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A Minimal Conserved Adjustment Scheme for Ocean Data Assimilation Systems

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Reference

- Chu, P.C., and C.W. Fan, 2010: A conserved minimal adjustment scheme for stabilization of hydrographic profiles. *Journal of Atmospheric and Oceanic Technology,* in press

Outline

- (1) False Static Instability
- (2) Adjustment Requirements
- (3) New Conserved Minimal Adjustment Scheme
- (4) Comparison to Existing Methods

(1) False Static Instability

- (T, S) climatological data

- (T, S) data assimilation

Criterion of Density Inversion

$$\Delta\rho = \rho_k^a(n+1) - \rho_k^a(n)$$

$n \rightarrow z$-level $\rightarrow z_n$

Here, n increases downward.

NODC Static Stability Criterion

- Density inversion → Depth-decrease of density of two consecutive z-levels

$$\Delta\rho \quad < 0.03 \text{ kg m}^{-3} \quad (-30 \text{ m} \leq z < 0)$$

$$< 0.02 \text{ kg m}^{-3} \quad (-400 \text{ m} < z < -30 \text{ m})$$

$$< 0 \text{ kg m}^{-3} \quad (z < -400 \text{ m})$$

Data Assimilation

→ Conducted in the Physical Space (**i**, **j**, **k**)

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W} \bullet \mathbf{d},$$

Innovation → $\mathbf{d} = \mathbf{y}_o - H(\mathbf{x}_b)$

Various ways → **W** - Matrix

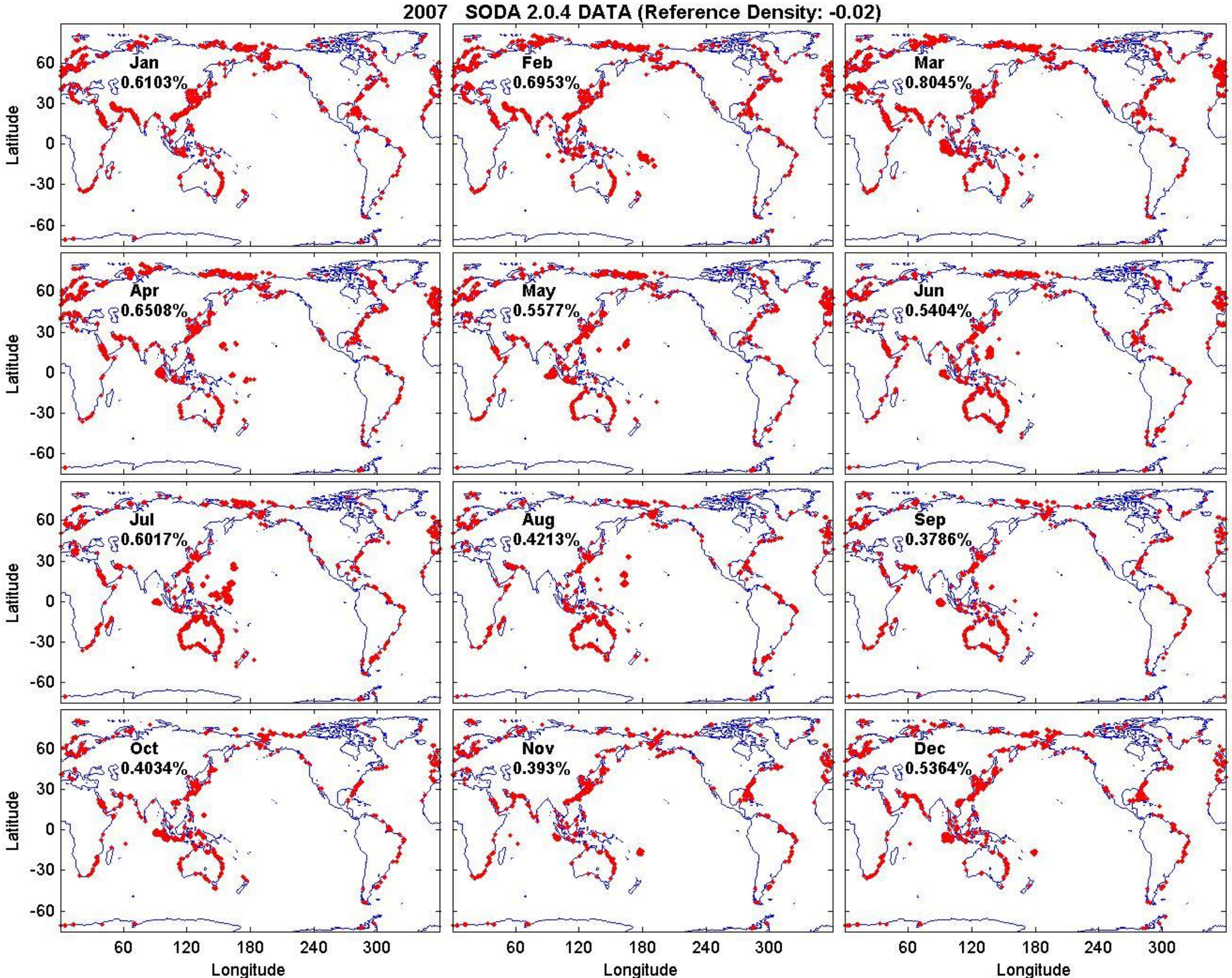
→ Different Data Assimilation Schemes

Major Methods of Ocean Data Assimilation

- Optimal Interpolation (OI)
- Kalman Filter
- Variational Methods

- Ocean observational (*T*, *S*) profile data has different sizes in vertical.
- The number of observational data may vary with horizontal level, i.e., more data points are assimilated in some levels than others.
- Due to nonlinearity of the Equation of State, such a treatment may lead to false static instability.

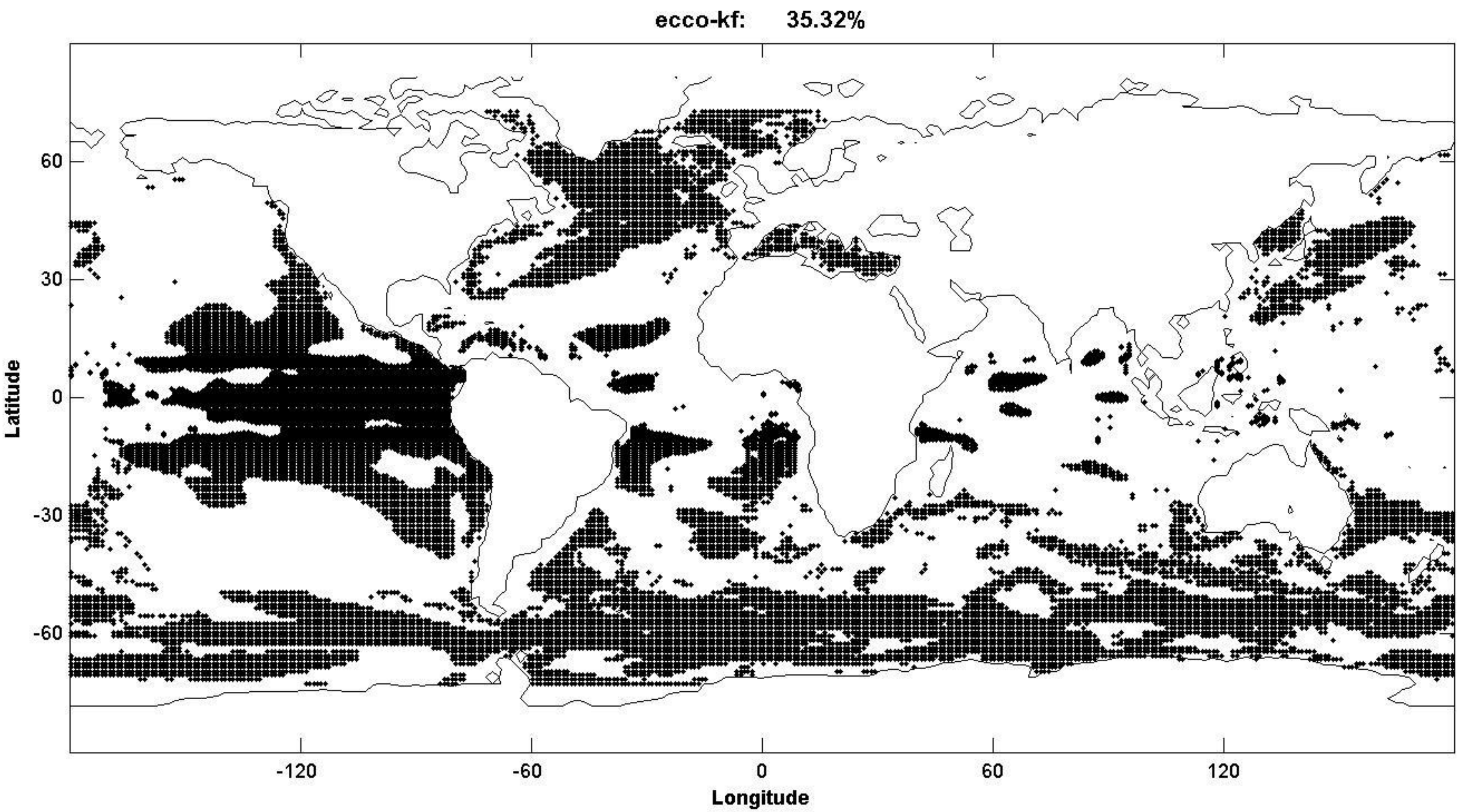
SODA 2007 (monthly)



JPL-ECCO 2007 (10-day)

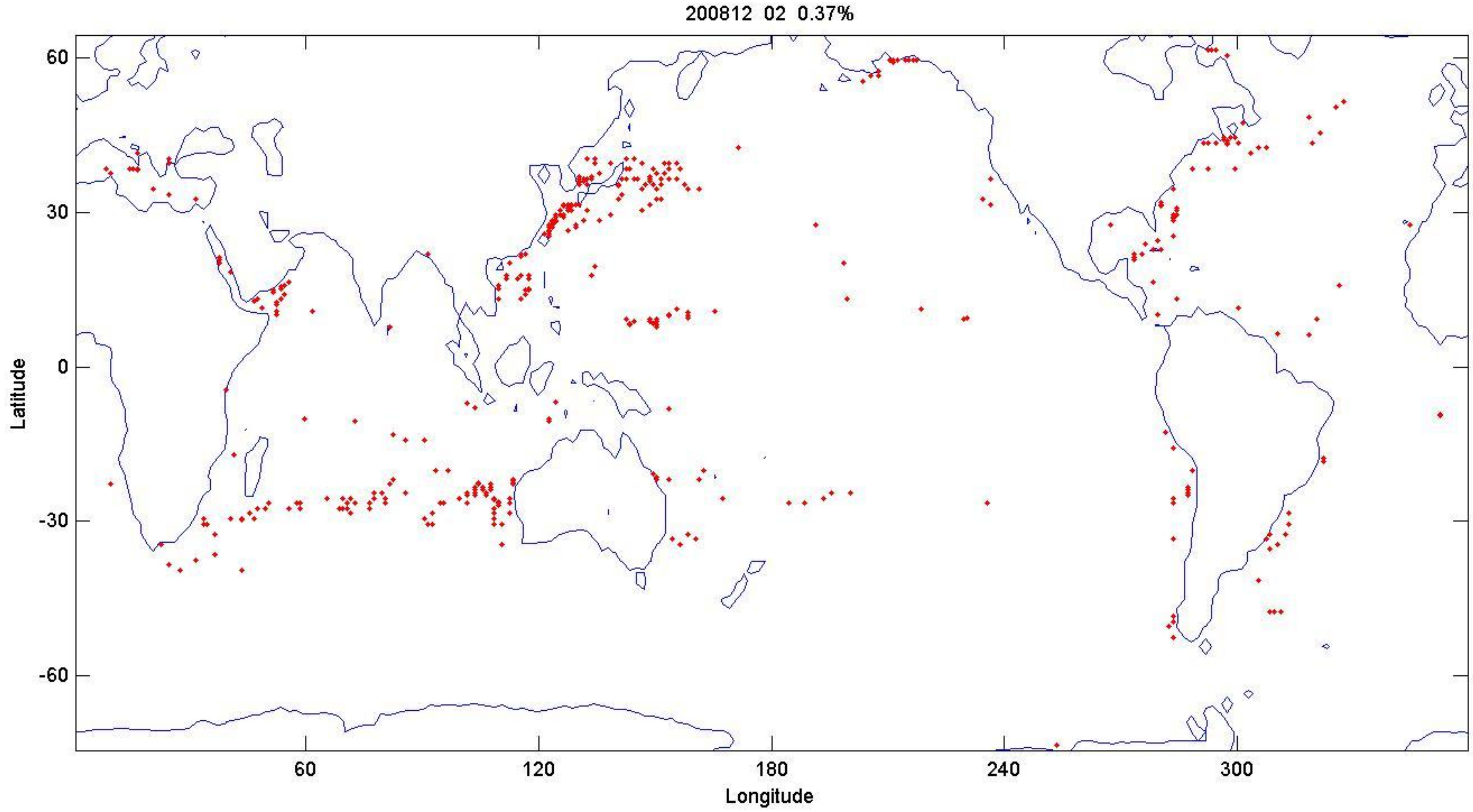
Centered on Dec 31, 2008

Unstable Profiles → 35.32%



GODAS December 2008

Unstable Profiles → 0.37%



(2) Requirements for Stabilization of (T, S) Casts

- (a) Minimal Adjustment → Relative root mean adjustment (RRMA)
- (b) Heat and Salt Conservation
- (c) Well-Posed (Easy to Get Results)

Relative root mean adjustment (RRMA)

$$\mathrm{RRMA} = \frac{\sqrt{\frac{1}{K}\sum_{k=1}^{K}(\Delta T_k)^2}}{\max(T_k) - \min(T_k)} + \frac{\sqrt{\frac{1}{K}\sum_{k=1}^{K}(\Delta S_k)^2}}{\max(S_k) - \min(S_k)}$$

(3) A Minimal Conserved Adjustment Scheme

Static stability (E)
Lynn and Reid (1968)

- Discrete samples (T_k, S_k) at depth z_k, k = 1, 2, …, K (k increasing downward)

$$E_k = \rho(S_{k+1}, T_{k+1}, z_k) - \rho(S_k, T_k, z_k),$$

$$E_k \geq E_{\min}, \quad k = 1, 2, \ldots, K$$

$E_{\min}$ is the reference value for the minimum static stability

Stabilization

- (a) stability increasing at unstable levels to
- $E_{k_l}^{*} = E_{\min}$ → $\Delta E_{k_l} = E_{\min} - E_{k_l}$
- (b) stability decreasing at stable levels

$$E_{k_l \pm m}^{*} = \begin{cases} E_{k_l \pm m} - \Delta E_{k_l} / 2^{m+1} & \text{if } E_{k_l \pm m} - \Delta E_{k_l} / 2^{m+1} \geq E_{\min} \\ E_{\min} & \text{if } E_{k_l \pm m} - \Delta E_{k_l} / 2^{m+1} < E_{\min} \end{cases}$$

- (c) normalization for conservation of stability for the cast.

$$I = \sum_{k=1}^{K} E_k, \quad I^{*} = \sum_{k=1}^{K} E_k^{*} \qquad E_k^{**} = \frac{I}{I^{*}} E_k^{*}$$

$2K$ Basic Algebraic Equations for Adjustment

$$\rho(S_{k+1}+\Delta S_{k+1}, T_{k+1}+\Delta T_{k+1}, z_k) - \rho(S_k+\Delta S_k, T_k+\Delta T_k, z_k) = E_k^{**},$$

$$k = 1,\ 2,\ \ldots,\ K\text{-}1.$$

Heat Conservation →

$$\sum_{k=1}^{K-1} \frac{(\Delta T_k + \Delta T_{k+1})}{2}(z_k - z_{k+1}) = 0$$

Salt Conservation →

Assigned (T, S) Adjustment Ratio γ_k →

$$\Delta T_k + \gamma_k \Delta S_k = 0$$

k = 1, 2, … K-1

$$\mathbf{P} \equiv \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ \vdots \\ \vdots \\ \vdots \\ p_{M-1} \\ p_M \end{bmatrix} = \begin{bmatrix} \Delta T_1 \\ \Delta S_1 \\ \Delta T_2 \\ \Delta S_2 \\ \vdots \\ \vdots \\ \vdots \\ \Delta T_K \\ \Delta S_K \end{bmatrix}, \qquad M = \mathbf{2} K.$$

2K Basic Algebraic Equations

$$\mathbf{F}(\mathbf{P}) = 0 \qquad (1)$$

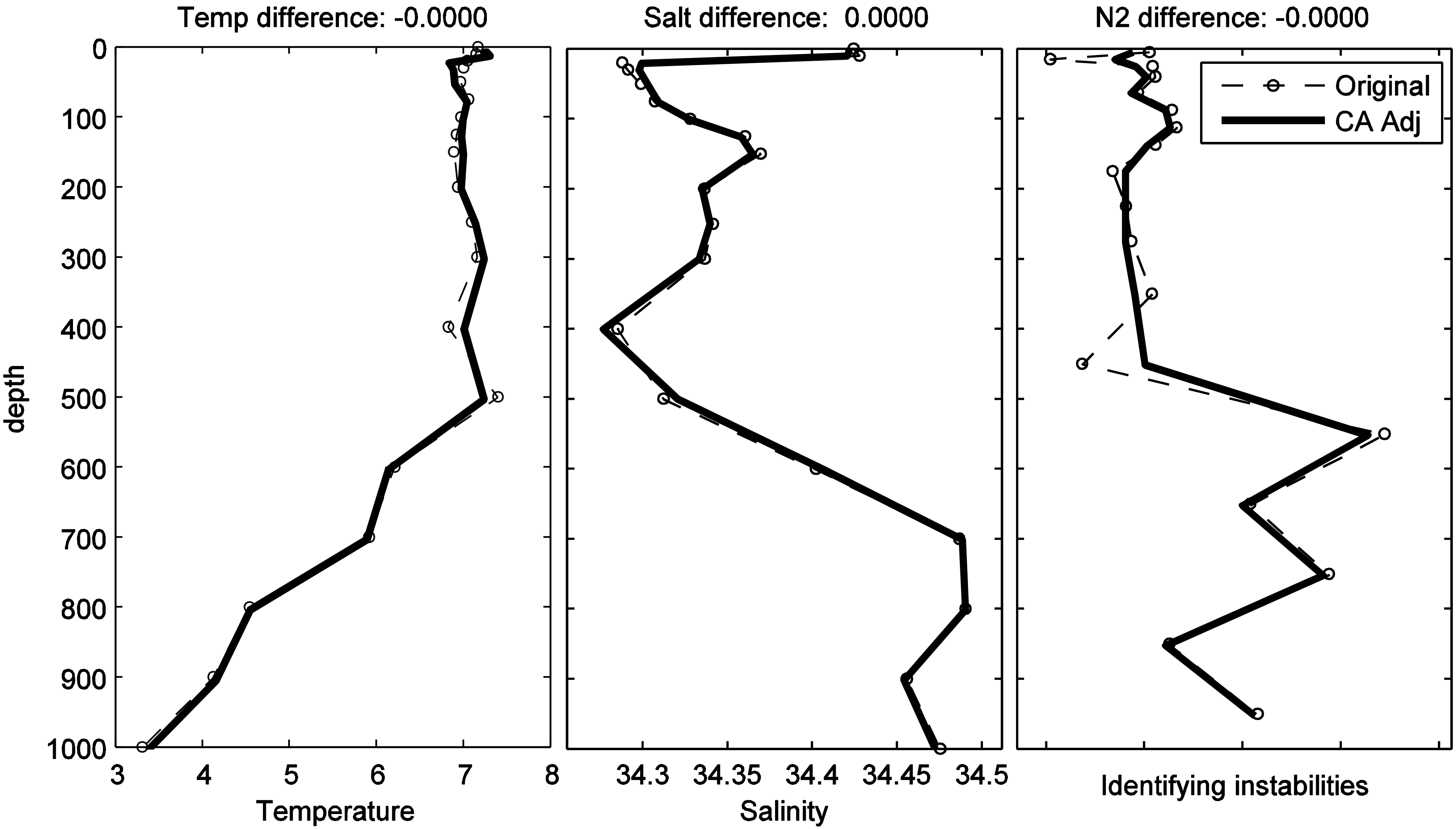
F has the dimension of 2*K*

Combination of (*K* -1) Nonlinear

and (*K*+1) Linear Equations

Grid-box 171.5°E, 53.5°S WOA98 profiles before stabilization (from Locarnini et al. 2006, Table B1). Here the symbol '*' in the last column indicates the static instability.

k	Depth (m)	T (°C)	S (ppt)	$\rho(S_{k+1}, T_{k+1}, z_k)$ (kg m^{-3})	$\rho(S_k, T_k, z_k)$ (kg m^{-3})	E_k (kg m^{-3})
1	0	7.1667	34.4243	26.9476	26.9423	0.0054
2	10	7.1489	34.4278	26.8982	26.9939	-0.0957*
3	20	7.0465	34.2880	26.9529	26.9443	0.0085
4	30	7.0050	34.2914	27.0104	26.9990	0.0114
5	50	6.9686	34.2991	27.0967	27.1028	-0.0061*
6	75	7.0604	34.3073	27.2406	27.2120	0.0286
7	100	6.9753	34.3280	27.3892	27.3560	0.0332
8	125	6.9218	34.3604	27.5164	27.5046	0.0117
9	150	6.8919	34.3697	27.6000	27.6316	-0.0316*
10	200	6.9363	34.3364	27.8123	27.8302	-0.0179*
11	250	7.0962	34.3415	28.0295	28.0421	-0.0126*
12	300	7.1622	34.3367	28.2684	28.2593	0.0092
13	400	6.8275	34.2852	28.6664	28.7281	-0.0618*
14	500	7.4001	34.3123	29.3699	29.1238	0.2461
15	600	6.2133	34.4022	29.9386	29.8292	0.1094
16	700	5.9186	34.4868	30.5869	30.3978	0.1891
17	800	4.5426	34.4904	31.0754	31.0488	0.0266
18	900	4.1263	34.4558	31.6539	31.5377	0.1162
19	1000	3.3112	34.4755		32.1176	



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5ºE, 53.5ºS using the analytical conserved method.

Performance

- Heat and Salt Conserved

- Minimal Adjustment

RRMA = 0.0482 < 0.0712 (JM Method)

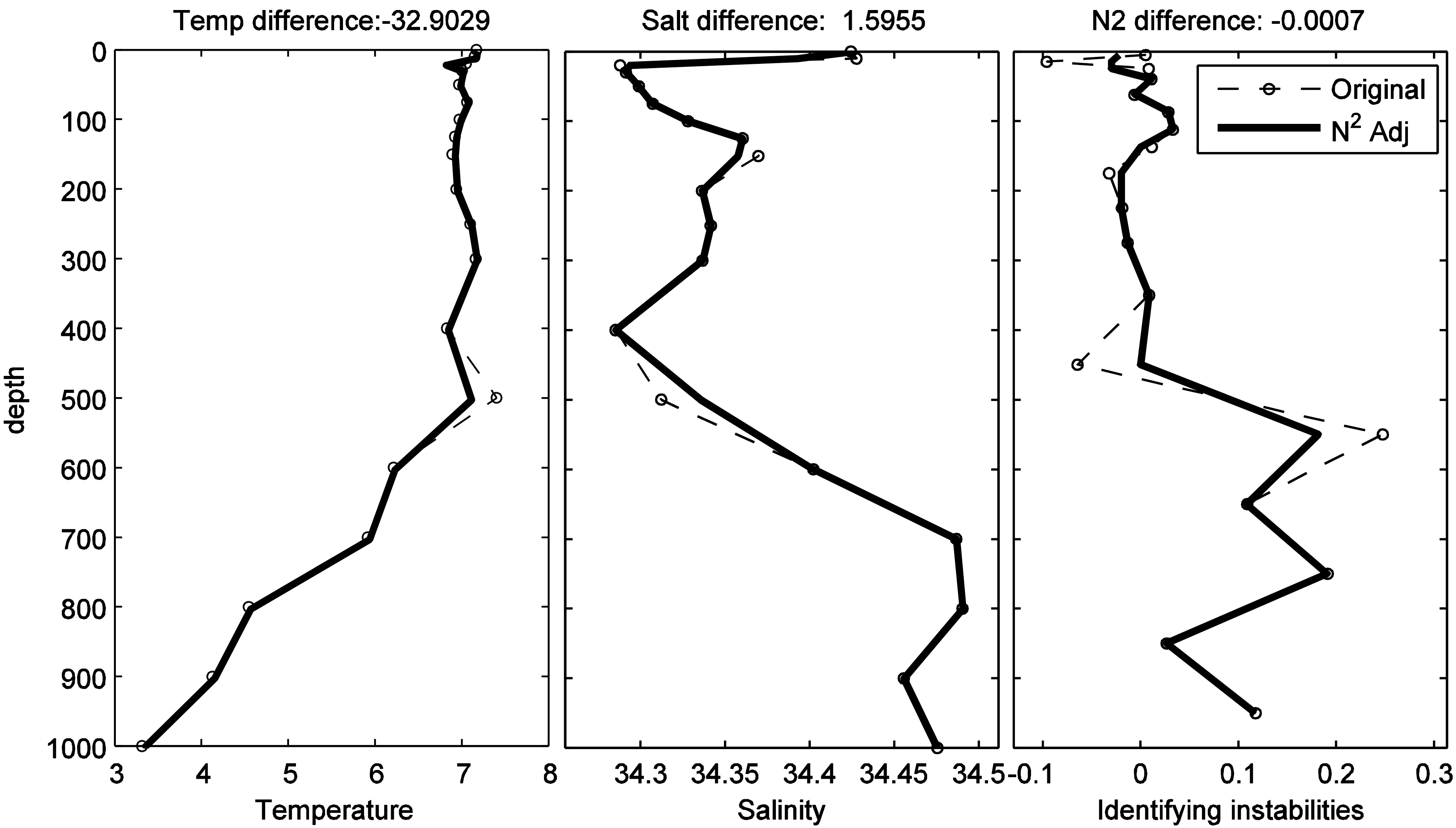
- Analytical → Well-Posed

(4) Comparison to Existing Methods

- (1) Minimal Adjustment Method (Jackett and McDougal,1995)
- (2) Convective Adjustment Method (Yin and Sarachik 1994)

World Ocean Atlas 2005 (Locarnini et al., 2006)

- Stabilization of (T, S) casts using the minimal adjustment   method (Jackett and McDougal,1995)



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5ºE, 53.5ºS using the JM method (Locarnini et al. 2006).

Heat and Salt Change for the (1^o X 1^o) grid cell

$$\Delta Q = -7.0411 \times 10^{17}\,\mathrm{J}$$

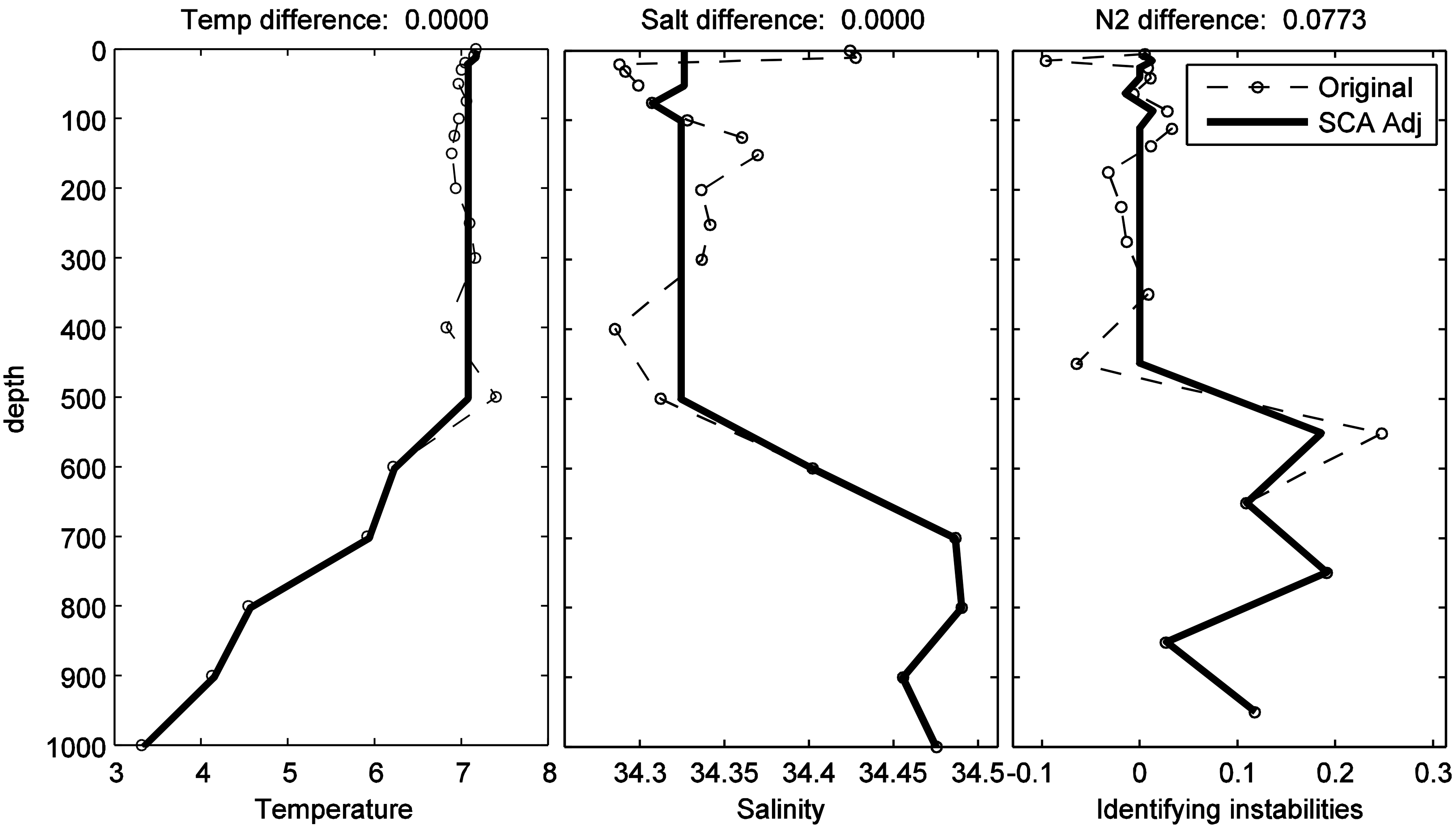
$$\Delta(\mathrm{salt}) \simeq -0.5443 \times 10^{10}\,\mathbf{kg}$$

RRMA =0.0712

Convective Adjustment in Ocean Models

- Whenever a water column is statically unstable, temperature and salinity are vertically adjusted to make the water column neutrally stable, with heat and salt conserved in the process.

- For ocean data assimilation, it may over-adjust since there is no convection. Unstable stratification is caused by combination of modeled and observed data.



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5^{o}E, 53.5^{o}S using the complete convective adjustment method (Yin and Sarachik 1994).

Performance of the Convective Adjustment Method

- Heat and Salt Conserved

- Large Adjustment

RRMA = 0.2192 > 0.0712 (JM Method)

> 0.0482 (Analytical Method)

Conclusions

- Minimal conserved adjustment is needed after (T, S) data analysis/assimilation
- The proposed method has the following features:
  - (1) Heat and salt conservation
  - (2) Removal of static instabilities with small (T, S) adjustments
  - (3) Minimal adjustment and analytical form