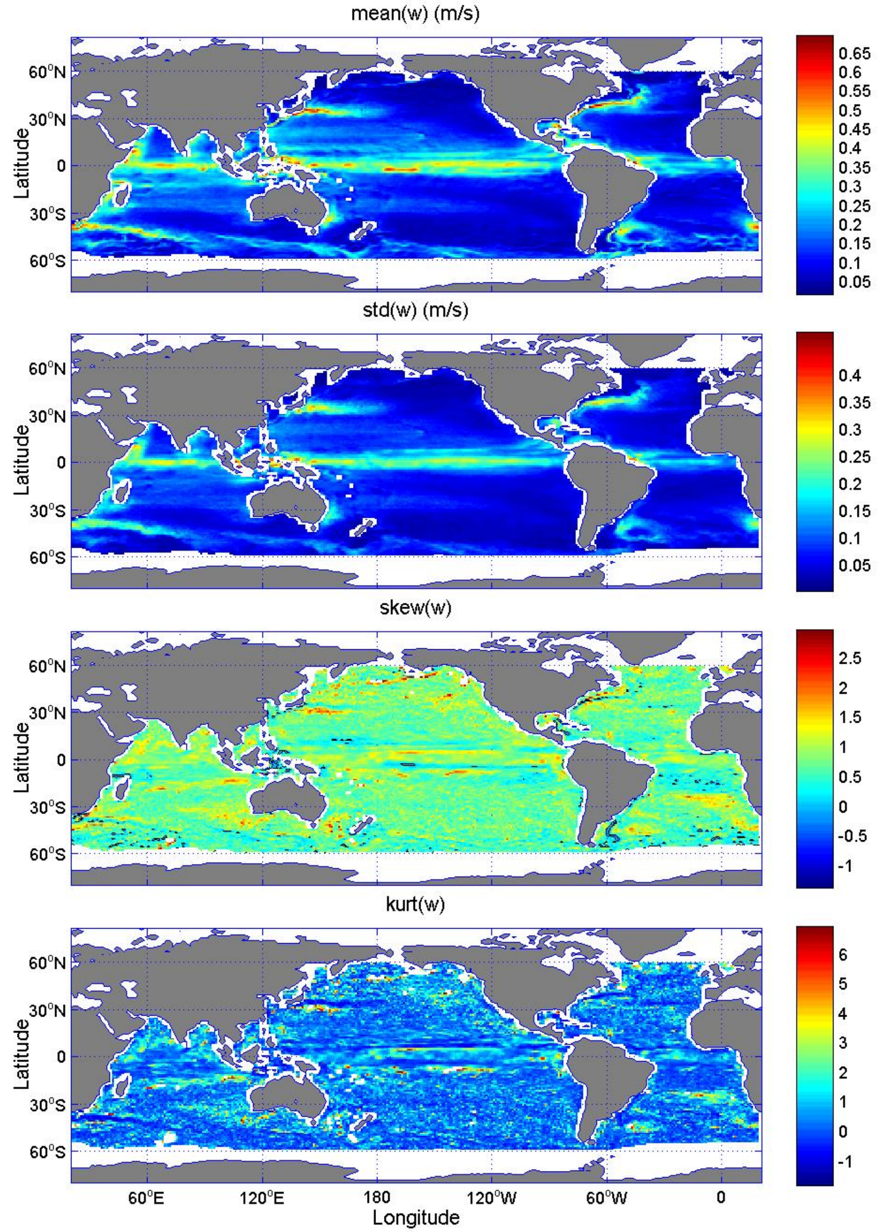
Statistical Characteristics of Global Surface Current Speed (1992-2008) (Chu 2008)

Mean

Standard Deviation

Skewness

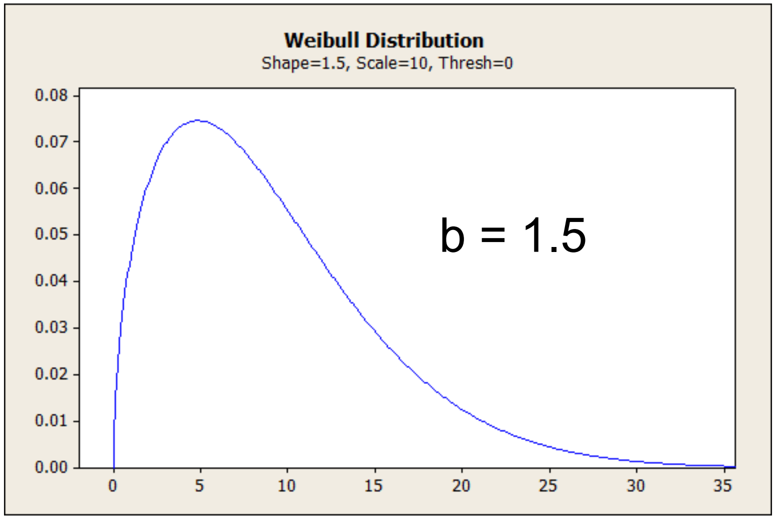
Kurtosis - 3

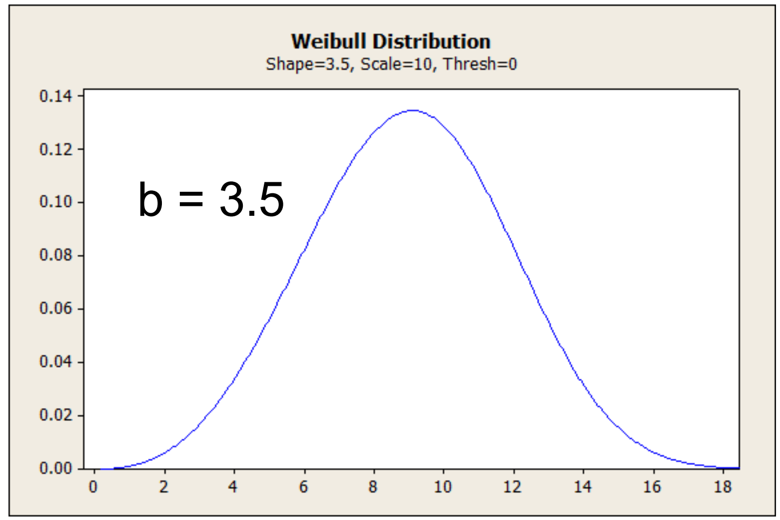


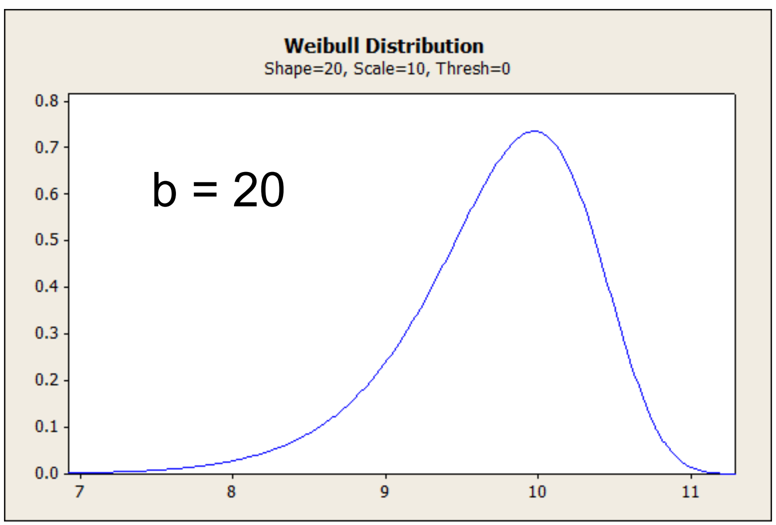
General Features

- Positive/Negative Skewness
- Kurtosis $\neq 3$

Weibull Distribution –
from Positive to Negative Skewness







$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{2}\right]$$

a = 10

Characteristics of the Weibull Distribution

$$b \simeq \left[\frac{\text{mean}(w)}{\text{std}(w)}\right]^{1.086}, \quad a = \frac{\text{mean}(w)}{\Gamma(1+1/b)}.$$

$$\text{skew}(w) = \frac{\Gamma\left(1+\frac{3}{b}\right) - 3\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) + 2\Gamma^3\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^{3/2}}$$

$$\text{kurt}(w) = \frac{\Gamma\left(1+\frac{4}{b}\right) - 4\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{3}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^2} + \frac{6\Gamma^2\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) - 3\Gamma^4\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^2} - 3$$

Γ → Gamma Function

Kurt ←→ Skew

- Since

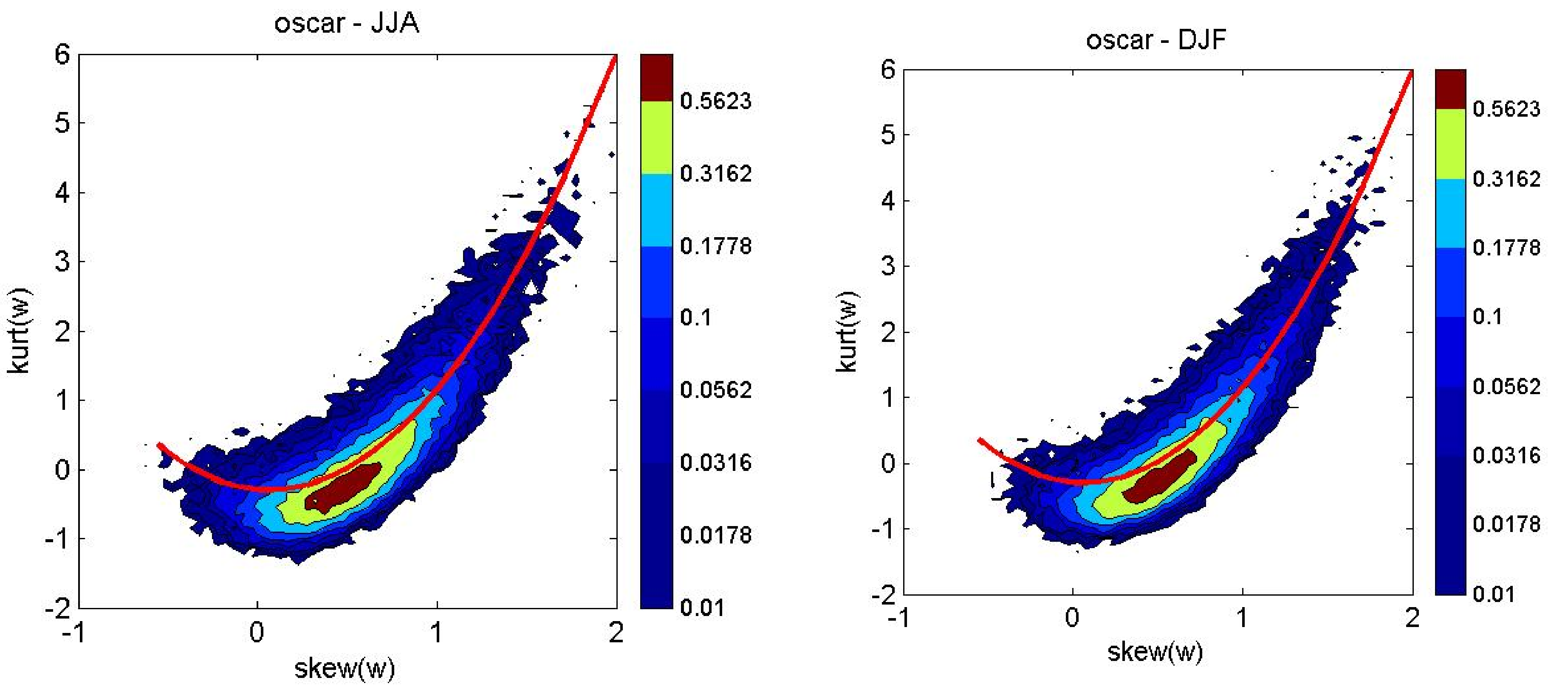
$$Skew = f_1(b), \quad Kurt = f_2(b)$$

→

$$Kurt = F(Skew)$$

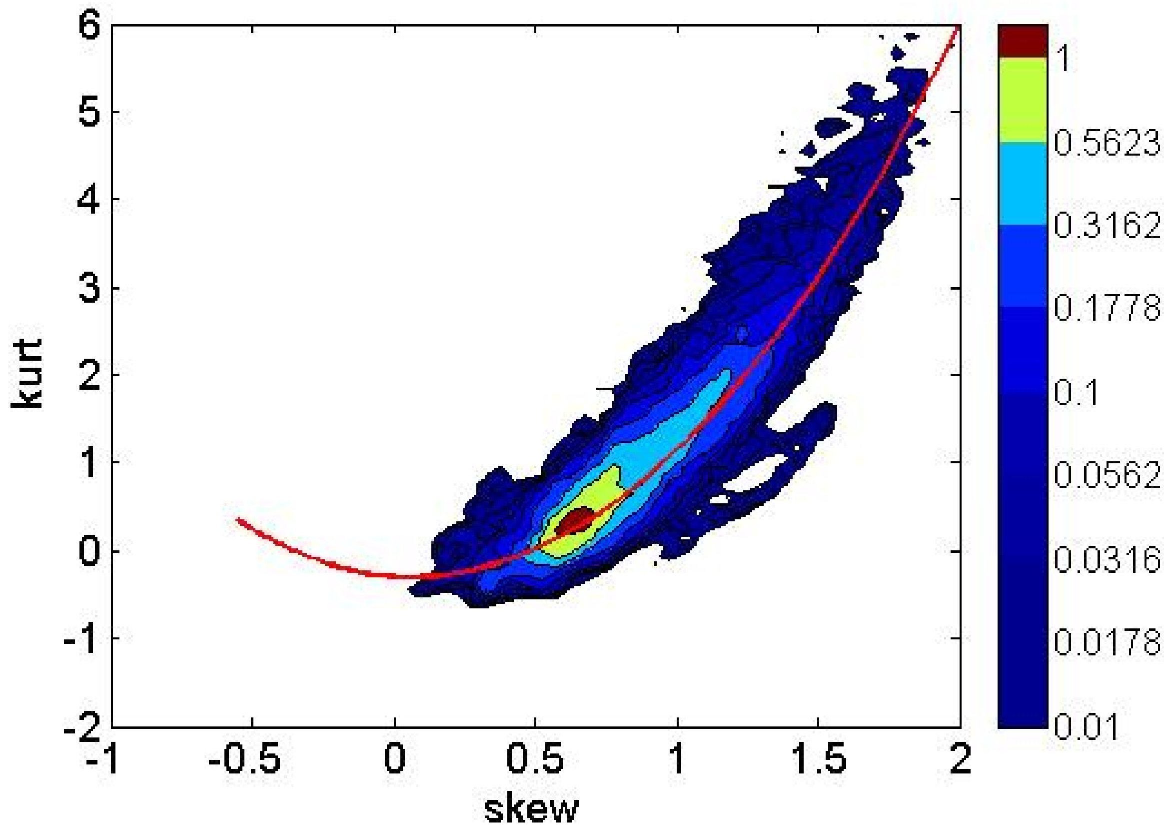
Kernel Density Estimates of Joint PDFs of kurtosis and *skewness* Surface Current Speed

(Red Curve → Weibull Distribution)



Kernel Density Estimates of Joint PDF of Kurtosis and Skewness – Significant Wave Heights

(Red Curve → Weibull Distribution)



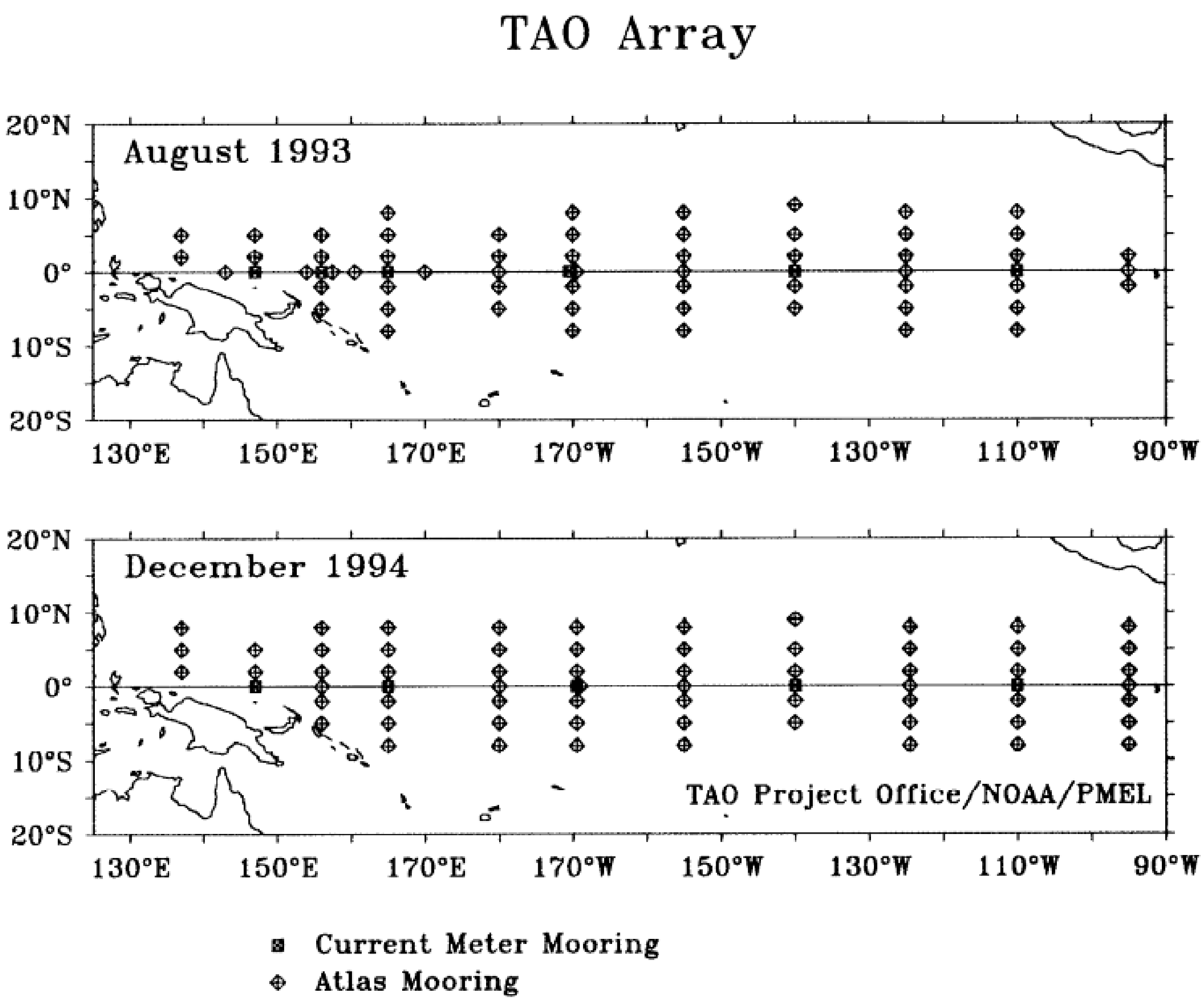


Fig. 1: The TOGA-TAO Array in August 1993 and in its final configuration December 1994. ATLAS moorings, ◇ and current-meter moorings, □.

0-50 m

Total

El Nino

La Nina

(Chu, 2008, GRL)

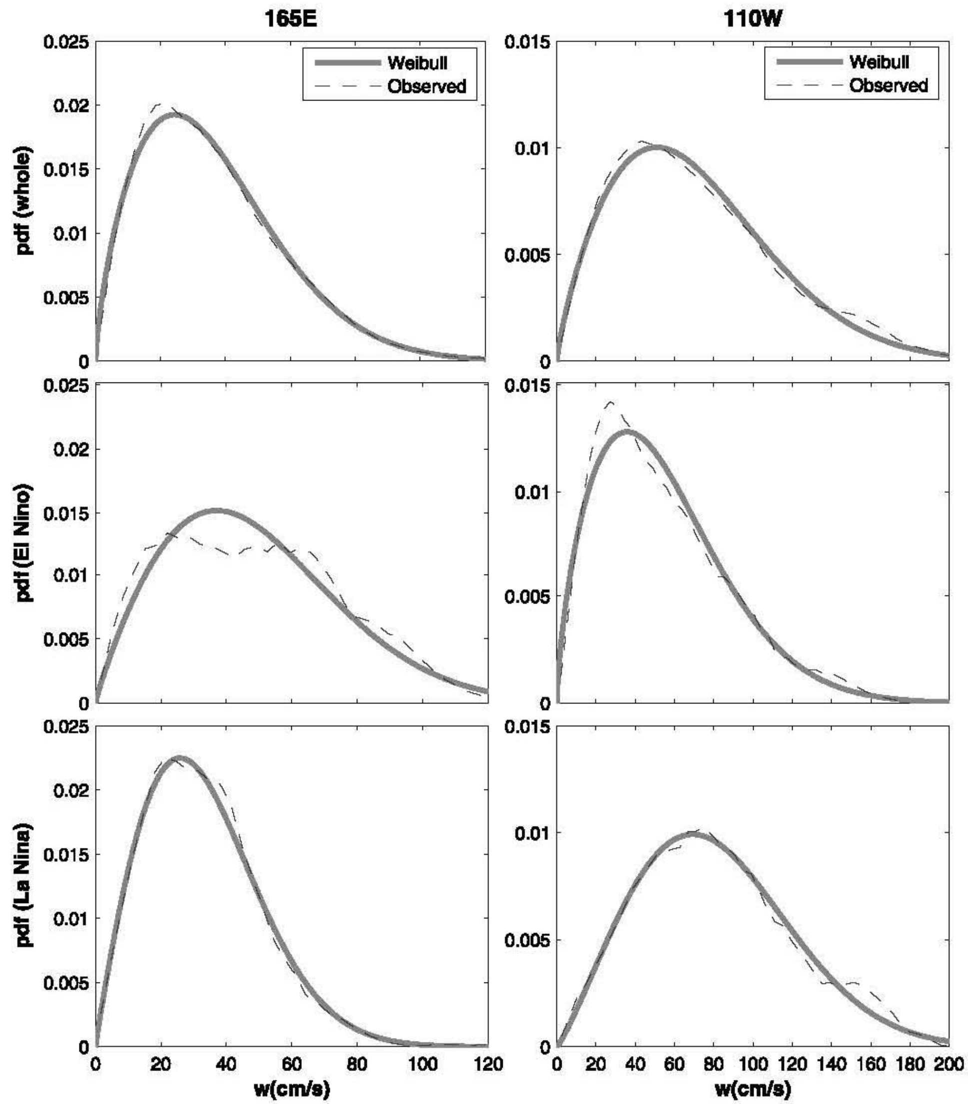


Figure 1. Comparison between observational *w*-PDFs (i.e., histogram, dashed curve) constructed from the TAO-ADCP data and Weibull distributions (solid curve) for upper layer (0–50 m) along the equator at: (left) 165°E, and (right) 110°W with the upper panels for the whole period, the middle panels for the major El Nino events, and the lower panels for the major La Nina events.

Why is the Weibull Distribution ?

Stochastic Dynamics for the Ocean Surface Currents

Vertically Averaged Horizontal Velocity (u, v) within the Mixed Layer -- Slab Model --

$$\frac{\partial u}{\partial t} = \frac{1}{h}\Lambda_u - \frac{K}{h^2}u \quad (1)$$

h → mixed layer depth

K → eddy viscosity

$$\frac{\partial v}{\partial t} = \frac{1}{h}\Lambda_v - \frac{K}{h^2}v \quad (2)$$

(τ_x, τ_y) → Surface wind stress

$$\Lambda_u \equiv fV_E + \frac{\tau_x}{\rho}, \quad \Lambda_v \equiv -fU_E + \frac{\tau_y}{\rho}$$

Ekman Transport (U_E, V_E)

$$(U_E, V_E) = \int_{-h}^{0} (\tilde{u} - u_g, \tilde{v} - v_g) dz,$$

($\tilde{u}, \tilde{v}$) → Vertically Varying Velocity

(u_g, v_g) → geostrophic velocity

(u_g, v_g) = 0 → Eqs(1) (2) → Wind-forced Slab model

Ensemble Mean and Stochastic Fluctuations of the Forcing

- (1) Ensemble Mean → Ekman Transport is determined by the surface wind stress →

$$\langle \Lambda_u \rangle = 0, \quad \langle \Lambda_v \rangle = 0$$

- (2) Fluctuations (Σ → strength)

$$\Lambda_u(t) = \langle \Lambda_u \rangle + \dot{W}_1(t) h \Sigma, \quad \Lambda_v(t) = \langle \Lambda_v \rangle + \dot{W}_2(t) h \Sigma \quad (3)$$

$$\langle \dot{W}_i(t_1) \dot{W}_j(t_2) \rangle = \delta_{ij} \delta(t_1 - t_2)$$

Stochastic Dynamic System

Eq(3) → Eqs(1) (2)

$$\frac{\partial u}{\partial t} = -\frac{K}{h^2} u + \dot{W}_1(t)\Sigma \qquad (4)$$

$$\frac{\partial v}{\partial t} = -\frac{K}{h^2} v + \dot{W}_2(t)\Sigma \qquad (5)$$

Fokker-Planck Equation for PDF of (*u*, *v*)

$$\frac{\partial p}{\partial t} = \left(\frac{\Sigma^2}{2}\right)\left(\frac{\partial^2 p}{\partial u^2} + \frac{\partial^2 p}{\partial v^2}\right) + \frac{\partial}{\partial u}\left[(\frac{K}{h^2}u)p\right] + \frac{\partial}{\partial v}\left[(\frac{K}{h^2}v)p\right] \qquad (6)$$

Polar Coordinate → $u = w\cos\varphi, \quad v = w\sin\varphi$

w → Current Speed

For Constant K → PDF of w → the Rayleigh Distribution

(Special Case of the Weibull Distribution)

$$p(w) = \frac{2w}{a^2}\exp\left[-\left(\frac{w}{a}\right)^2\right], \quad a \equiv \frac{\Sigma h}{\sqrt{K}}$$

For Non-Constant *K* → Weibull Distribution

$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{2}\right]$$

$$\mathrm{mean}(w) = a\Gamma\left(1+\frac{1}{b}\right)$$

$$\mathrm{std}(w) = a\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^{2}\left(1+\frac{1}{b}\right)\right]^{1/2}$$

Γ → Gamma Function

Summary

- The ***Weibull distribution*** provides a reasonable empirical approximation to the PDF of the near surface current speeds for the global oceans.