20th Conference on Probability and Statistics in the Atmospheric Sciences
American Meteorological Society, Atlanta, January 16-21, 2010

# Statistical Structure of Global Significant Wave Heights

*Peter C. Chu[1], George Galanis[2], Y.H. Kuo[1]*
[1]Naval Postgraduate School, USA
[2]University of Athens, Greece
pcchu@nps.edu,
http://faculty.nps.edu/pcchu
tel: 831-656-3688, fax: 831-656-3686

# Significant Wave Height ($H_s$)

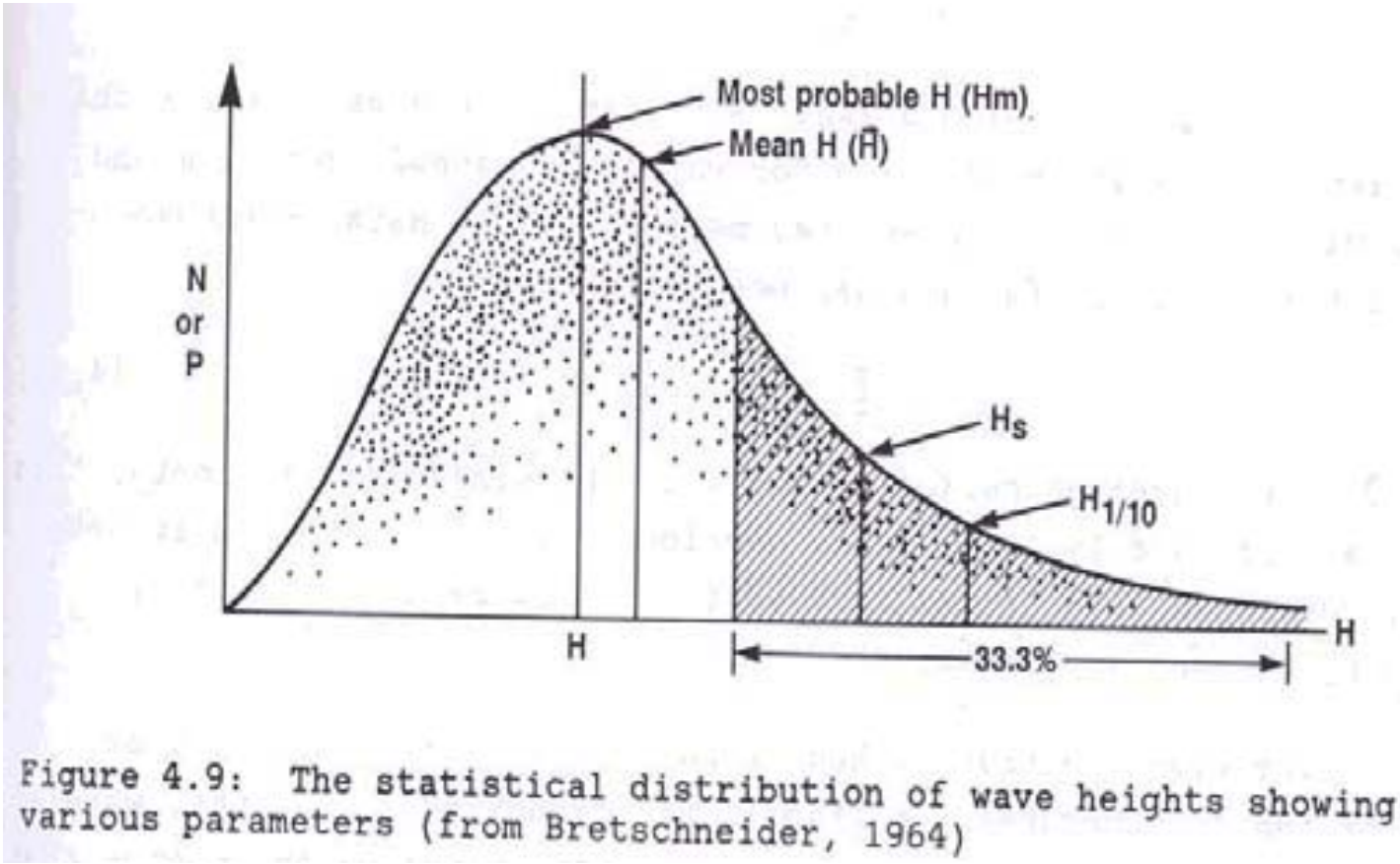
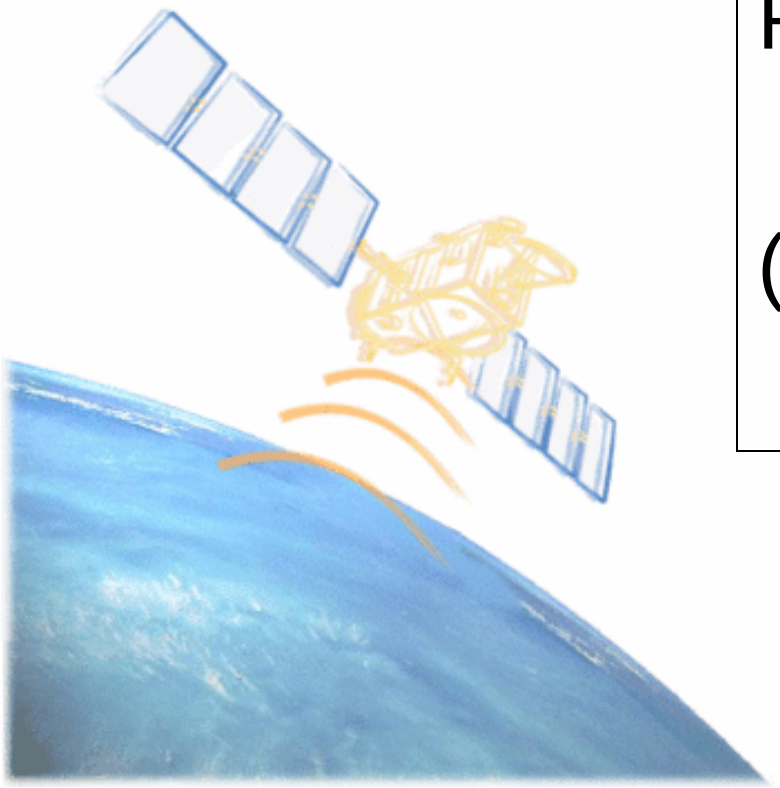



Figure 4.9: The statistical distribution of wave heights showing various parameters (from Bretschneider, 1964)

average height of the highest one-third waves in a wave spectrum

# Ocean Significant Wave Heights



Radar Altimetry Tutorial

Merged all Satellite

(CNES, ESA, NASA, NOAA, US NAVY)

- Mean

$$Mean = \mu = \frac{1}{N} \cdot \sum_{i=1}^{N} swh(i)$$

- Stand Deviation

$$\sigma = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} \left( swh(i) - \mu \right)^2}$$

- Coefficient of variation

$$c_v = \frac{\sigma}{\mu}$$

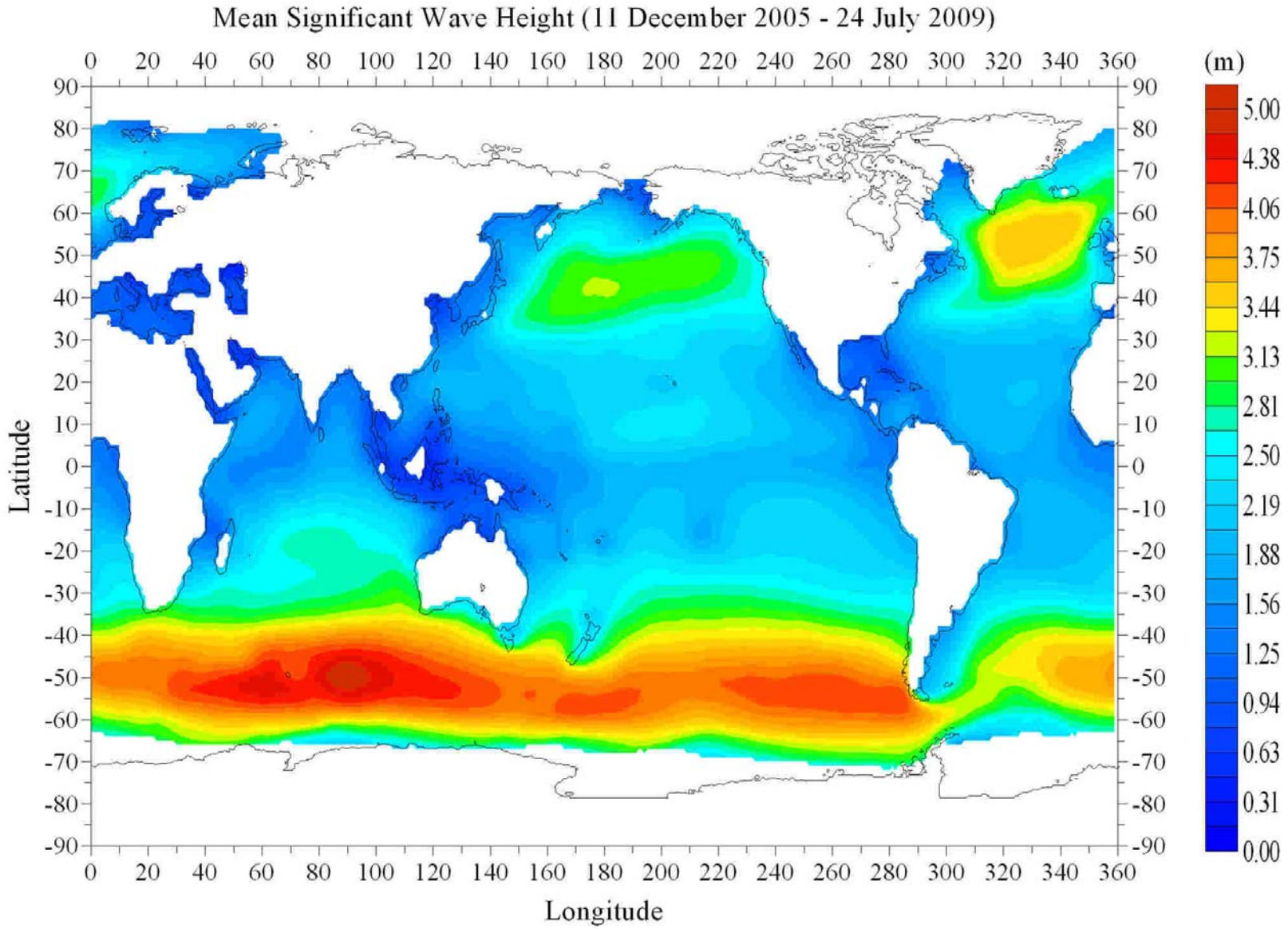
- Skewness

$$g_1 = \frac{\frac{1}{N} \cdot \sum_{i=1}^{N} \left( swh(i) - \mu \right)^3}{\sigma^3}$$

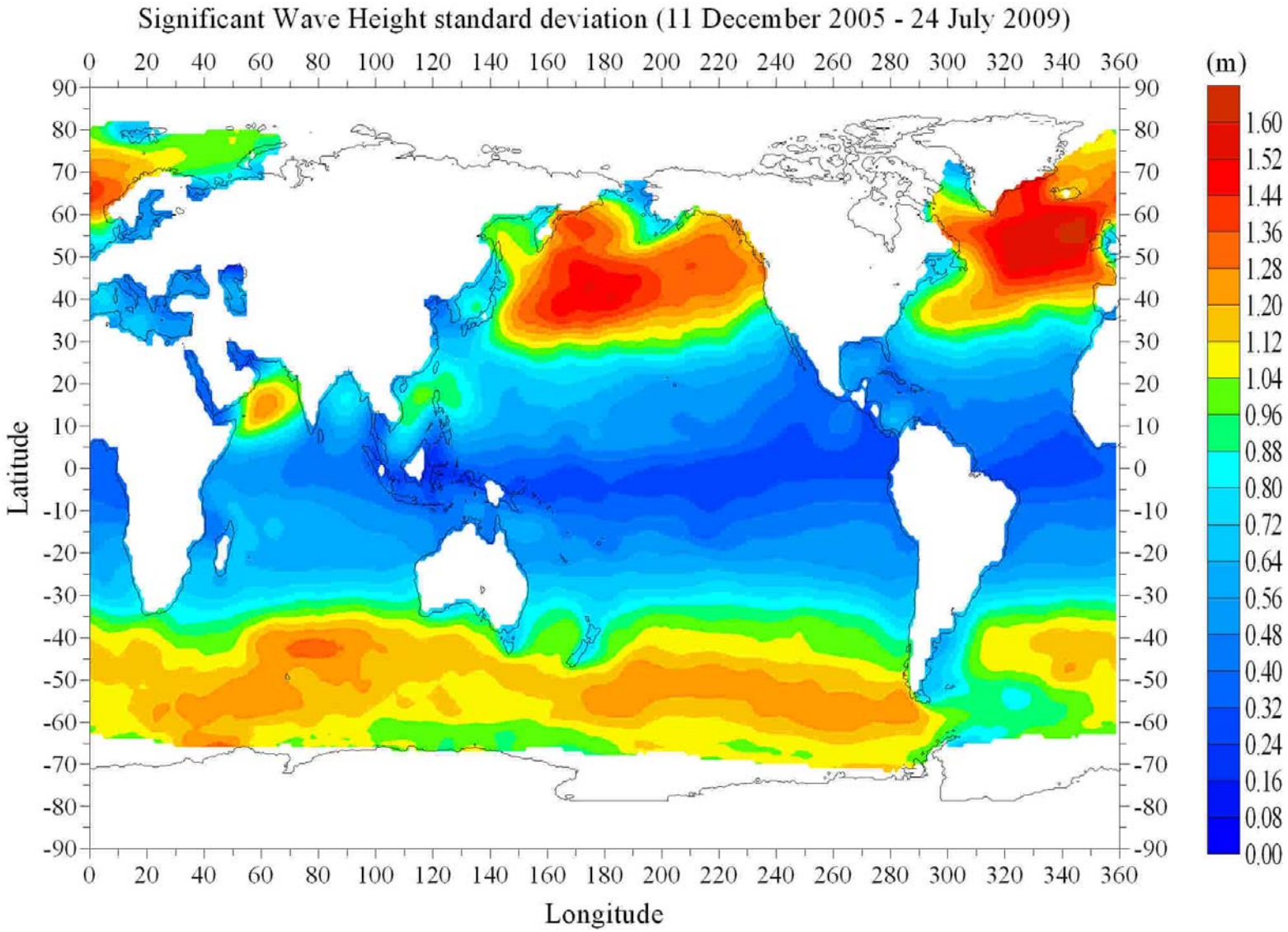
- Kurtosis

$$g_2 = \frac{\frac{1}{N} \cdot \sum_{i=1}^{N} \left( swh(i) - \mu \right)^4}{\sigma^4} - 3$$

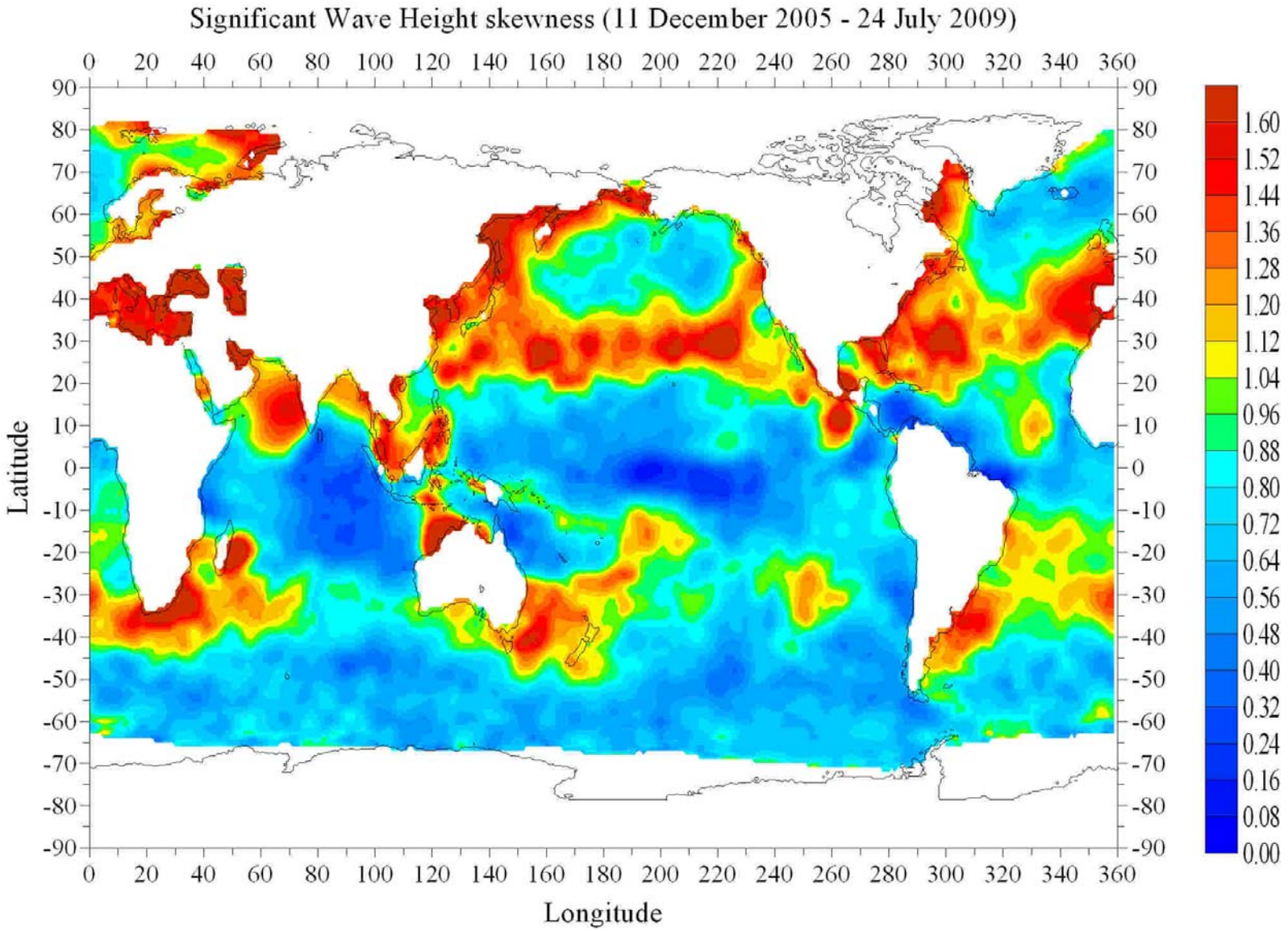
# Mean Significant Wave Height



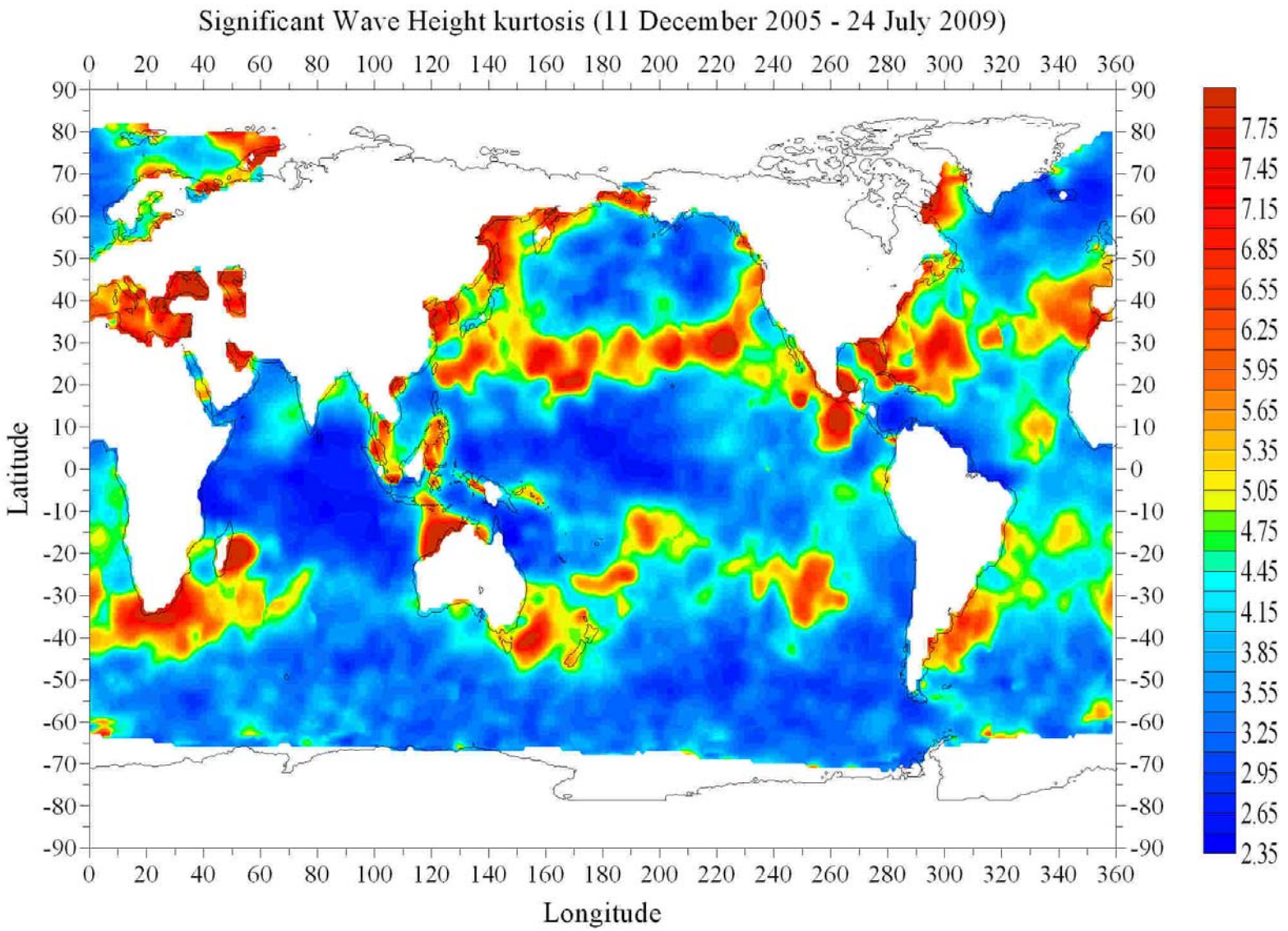
# Standard Deviation



# Skewness


Significant Wave Height skewness (11 December 2005 - 24 July 2009)
Latitude
Longitude
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360
90 80 70 60 50 40 30 20 10 0 -10 -20 -30 -40 -50 -60 -70 -80 -90
1.60
1.52
1.44
1.36
1.28
1.20
1.12
1.04
0.96
0.88
0.80
0.72
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00

# Kurtosis


Significant Wave Height kurtosis (11 December 2005 - 24 July 2009)
Latitude
Longitude
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360
90 80 70 60 50 40 30 20 10 0 -10 -20 -30 -40 -50 -60 -70 -80 -90
7.75
7.45
7.15
6.85
6.55
6.25
5.95
5.65
5.35
5.05
4.75
4.45
4.15
3.85
3.55
3.25
2.95
2.65
2.35

# Two Parameter Weibull Distribution

$$p(x) = \frac{b}{a}\left(\frac{x}{a}\right)^{b-1} \exp\left[-\left(\frac{x}{a}\right)^{2}\right]$$

# Weibull Distribution

$$\text{mean}(w) = a\Gamma\left(1+\frac{1}{b}\right),$$

$$\text{std}(w) = a\left[\Gamma\left(1+\frac{2}{b}\right)-\Gamma^2\left(1+\frac{1}{b}\right)\right]^{1/2}$$

$\Gamma$ → Gamma Function

$$\text{skew}(w) = \frac{\Gamma\left(1+\frac{3}{b}\right) - 3\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) + 2\Gamma^3\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^{3/2}}$$

$$\text{kurt}(w) = \frac{\Gamma\left(1+\frac{4}{b}\right) - 4\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{3}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^2}$$

$$+ \frac{6\Gamma^2\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) - 3\Gamma^4\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^2} - 3$$

# Kurt ←→ Skew

- Since

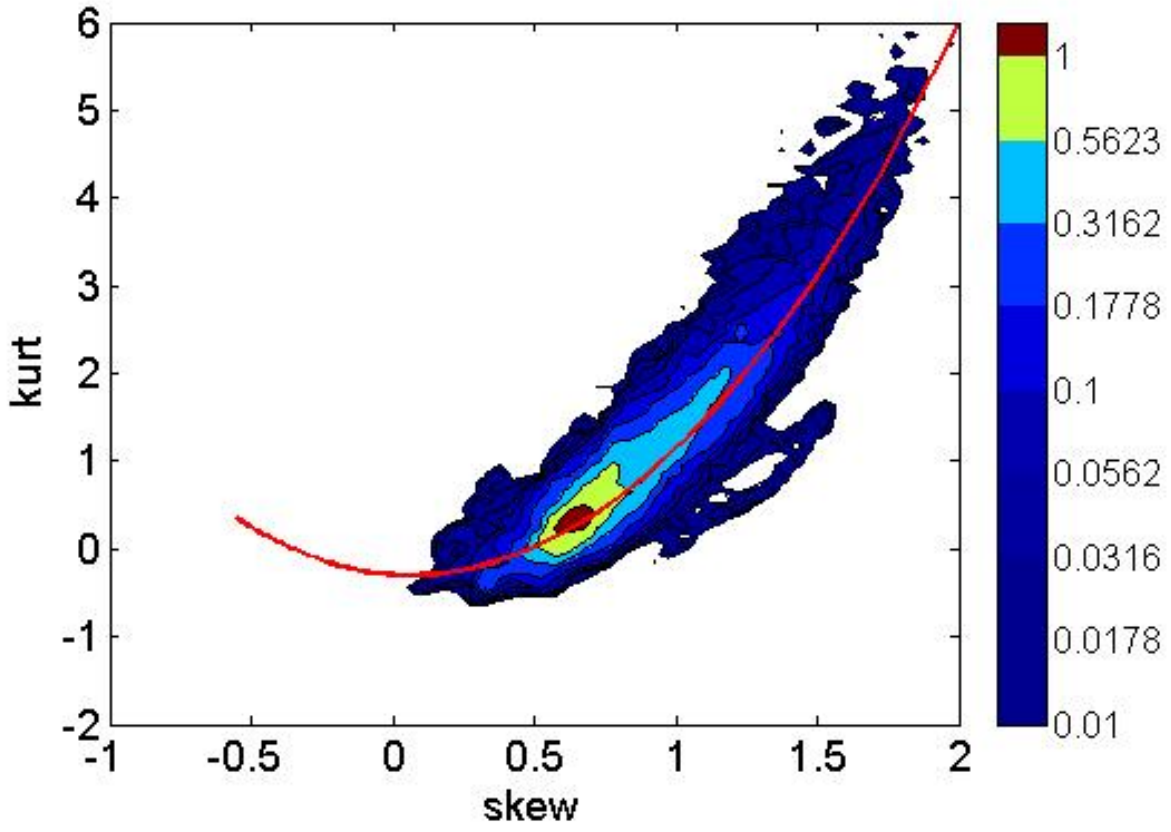
$$\text{Skew} = f_1(b), \quad \text{Kurt} = f_2(b)$$

→

$$\text{Kurt} = F(\text{Skew})$$

# Kernel Density Estimates of Joint PDF of Kurtosis and Skewness

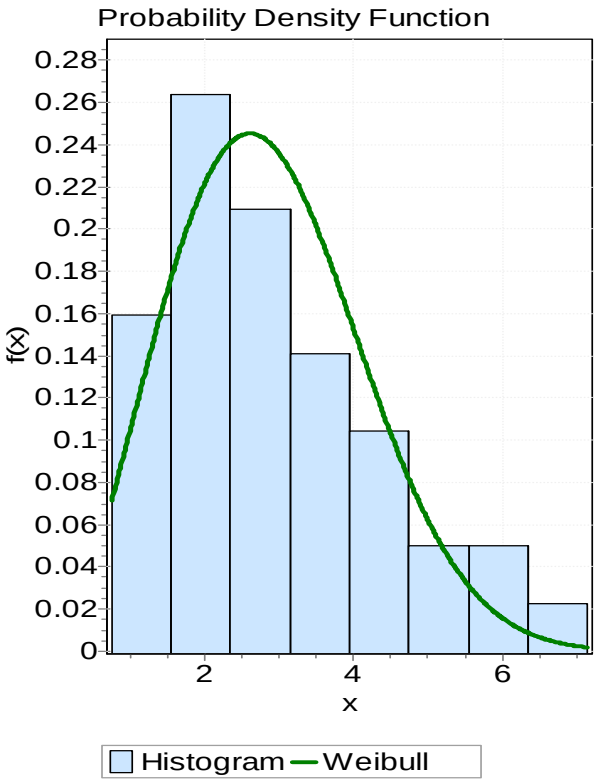
(Red Curve → Weibull Distribution)

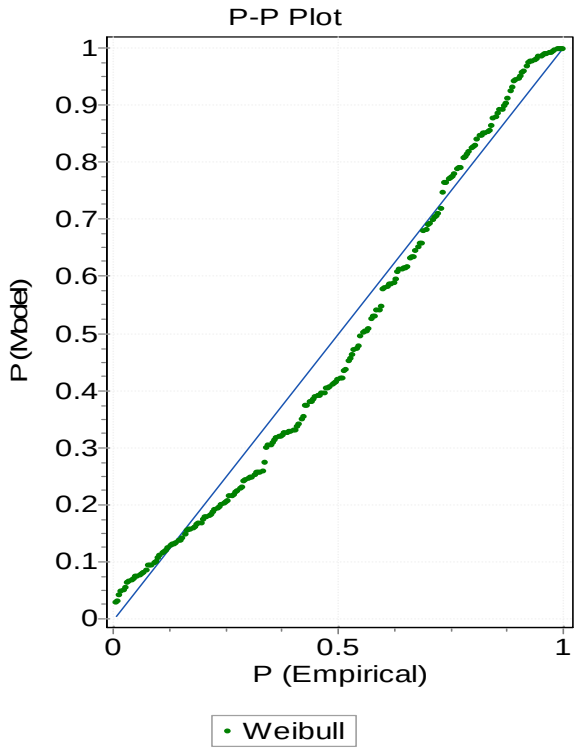


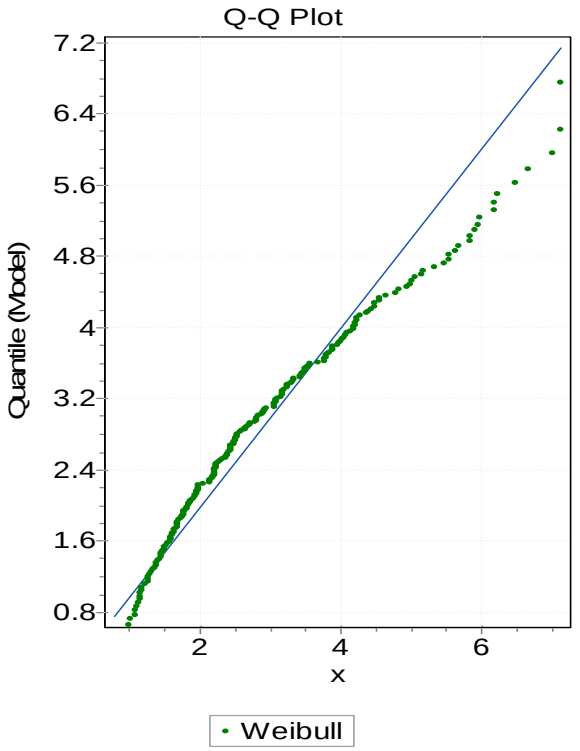
# Seasonal Variation (2008) in North Atlantic

| Statistical Parameter | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Range*** | 7.89 | 8.15 | 8.37 | 7.27 | 6.45 | 4.00 | 4.36 | 6.05 | 5.48 | 7.21 | 6.29 | 7.74 |
| ***Mean*** | 2.87 | 2.71 | 2.54 | 2.24 | 1.67 | 1.55 | 1.46 | 1.56 | 1.78 | 2.21 | 2.33 | 2.75 |
| ***Std*** | 1.44 | 1.38 | 1.21 | 0.98 | 0.73 | 0.61 | 0.63 | 0.71 | 0.88 | 1.15 | 1.09 | 1.40 |
| ***Coef. of Variation*** | 0.50 | 0.51 | 0.48 | 0.44 | 0.44 | 0.39 | 0.43 | 0.46 | 0.49 | 0.52 | 0.47 | 0.51 |
| ***Skewness*** | 1.03 | 1.07 | 1.19 | 0.93 | 1.27 | 0.54 | 0.74 | 1.56 | 1.25 | 1.37 | 1.09 | 1.07 |
| ***Kurtosis*** | 0.89 | 0.98 | 1.91 | 1.56 | 3.58 | 0.79 | 1.45 | 5.56 | 1.87 | 2.11 | 1.26 | 0.92 |

# Weibull - January 2008



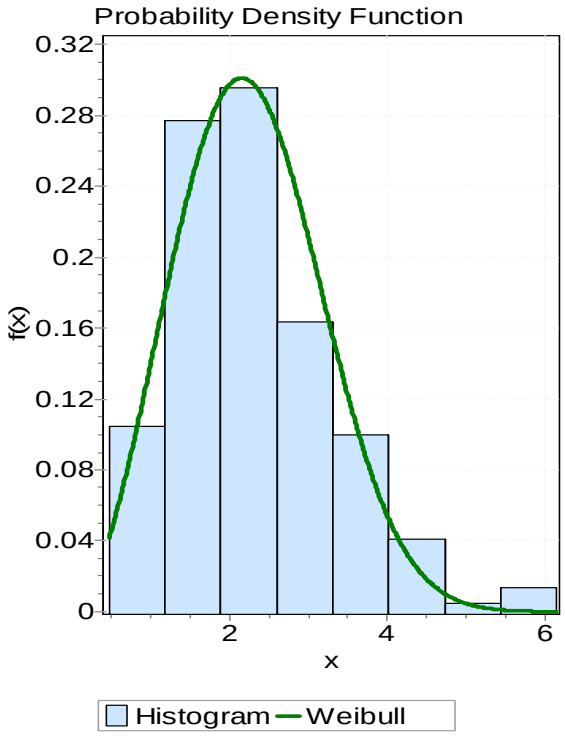



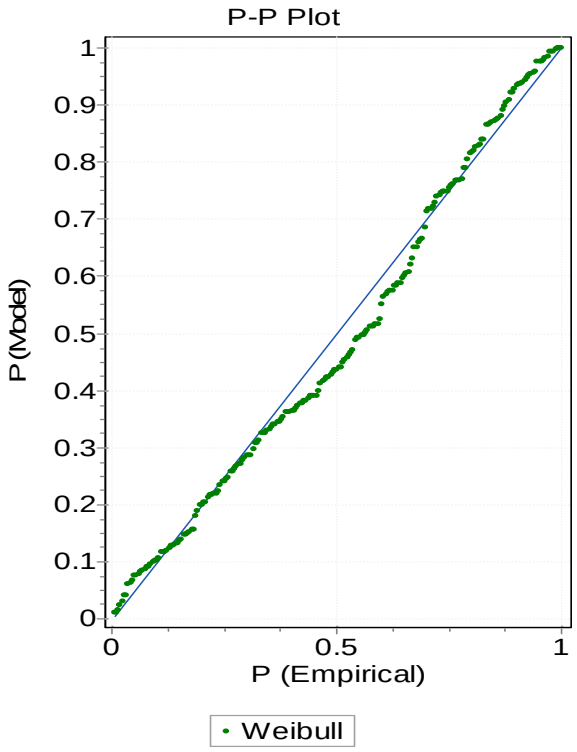

PDF

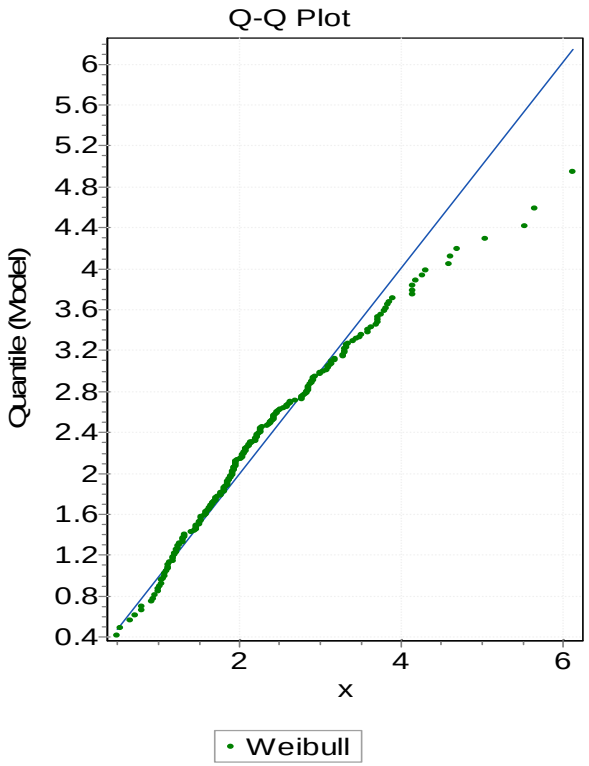
P-P Plot

Q-Q Plot

# Weibull - April 2008



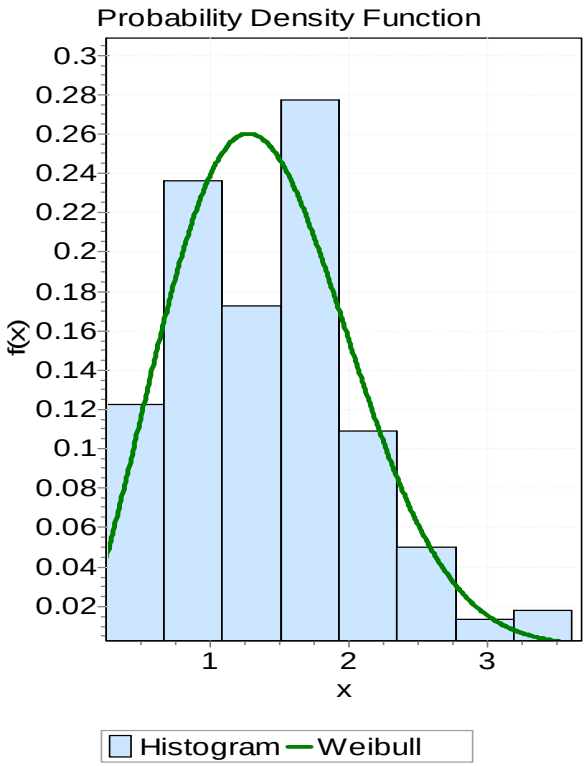



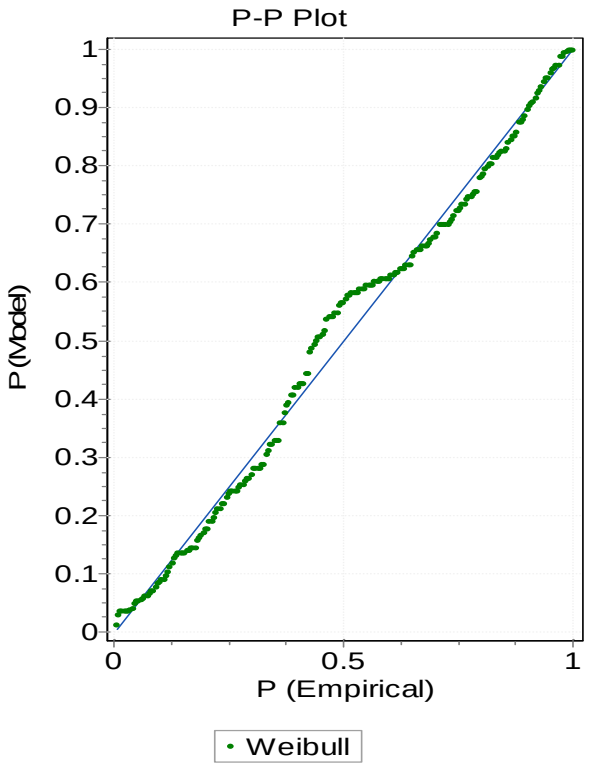

PDF

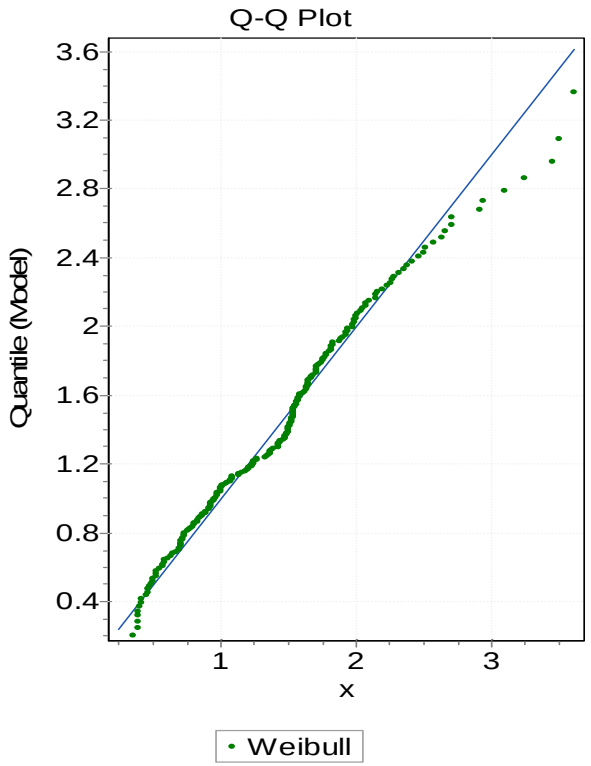
P-P Plot

Q-Q Plot

# Weibull – July 2008



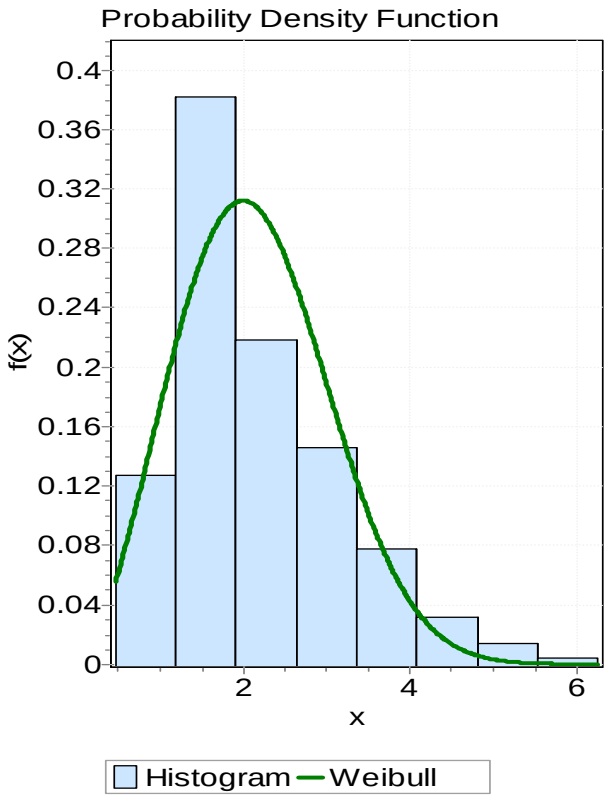



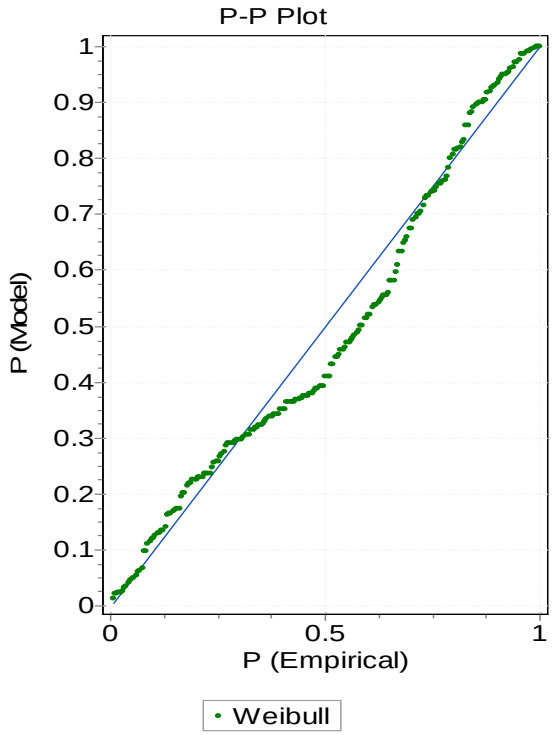

PDF

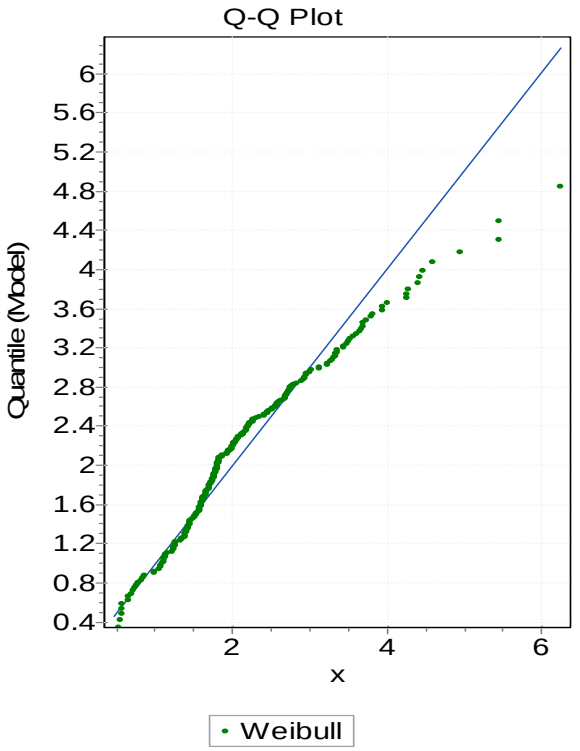
P-P Plot

Q-Q Plot

# Weibull - October 2008




PDF

P-P Plot

Q-Q Plot

# Conclusions

- The ***Weibull distribution*** provides a reasonable empirical approximation to the PDF of the significant wave heights for the global oceans.