Change of Upper Ocean Multifractal Structure due to Internal Soliton Propagation

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Upper Ocean Dynamics

from http://www.hpl.umces.edu/ocean/sml_main.htm
High-Order Structure Function

\[ T_i = T(x_i), \quad x_i = il, \quad i = 0, 1, \ldots, \Lambda, \quad L = \Lambda l, \]

\[ |\Delta T(x_i, rl)| = |T(x_{i+r}) - T(x_i)|, \quad i = 0, 1, \ldots, \Lambda - r \]

\[ S(r, q) = \left\langle |\Delta T(x, rl)|^q \right\rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} |\Delta T(x_i, rl)|^q. \]

Here, \( r \) is the lag, \( q \) is the order of the structure function.

\( S(r, 1) \) is the commonly used structure function.

\[ S(1,1) = \frac{1}{\Lambda - 1} \sum_{i=0}^{\Lambda - 1} |T(x_{i+1}) - T(x_i)| \quad \text{S}(1,1) \text{ is the mean gradient} \]
Power Law

\[ S(r,q) \sim r^{\zeta(q)} \]

- Simple self-similarity
  e.g., Gaussian Turbulence

- Multifractal structure

\[ \zeta(q) = qH \]

\[ \zeta(q) \neq qH. \]

\[ H = 1/3 \]

Kolmogorov (1941)
Two Questions

• Does the upper ocean have self-similarity feature or multifractal structure?

• What is the effect of the internal wave/soliton propagation?

Chu (Chaos, Solitons, and Fractals, 2004)
Chu and Hsieh (Journal of Oceanography, 2007)
Generation of Internal Waves and Solitons near Taiwan

Likely generation site for observed internal solitons
(Figure taken from Jackson and Apel [2004])

Philippine Sea
Data Observation

- Coastal Monitoring Buoy (CMB)
  - DB 4280
  - U.S. Naval Oceanographic Office
  - July 28 - August 7, 2005
  - Atmosphere data and Ocean data at
    1, 3, 5, 18, and 20 meters
  - Record intervals - 10 minutes

- Thermistors
  - SBE 39
  - Attached at 15 depths from 25 to 140 meters.
  - Records intervals - 15 seconds.
- Latitude - 22°17′N - 23°15′N
- Longitude - 124°14′E - 124°49′E
- Distance - 229.14 Km
- Velocity - 3.82m/15s
Isopycnal Displacement
(Desaubles and Gregg, 1981, JPO)

\[ T'(t, z) = T(t, z) - \overline{T}(z), \]

\[ \eta(t, z) = -\frac{T'(t, z)}{d\overline{T}/dz}, \]
Three Types

(a) Weak internal waves
   Turbulence-Dominated (T)
   (0000- 0500 GMT August 1)

(b) Internal Wave (IW)
   (1000-1500 GMT July 29)

(c) Internal Soliton - turbulence (IS)
   (0700-1200 GMT July 30)
Turbulence-dominated (T) (0000-0500 GMT August 1)
Isopycnal Displacement
turbulence-Dominated (00-05 GMT Aug 1)
Structure Function (Power Law)
T-Type

\[ S(r, q) \propto r^{\zeta(q)}, \]
\[ \zeta(q) = q/3 \]

Uniform and Isotropic
No Multifractal Structure
Internal Wave (IW-Type)
(1000-1500 GMT July 29)
Isopycnal Displacement
IW-turbulence (10-15 GMT July 29)
High-Order Structure Functions
IW-T type (Power Law)

\[ S(r, q) \propto r^{\zeta(q)}, \]
\[ \zeta(q) = H(q)q, \]
\[ H(q) \neq 1/3 \]

Multi-fractal Structure
Internal Soliton (IS-Type)
(0700-1200 GMT July 30)
Isopycnal Displacement
IS-turbulence (07-12 GMT July 30)

Frequency is around 4 CPH
Structure Function (No Power Law) 
IS-T type

Power law breaks at 8 min, near half period (4 CPH) of the internal solitons
Conclusions

• (1) Three types of thermal variability (IW, IS, and T) are identified.

• (2) Without internal waves/internal solitons, the upper layer of the Philippine Sea shows simple self similarity.

• (3) With the internal wave propagation, multifractal structures are found in the upper layer of the western Philippine Sea.

• (4) The internal solitons destroy the power law characteristics in the structure function at the lag of 8 min, which is nearly half period of the IS (with frequency of 4 CPH).
Possible Reason for Preservation of the Power Law in Evident Internal Wave Propagation

Using the Hamiltonian formulation, Lvov and Tabak (2001) modified the Garrett-Munk spectrum into

\[
E(k,m) = \frac{2 f N E (m/m^*) A(m/m^*)}{\pi \left( N^2 k^2 + f^2 m^2 \right)},
\]

\[
m^* \equiv \gamma (\omega^2 - f^2)^{-\delta/2}, \quad A(\lambda) \equiv \frac{t - 1}{(1 + \lambda)^t}
\]

Internal waves and wave turbulence have similar spectrum.
Possible Reason for Break of the Power Law in Internal Soliton Propagation

- The internal solitary waves are a class of nonsinusoidal, nonlinear, more-or-less isolated waves of complex shape that maintain their coherence. Their energy spectrum is totally different from the internal wave spectrum.
References
