A Minimal Conserved Adjustment Scheme for Ocean Data Assimilation Systems

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Reference

Outline

• (1) False Static Instability

• (2) Adjustment Requirements

• (3) New Conserved Minimal Adjustment Scheme

• (4) Comparison to Existing Methods
(1) False Static Instability

- \((T, S)\) climatological data
- \((T, S)\) data assimilation
Criterion of Density Inversion

\[ \Delta \rho = \rho_k^a (n + 1) - \rho_k^a (n) \]

\[ n \rightarrow \text{z-level} \rightarrow z_n \]

Here, \( n \) increases downward.
NODC Static Stability Criterion

- Density inversion $\rightarrow$ Depth-decrease of density of two consecutive $z$-levels

\[
\Delta \rho < 0.03 \text{ kg m}^{-3} \quad (-30 \text{ m} \leq z < 0)
\]

\[
< 0.02 \text{ kg m}^{-3} \quad (-400 \text{ m} < z < -30 \text{ m})
\]

\[
< 0 \text{ kg m}^{-3} \quad (z < -400 \text{ m})
\]
Data Assimilation

→ Conducted in the Physical Space \((i, j, k)\)

\[
x_a = x_b + W \cdot d,
\]

Innovation → \(d = y_o - H(x_b)\)

Various ways → \(W\) - Matrix

→ Different Data Assimilation Schemes
Major Methods of Ocean Data Assimilation

- Optimal Interpolation (OI)
- Kalman Filter
- Variational Methods
• Ocean observational \((T, S)\) profile data has different sizes in vertical.

• The number of observational data may vary with horizontal level, i.e., more data points are assimilated in some levels than others.

• Due to nonlinearity of the Equation of State, such a treatment may lead to false static instability.
SODA 2007 (monthly)
JPL-ECCO 2007 (10-day)
Centered on Dec 31, 2008
Unstable Profiles → 35.32%
GODAS December 2008
Unstable Profiles → 0.37%
(2) Requirements for Stabilization of (T, S) Casts

- (a) Minimal Adjustment $\Rightarrow$ Relative root mean adjustment (RRMA)

- (b) Heat and Salt Conservation

- (c) Well-Posed (Easy to Get Results)
Relative root mean adjustment (RRMA)

$$RRMA = \frac{\sqrt{\frac{1}{K} \sum_{k=1}^{K} (\Delta T_k)^2}}{\max(T_k) - \min(T_k)} + \frac{\sqrt{\frac{1}{K} \sum_{k=1}^{K} (\Delta S_k)^2}}{\max(S_k) - \min(S_k)}$$
(3) A Minimal Conserved Adjustment Scheme
Static stability \((E)\)

Lynn and Reid (1968)

- Discrete samples \((T_k, S_k)\) at depth \(z_k, k = 1, 2, \ldots, K\) \((k\) increasing downward)
Stabilization

- (a) stability increasing at unstable levels to

\[ E_{k_i}^* = E_{\text{min}} \rightarrow \Delta E_{k_i} = E_{\text{min}} - E_{k_i} \]

- (b) stability decreasing at stable levels

\[ E_{k_i \pm m}^* = \begin{cases} 
E_{k_i \pm m} - \Delta E_{k_i} / 2^{m+1} & \text{if } E_{k_i \pm m} - \Delta E_{k_i} / 2^{m+1} \geq E_{\text{min}} \\
E_{\text{min}} & \text{if } E_{k_i \pm m} - \Delta E_{k_i} / 2^{m+1} < E_{\text{min}}
\end{cases} \]

- (c) normalization for conservation of stability for the cast.

\[ I = \sum_{k=1}^{K} E_k, \quad I^* = \sum_{k=1}^{K} E_k^* \quad \quad E_{k_i}^{**} = \frac{I}{I^*} E_k^* \]
2K Basic Algebraic Equations for Adjustment

\[ \rho(S_{k+1} + \Delta S_{k+1}, T_{k+1} + \Delta T_{k+1}, z_k) - \rho(S_k + \Delta S_k, T_k + \Delta T_k, z_k) = E_k^{**}, \]

\[ k = 1, 2, ..., K - 1. \]

Heat Conservation $\rightarrow$

\[ \sum_{k=1}^{K-1} \frac{(\Delta T_k + \Delta T_{k+1})}{2} (z_k - z_{k+1}) = 0 \]

Salt Conservation $\rightarrow$

Assigned (T, S) Adjustment Ratio $\rightarrow$

\[ \Delta T_k + \gamma_k \Delta S_k = 0 \]

\[ k = 1, 2, ..., K-1 \]
\[ \mathbf{P} \equiv \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ \vdots \\ \vdots \\ P_{M-1} \\ P_M \end{bmatrix} = \begin{bmatrix} \Delta T_1 \\ \Delta S_1 \\ \Delta T_2 \\ \Delta S_2 \\ \vdots \\ \vdots \\ \Delta T_K \\ \Delta S_K \end{bmatrix}, \quad M = 2K. \]
2K Basic Algebraic Equations

\[ F(P) = 0 \quad (1) \]

\( F \) has the dimension of \( 2K \)

Combination of \((K-1)\) Nonlinear

and \((K+1)\) Linear Equations
Grid-box 171.5°E, 53.5°S WOA98 profiles before stabilization (from Locarnini et al. 2006, Table B1). Here the symbol ‘*’ in the last column indicates the static instability.

<table>
<thead>
<tr>
<th>k</th>
<th>Depth (m)</th>
<th>T (°C)</th>
<th>S (ppt)</th>
<th>$\rho(S_{k+1}, T_{k+1}, z_k)$ (kg m$^{-3}$)</th>
<th>$\rho(S_k, T_k, z_k)$ (kg m$^{-3}$)</th>
<th>$E_k$ (kg m$^{-3}$)</th>
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Original (dashed) and adjusted (solid) profiles temperature ($T_k$), salinity ($S_k$), and static stability ($E_k$) at the grid box $171.5^\circ$E, $53.5^\circ$S using the analytical conserved method.
Performance

- Heat and Salt Conserved
- Minimal Adjustment

\[ \text{RRMA} = 0.0482 < 0.0712 \ (\text{JM Method}) \]

- Analytical $\rightarrow$ Well-Posed
(4) Comparison to Existing Methods

• (1) Minimal Adjustment Method (Jackett and McDougal, 1995)

• (2) Convective Adjustment Method (Yin and Sarachik 1994)
World Ocean Atlas 2005
(Locarnini et al., 2006)

• Stabilization of (T, S) casts using the minimal adjustment method (Jackett and McDougal, 1995)
Original (dashed) and adjusted (solid) profiles temperature ($T_k$), salinity ($S_k$), and static stability ($E_k$) at the grid box 171.5°E, 53.5°S using the JM method (Locarnini et al. 2006).
Heat and Salt Change for the (1° X 1°) grid cell

\[ \Delta Q = -7.0411 \times 10^{17} \text{J} \]

\[ \Delta \text{(salt)} = -0.5443 \times 10^{10} \text{kg} \]

RRMA = 0.0712
Convective Adjustment in Ocean Models

• Whenever a water column is statically unstable, temperature and salinity are vertically adjusted to make the water column neutrally stable, with heat and salt conserved in the process.

• For ocean data assimilation, it may over-adjust since there is no convection. Unstable stratification is caused by combination of modeled and observed data.
Original (dashed) and adjusted (solid) profiles temperature ($T_k$), salinity ($S_k$), and static stability ($E_k$) at the grid box 171.5°E, 53.5°S using the complete convective adjustment method (Yin and Sarachik 1994).
Performance of the Convective Adjustment Method

• Heat and Salt Conserved

• Large Adjustment

\[ RRMA = 0.2192 > 0.0712 \text{ (JM Method)} \]
\[ > 0.0482 \text{ (Analytical Method)} \]
Conclusions

• Minimal conserved adjustment is needed after (T, S) data analysis/assimilation

• The proposed method has the following features:

  – (1) Heat and salt conservation
  
  – (2) Removal of static instabilities with small (T, S) adjustments
  
  – (3) Minimal adjustment and analytical form