Near-Surface Ocean Currents

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• Magnitude of Near Surface Currents
  10 cm/s

• What are the statistical features?
Dealing with Uncertainty

- Gaussian Distribution is often used
Gaussian Distribution

- Mean = \( \mu \)
- Standard Deviation = \( \sigma \)
- Skewness = 0
- Kurtosis – 3 = 0

\[
p(x) = \frac{1}{\sqrt{2\pi \sigma^2}} \cdot \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right]
\]
Gaussian Process

- Gaussian process is much easier to analyze, so is useful to produce a first approximation to reality. A frequent mistake is to confuse first approximation with the truth.
Satellite Altimeters (JASON-1, GFO, ENVISAT)
Scatterometer (QSCAT)

→
Ocean Surface Velocity
Ocean Surface currents data available for whole world oceans at [www.oscar.noa](http://www.oscar.noa)

Data continuously available every 5 days

Mean

Standard Deviation

Skewness

Kurtosis - 3
General Features

• Positive/Negative Skewness

• Kurtosis ≠ 3
Weibull Distribution – from Positive to Negative Skewness

\[
p(w) = \frac{b}{a} \left( \frac{w}{a} \right)^{b-1} \exp \left[ -\left( \frac{w}{a} \right)^2 \right]
\]

- \( a = 10 \)
- \( b = 1.5 \)
- \( b = 3.5 \)
- \( b = 20 \)
Characteristics of the Weibull Distribution

\[ b = \left[ \frac{\text{mean}(w)}{\text{std}(w)} \right]^{1.086}, \quad a = \frac{\text{mean}(w)}{\Gamma(1 + 1/b)}. \]

\[ \text{skew}(w) = \frac{\Gamma \left( 1 + \frac{3}{b} \right) - 3 \Gamma \left( 1 + \frac{1}{b} \right) \Gamma \left( 1 + \frac{2}{b} \right) + 2 \Gamma^3 \left( 1 + \frac{1}{b} \right)}{\left[ \Gamma \left( 1 + \frac{2}{b} \right) - \Gamma^2 \left( 1 + \frac{1}{b} \right) \right]^{3/2}}. \]

\[ \text{kurt}(w) = \frac{\Gamma \left( 1 + \frac{4}{b} \right) - 4 \Gamma \left( 1 + \frac{1}{b} \right) \Gamma \left( 1 + \frac{3}{b} \right)}{\left[ \Gamma \left( 1 + \frac{2}{b} \right) - \Gamma^2 \left( 1 + \frac{1}{b} \right) \right]^2} + \frac{6 \Gamma^2 \left( 1 + \frac{1}{b} \right) \Gamma \left( 1 + \frac{2}{b} \right) - 3 \Gamma^4 \left( 1 + \frac{1}{b} \right)}{\left[ \Gamma \left( 1 + \frac{2}{b} \right) - \Gamma^2 \left( 1 + \frac{1}{b} \right) \right]^2} - 3. \]

\[ \Gamma \rightarrow \text{Gamma Function} \]
Kurt ↔ Skew

• Since

\[ \text{Skew} = f_1(b), \quad \text{Kurt} = f_2(b) \]

→

\[ \text{Kurt} = F(\text{Skew}) \]
Kernel Density Estimates of Joint PDFs of kurtosis and skewness Surface Current Speed

(Red Curve → Weibull Distribution)
Kernel Density Estimates of Joint PDF of Kurtosis and Skewness – Significant Wave Heights

(Red Curve → Weibull Distribution)
Fig. 1: The TOGA-TAO Array in August 1993 and in its final configuration December 1994. ATLAS moorings, ◇ and current-meter moorings, □.
0-50 m

Total

El Nino

La Nina

(Chu, 2008, GRL)

Figure 1. Comparison between observational w-PDFs (i.e., histogram, dashed curve) constructed from the TAO-ADCP data and Weibull distributions (solid curve) for upper layer (0–50 m) along the equator at: (left) 165°E, and (right) 110°W with the upper panels for the whole period, the middle panels for the major El Nino events, and the lower panels for the major La Nina events.
Why is the Weibull Distribution?

Stochastic Dynamics for the Ocean Surface Currents
Vertically Averaged Horizontal Velocity \((u, v)\) within the Mixed Layer

-- Slab Model --

\[
\frac{\partial u}{\partial t} = \frac{1}{h} \Lambda_u - \frac{K}{h^2} u \quad (1)
\]

\[
\frac{\partial v}{\partial t} = \frac{1}{h} \Lambda_v - \frac{K}{h^2} v \quad (2)
\]

\(h \rightarrow\) mixed layer depth

\(K \rightarrow\) eddy viscosity

\((\tau_x, \tau_y) \rightarrow\) Surface wind stress

\[\Lambda_u \equiv f V_E + \frac{\tau_x}{\rho}, \quad \Lambda_v \equiv -f U_E + \frac{\tau_y}{\rho}\]
Ekman Transport \((U_E, V_E)\)

\[
(U_E, V_E) = \int_{-h}^{0} (\tilde{u} - u_g, \tilde{v} - v_g) \, dz,
\]

\((\tilde{u}, \tilde{v}) \rightarrow \text{Vertically Varying Velocity}\)

\((u_g, v_g) \rightarrow \text{geostrophic velocity}\)

\((u_g, v_g) = 0 \rightarrow \text{Eqs}(1) \,(2) \rightarrow \text{Wind-forced Slab model}\)
Ensemble Mean and Stochastic Fluctuations of the Forcing

• (1) Ensemble Mean $\rightarrow$ Ekman Transport is determined by the surface wind stress $\rightarrow$

$$\langle \Lambda_u \rangle = 0, \quad \langle \Lambda_v \rangle = 0$$

• (2) Fluctuations ($\Sigma \rightarrow$ strength)

$$\Lambda_u(t) = \langle \Lambda_u \rangle + \dot{W}_1(t) h \Sigma, \quad \Lambda_v(t) = \langle \Lambda_v \rangle + \dot{W}_2(t) h \Sigma \quad (3)$$

$$\langle \dot{W}_i(t_1) \dot{W}_j(t_2) \rangle = \delta_{ij} \delta(t_1 - t_2)$$
Stochastic Dynamic System

Eq(3) $\rightarrow$ Eqs(1) (2)

\[
\frac{\partial u}{\partial t} = -\frac{K}{h^2} u + \dot{W}_1(t)\Sigma
\]

(4)

\[
\frac{\partial v}{\partial t} = -\frac{K}{h^2} v + \dot{W}_2(t)\Sigma
\]

(5)
Fokker-Planck Equation for PDF of \((u, v)\)

\[
\frac{\partial p}{\partial t} = \left( \frac{\Sigma^2}{2} \right) \left( \frac{\partial^2 p}{\partial u^2} + \frac{\partial^2 p}{\partial v^2} \right) + \frac{\partial}{\partial u} \left[ \left( \frac{K}{h^2} u \right) p \right] + \frac{\partial}{\partial v} \left[ \left( \frac{K}{h^2} v \right) p \right]
\]

(6)

Polar Coordinate \(\rightarrow\) \(u = w \cos \phi, \quad v = w \sin \phi\)

\(w \rightarrow\) Current Speed
For Constant $K \rightarrow$

PDF of $w \rightarrow$ the Rayleigh Distribution

(Special Case of the Weibull Distribution)

$$p(w) = \frac{2w}{a^2} \exp \left[ -\left( \frac{w}{a} \right)^2 \right], \quad a \equiv \frac{\Sigma h}{\sqrt{K}}$$
For Non-Constant $K \rightarrow$ Weibull Distribution

\[ p(w) = \frac{b}{a} \left( \frac{w}{a} \right)^{b-1} \exp \left[ -\left( \frac{w}{a} \right)^2 \right] \]

\[ \text{mean}(w) = a \Gamma \left( 1 + \frac{1}{b} \right) \]

\[ \text{std}(w) = a \left[ \Gamma \left( 1 + \frac{2}{b} \right) - \Gamma^2 \left( 1 + \frac{1}{b} \right) \right]^{1/2} \]

$\Gamma \rightarrow$ Gamma Function
Summary

- The **Weibull distribution** provides a reasonable empirical approximation to the PDF of the near surface current speeds for the global oceans.