

Change of Upper Ocean Multifractal Structure due to Internal Soliton Propagation

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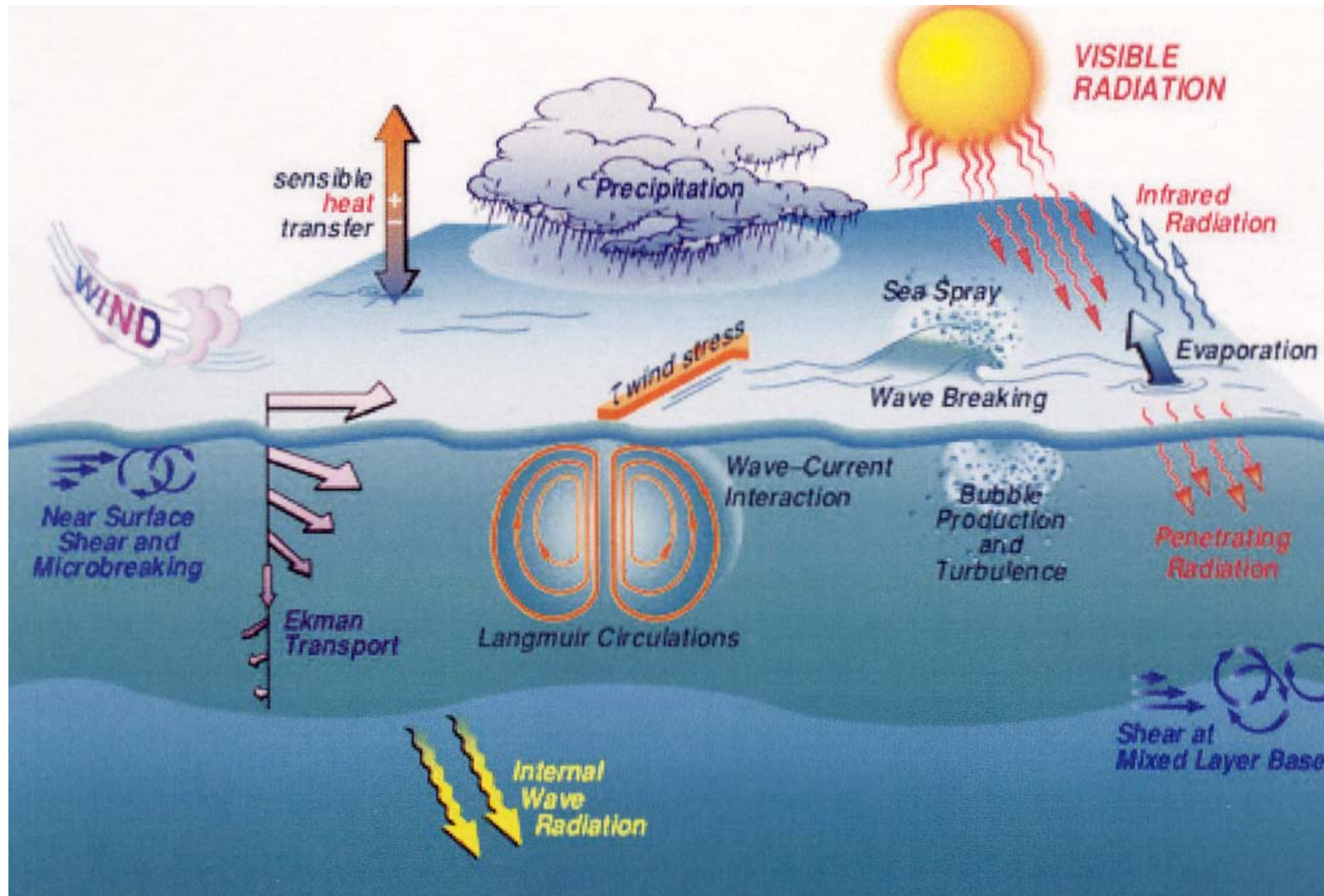
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Collaborator: Kevin Mahoney (NAVO)

AGU Fall Meeting, San Francisco, December 9, 2011

Upper Ocean Dynamics

from http://www.hpl.umces.edu/ocean/sml_main.htm



High-Order Structure Function

$$T_i = T(x_i), \quad x_i = il, \quad i = 0, 1, \dots, \Lambda, \quad L = \Lambda l,$$

$$|\Delta T(x_i, rl)| = |T(x_{i+r}) - T(x_i)|, \quad i = 0, 1, \dots, \Lambda - r$$

$$S(r, q) \equiv \left\langle |\Delta T(x, rl)|^q \right\rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda-r} |\Delta T(x_i, rl)|^q .$$

Here, r is the lag, q is the order of the structure function.

$S(r, 1)$ is the commonly used structure function.

$$S(1, 1) = \frac{1}{\Lambda - 1} \sum_{i=0}^{\Lambda-1} |T(x_{i+1}) - T(x_i)| \quad \mathbf{S(1,1) is the mean gradient .}$$

Power Law

$$S(r, q) \sim r^{\zeta(q)}$$

- Simple self-similarity
e.g., Gaussian Turbulence
- Multifractal structure

$$\zeta(q) = qH$$

$$\zeta(q) \neq qH .$$

$$H = 1/3$$

Kolmogorov (1941)

Two Questions

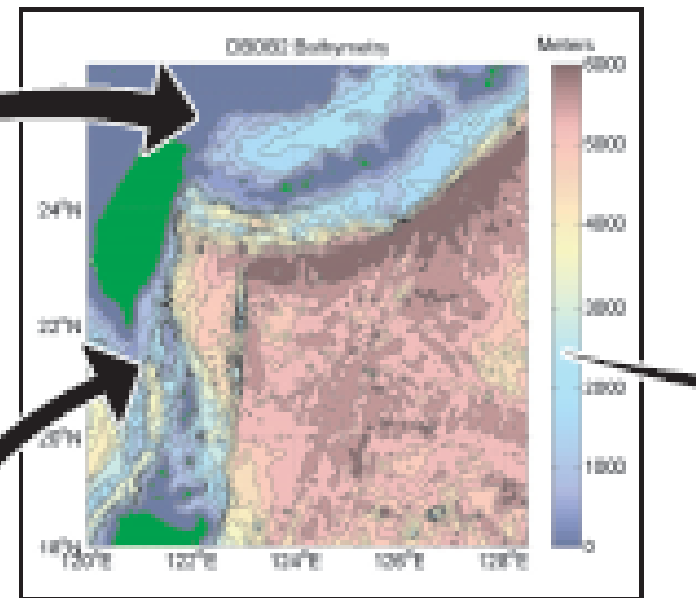
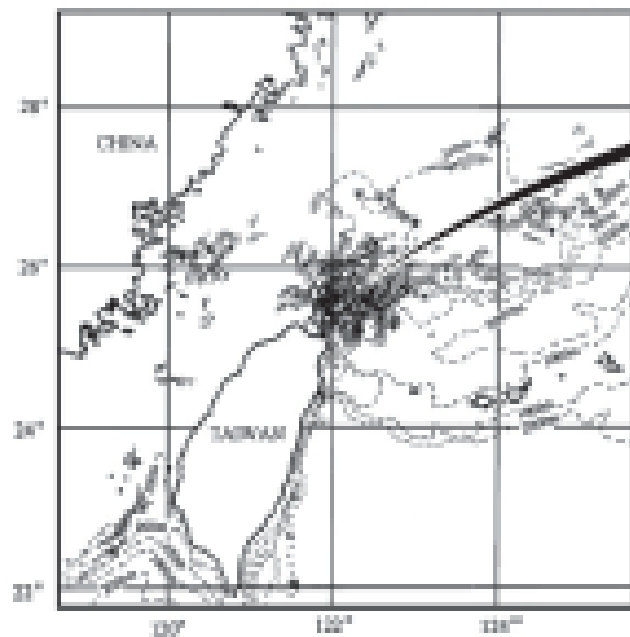
- Does the upper ocean have self-similarity feature or multifractal structure ?
- What is the effect of the internal wave/soliton propagation?

Chu (Chaos, Solitons, and Fractals, 2004)

Chu and Hsieh (Journal of Oceanography, 2007)

Generation of Internal Waves and Solitons near Taiwan

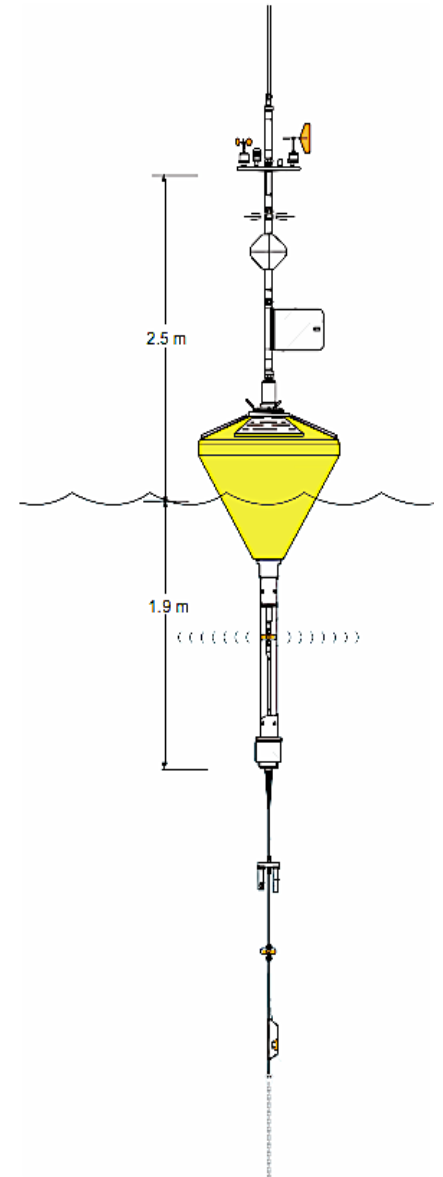
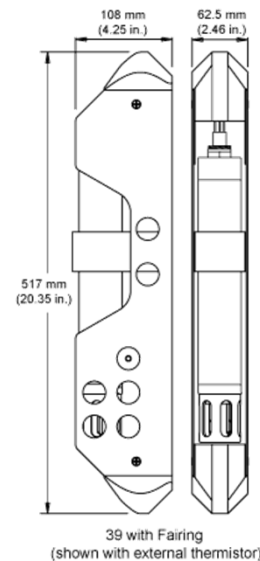
Likely generation site for observed internal solitons
(Figure taken from Jackson and Apel [2004])



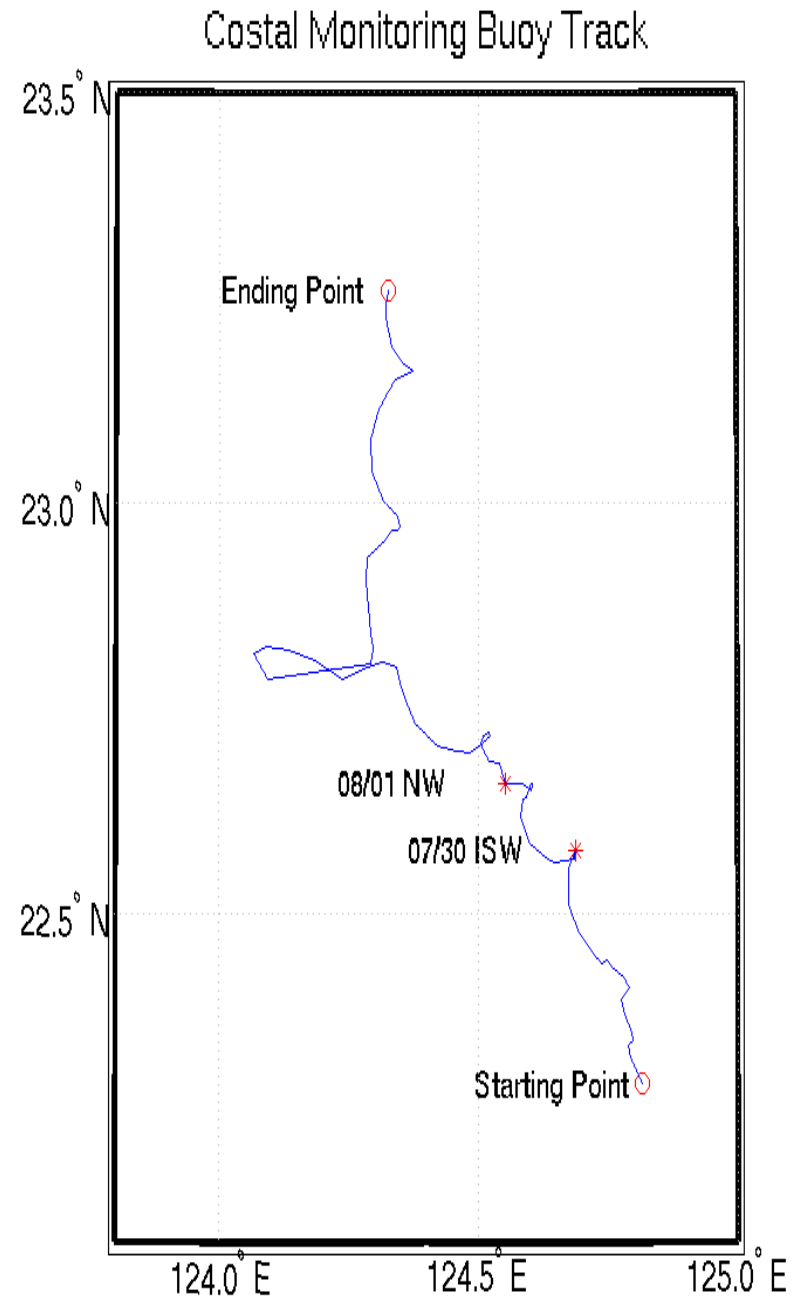
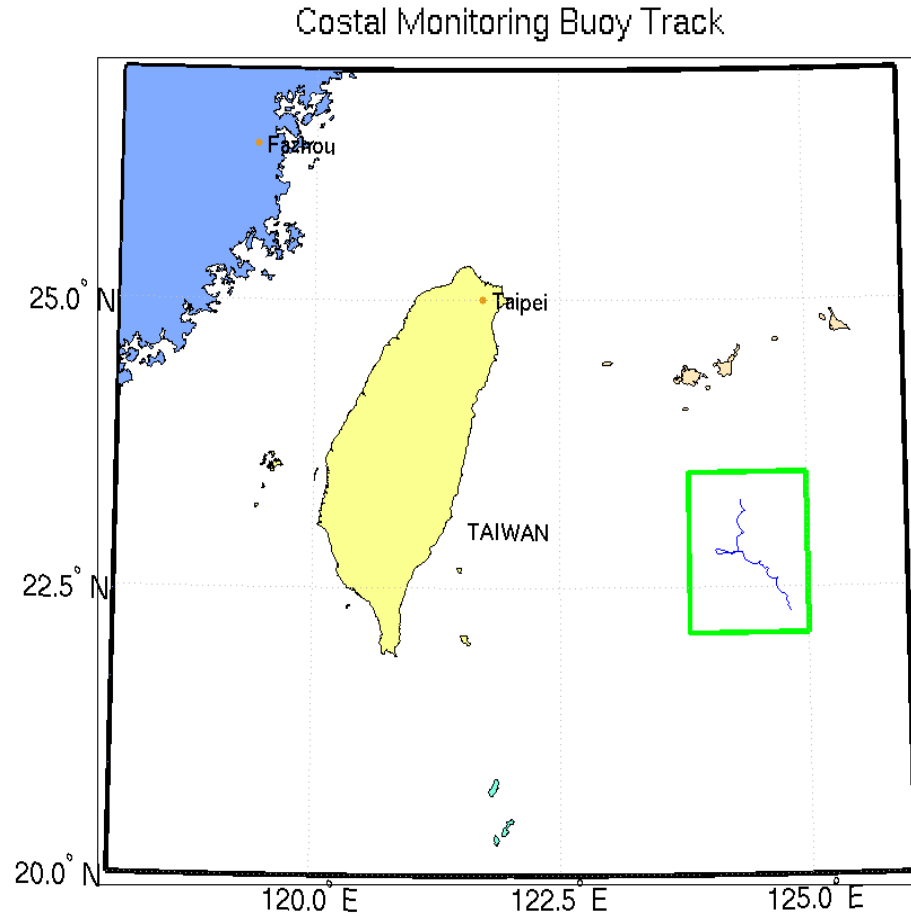
Philippine Sea

Data Observation

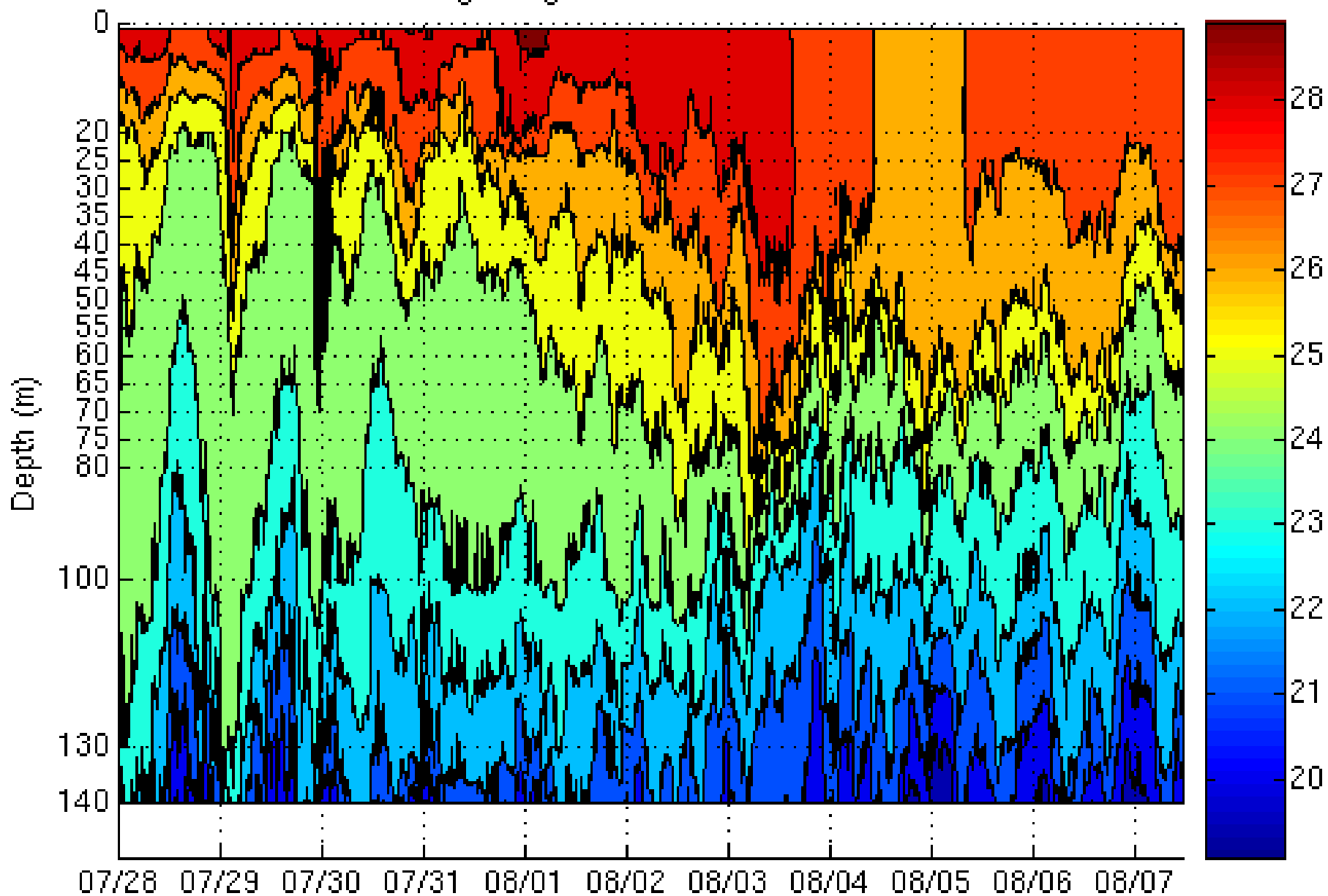
- Coastal Monitoring Buoy (CMB)
 - DB 4280
 - U.S. Naval Oceanographic Office
 - July 28 - August 7, 2005
 - Atmosphere data and Ocean data at 1,3,5,18, and 20 meters
 - Record intervals - 10 minutes
- **Thermistors**
 - SBE 39
 - Attached at **15 depths** from 25 to 140 meters.
 - **Records intervals - 15 seconds.**



- Latitude - $22^{\circ}17'N$ - $23^{\circ}15'N$
- Longitude - $124^{\circ}14'E$ - $124^{\circ}49'E$
- Distance - 229.14 Km
- Velocity - 3.82m/ 15s



Temperature : Contoured at 1 ° C
Beginning 28-Jul-2005 09:08:00



Isopycnal Displacement

(Desaubles and Gregg, 1981, JPO)

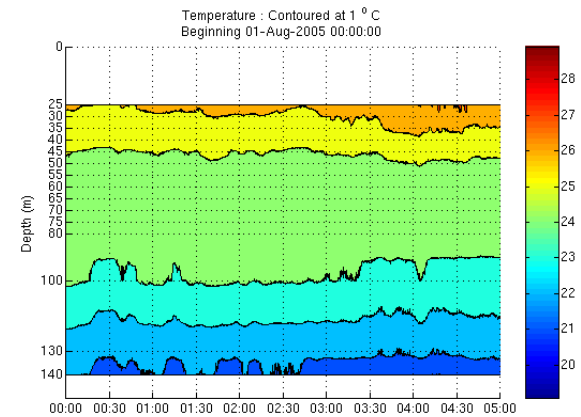
$$T'(t, z) = T(t, z) - \bar{T}(z),$$

$$\eta(t, z) = -\frac{T'(t, z)}{d\bar{T} / dz},$$

Three Types

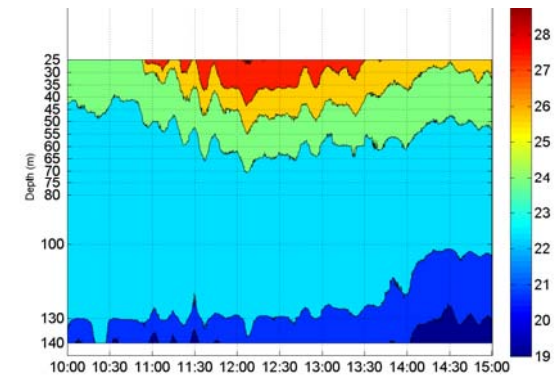
(a) Weak internal waves

Turbulence-Dominated (T)
(0000- 0500 GMT August 1)



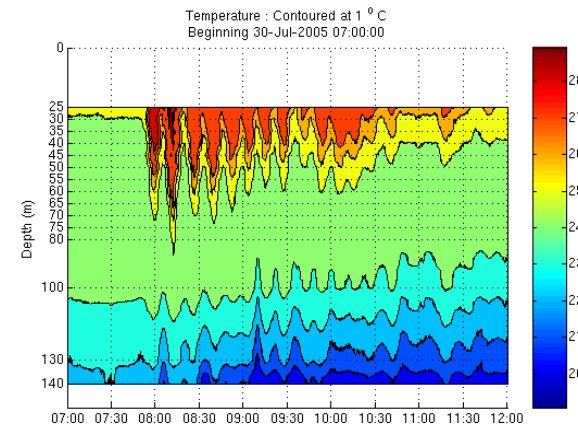
(b) Internal Wave (IW)

(1000-1500 GMT July 29)



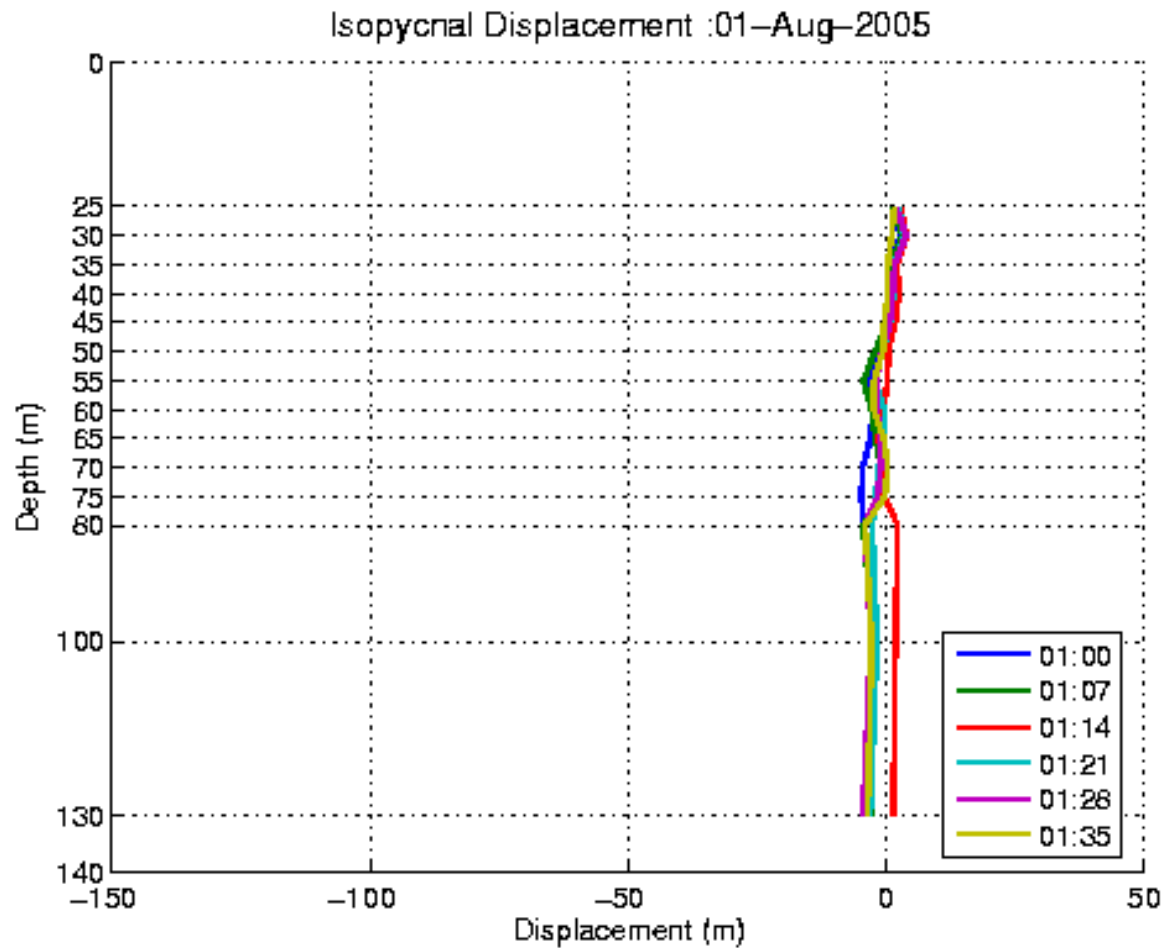
(c) Internal Soliton -
turbulence (IS)

(0700-1200 GMT July 30)

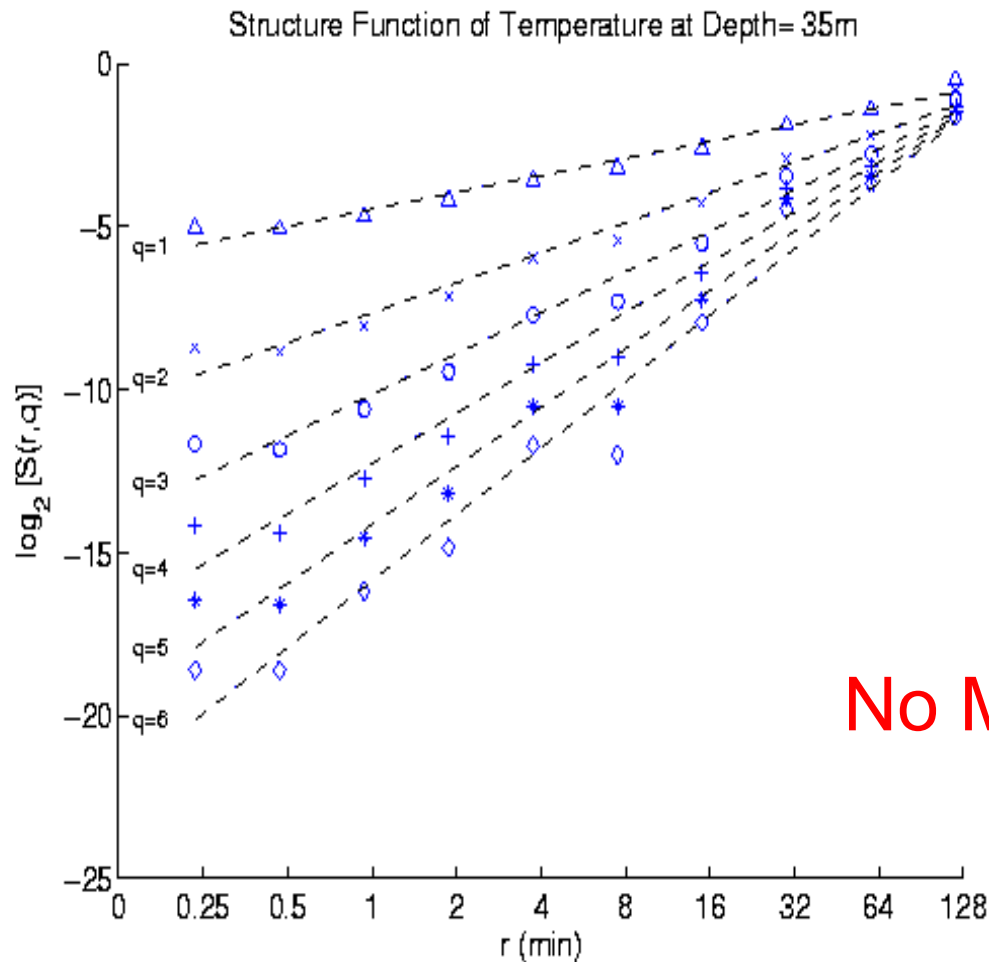


Isopycnal Displacement

turbulence-Dominated (00-05 GMT Aug 1)



Structure Function (Power Law) T-Type



$$S(r, q) \propto r^{\zeta(q)},$$

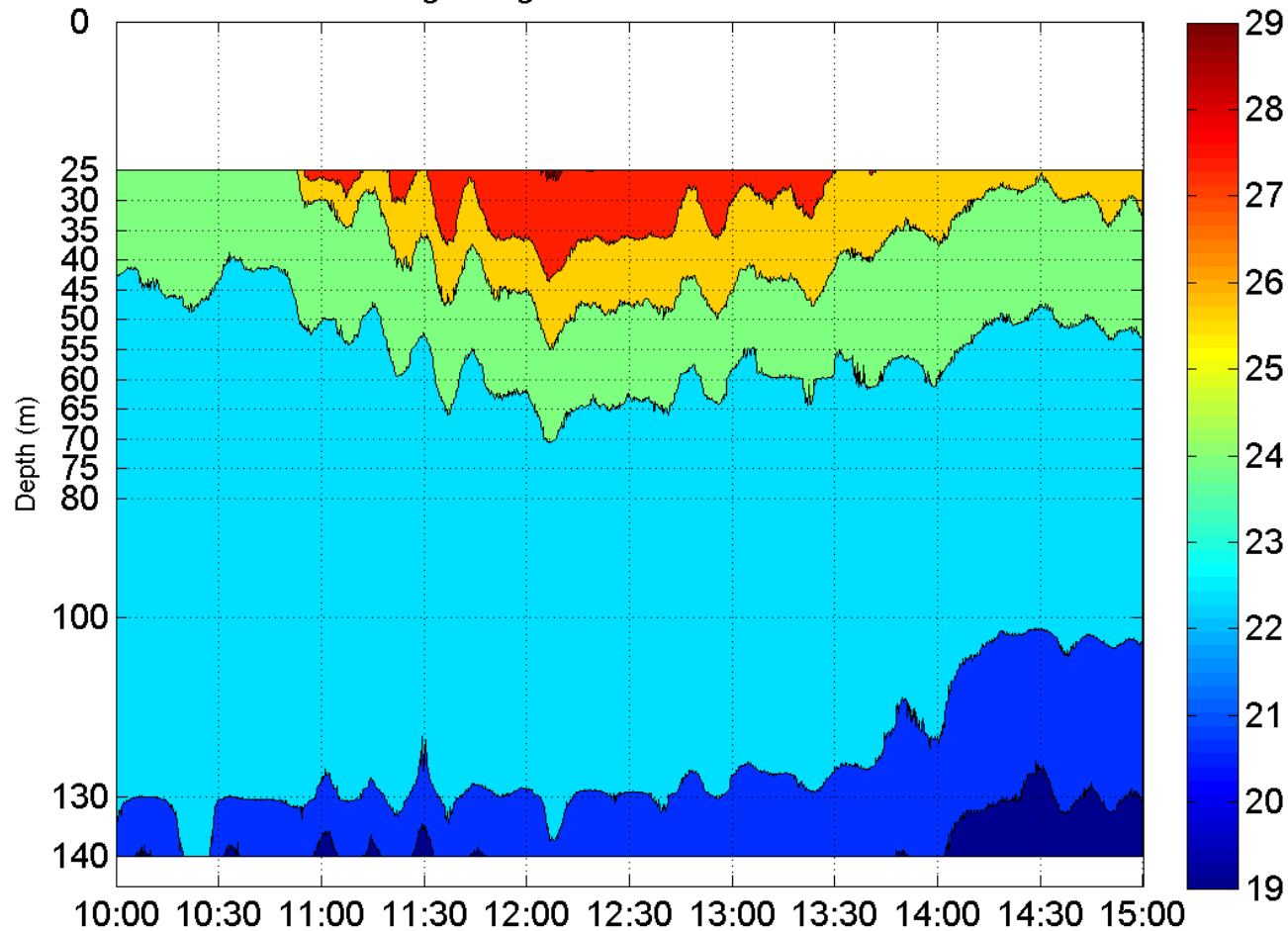
$$\zeta(q) = q/3$$

Uniform and Isotropic

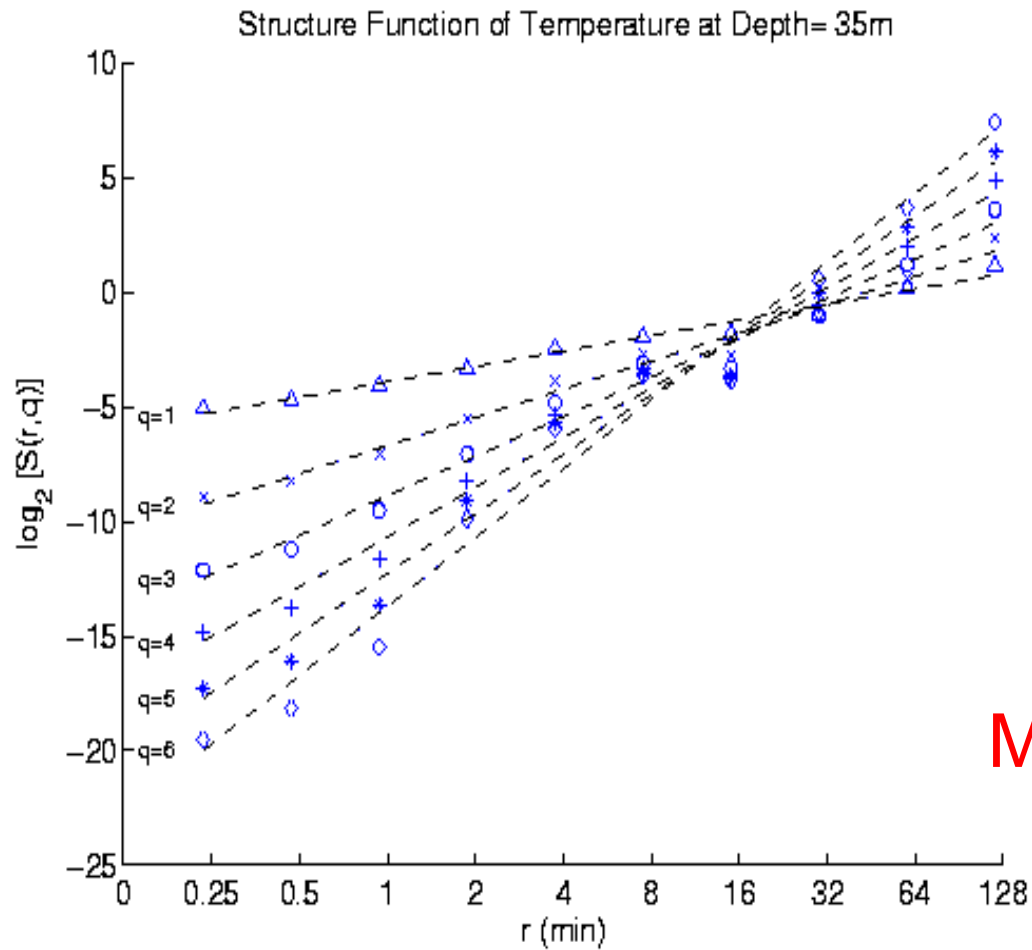
No Multifractal Structure

Internal Wave (IW-Type) (1000-1500 GMT July 29)

Temperature: Contoured at 1 °C
Beginning 29-Jul-2005 10:00:00



High-Order Structure Functions IW-T type (Power Law)



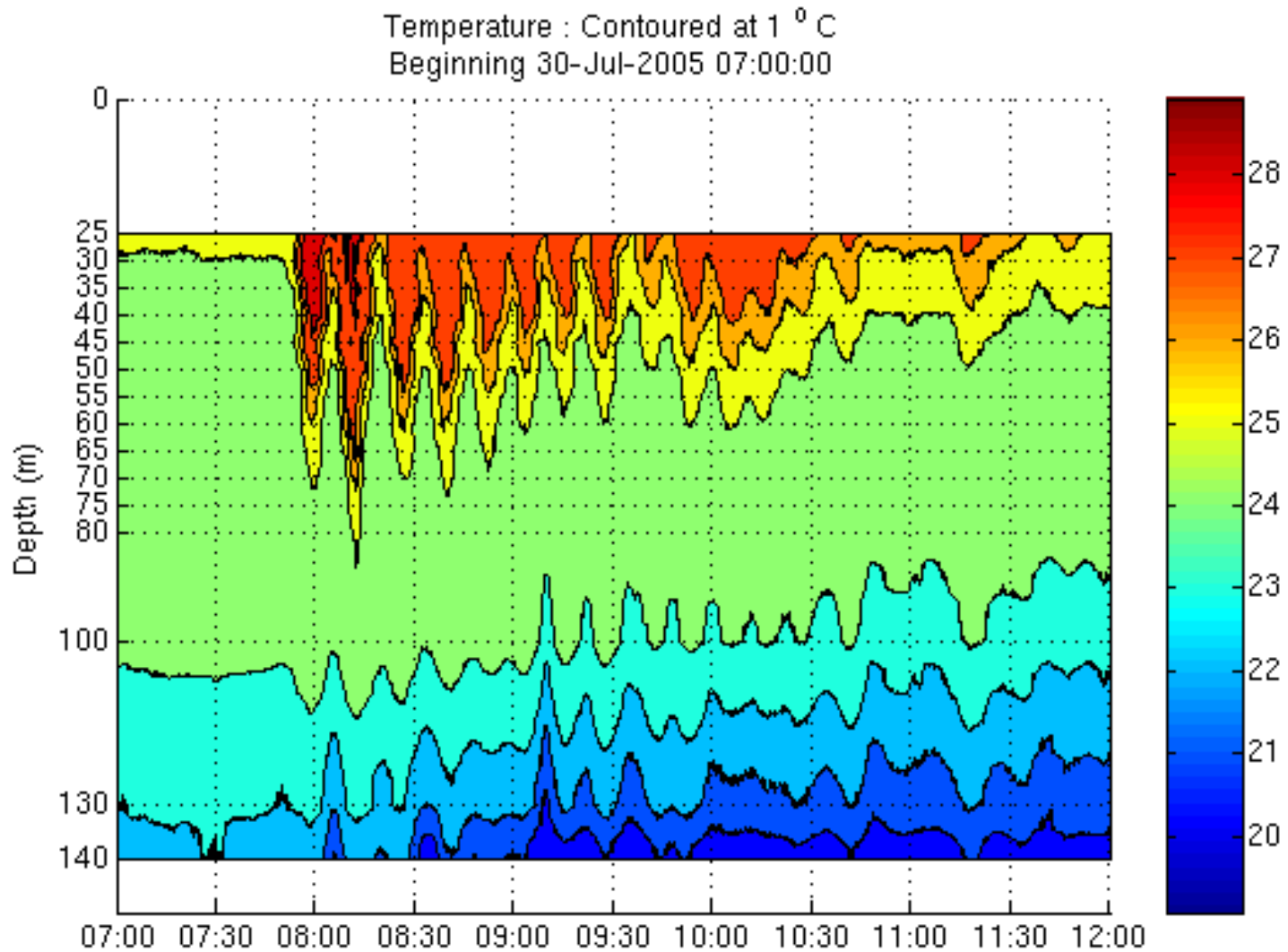
$$S(r, q) \propto r^{\zeta(q)},$$

$$\zeta(q) = H(q)q,$$

$$H(q) \neq 1/3$$

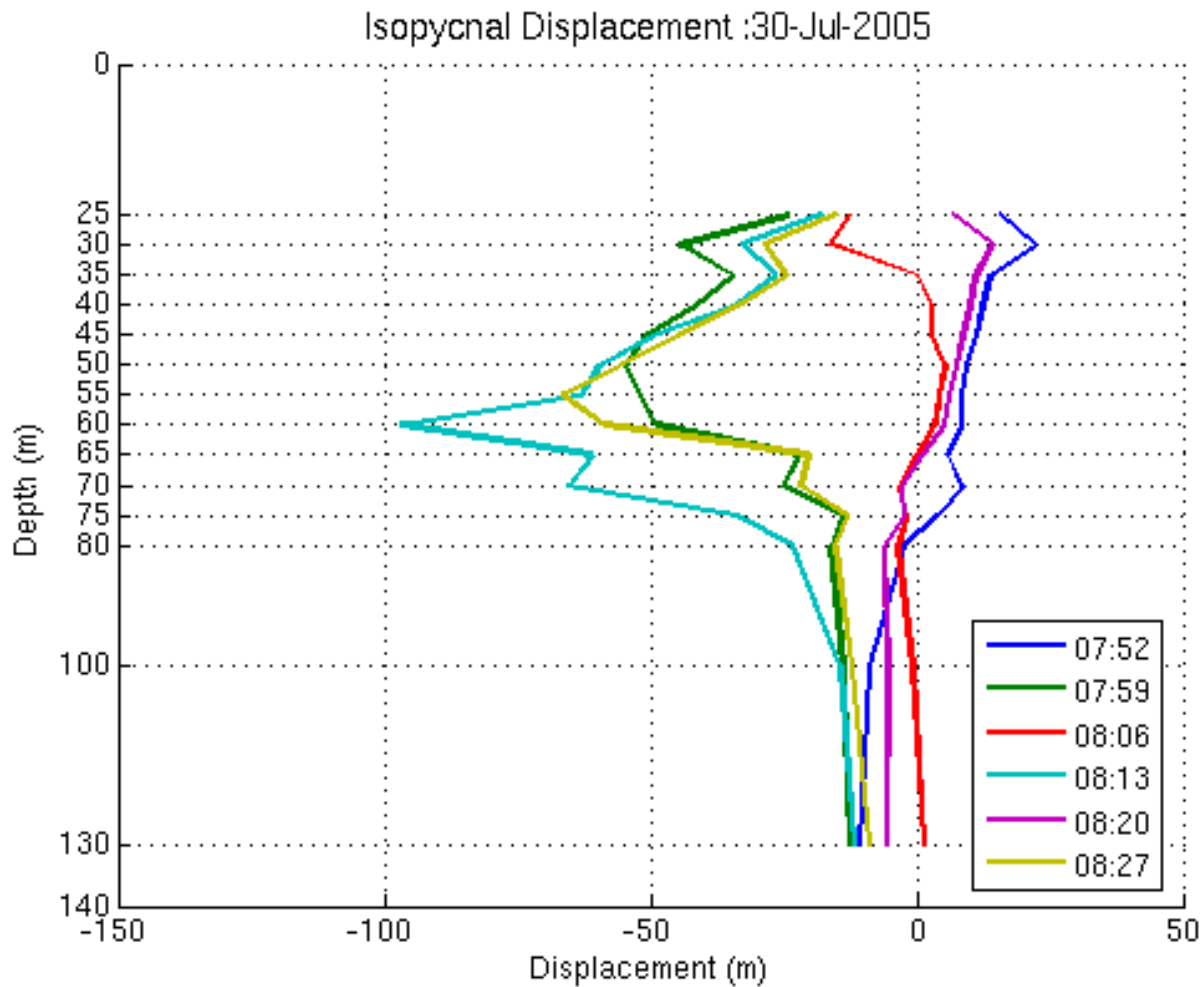
Multi-fractal Structure

Internal Soliton (IS-Type) (0700-1200 GMT July 30)



Isopycnal Displacement

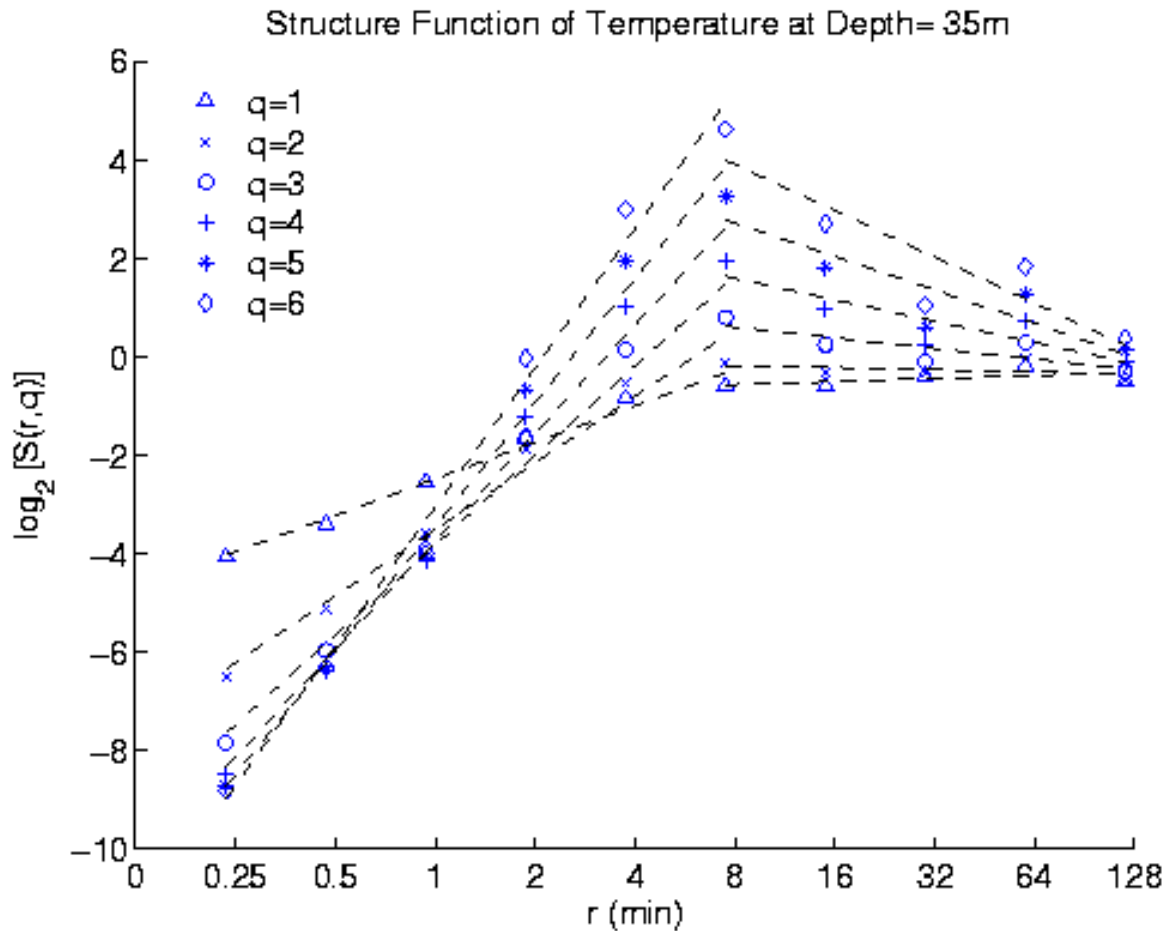
IS-turbulence (07-12 GMT July 30)



Frequency
is around

4 CPH

Structure Function (No Power Law) IS-T type



Power law breaks
at 8 min, near half
period (4 CPH) of
the internal solitons

Conclusions

- (1) Three types of thermal variability (IW, IS, and T) are identified.
- (2) Without internal waves/internal solitons, the upper layer of the Philippine Sea shows **simple self similarity**.
- (3) With the internal wave propagation, **multifractal structures** are found in the upper layer of the western Philippine Sea.
- (4) The internal solitons **destroy the power law** characteristics in the structure function at the lag of 8 min, which is nearly half period of the IS (with frequency of 4 CPH).

Possible Reason for Preservation of the Power Law in Evident Internal Wave Propagation

Using the Hamiltonian formulation, Lvov and Tabak (2001) modified the Garrett-Munk spectrum into

$$E(k, m) = \frac{2fNE}{\pi} \frac{(m/m^*)A(m/m^*)}{N^2k^2 + f^2m^2},$$

$$m^* \equiv \gamma(\omega^2 - f^2)^{-\delta/2}, \quad A(\lambda) \equiv \frac{t-1}{(1+\lambda)^t}$$

Internal waves and wave turbulence have similar spectrum.

Possible Reason for Break of the Power Law in Internal Soliton Propagation

- The internal solitary waves are a class of nonsinusoidal, nonlinear, more-or-less isolated waves of complex shape that maintain their coherence. Their energy spectrum is totally different from the internal wave spectrum.

References

- Chu, P.C., 2004: Multifractal thermal characteristics of the southwestern GIN Sea upper layer, [*Chaos, Solitons and Fractals*](#), **19**, 275-284.
- Chu, P.C. and C. P. Hsieh, 2007: Change of Multifractal Thermal Characteristics in the Western Philippine Sea Upper Layer during Internal Wave-Soliton Propagation. [*Journal of Oceanography*](#), **63**, 927-939.