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False Static Instability in (T, S) Data Assimilation

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Major Methods of Ocean Data Assimilation

- Optimal Interpolation (OI)
- Kalman Filter
- Variational Methods

Ocean Variables

$\mathbf{x}_t$ → "true"

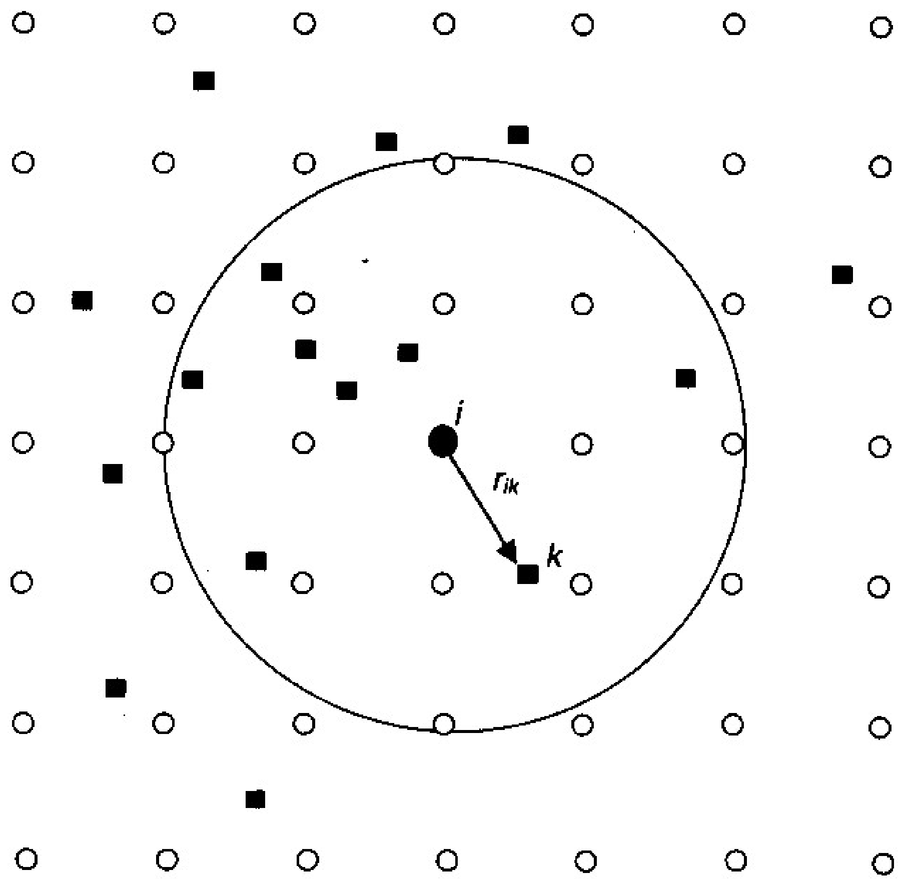
$\mathbf{x}_a$ → analysis (model variables)

$\mathbf{x}_b$ → background (or forecast)

$(\mathbf{x}_t, \mathbf{x}_a, \mathbf{x}_b)$ → grid point (i)

$\mathbf{y}_o$ → Observation → grid point (k)

Grid point (i)
Observational Point (k)



Data Analysis/Data Assimilation

→ Conducted in the Physical Space (**i**, **j**, **k**)

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W} \bullet \mathbf{d},$$

Innovation → $\mathbf{d} = \mathbf{y}_o - H(\mathbf{x}_b)$

Various ways → **W** - Matrix

→ Different Data Assimilation Schemes

W → Gain Matrix (OI)

$$\min\,(\boldsymbol{\varepsilon}_a^T \bullet \boldsymbol{\varepsilon}_a) \quad \rightarrow$$

$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$

→Depends on **B**

R (usually assumed given)

W – Gain Matrix (3D-Var)

Minimizing Cost Function → $\nabla J(\mathrm{x}) = 0$

$$2J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b)$$
$$+[\mathbf{y}_o - H(\mathbf{x})]^T \mathbf{R}^{-1}[\mathbf{y}_o - H(\mathbf{x})]$$

$$\mathbf{W} = (\mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathrm{P}^{-1}$$

→Depends on **B**

R (usually given)

W – Gain Matrix (Kalman Filter)

W – Matrix is called Kalman Gain

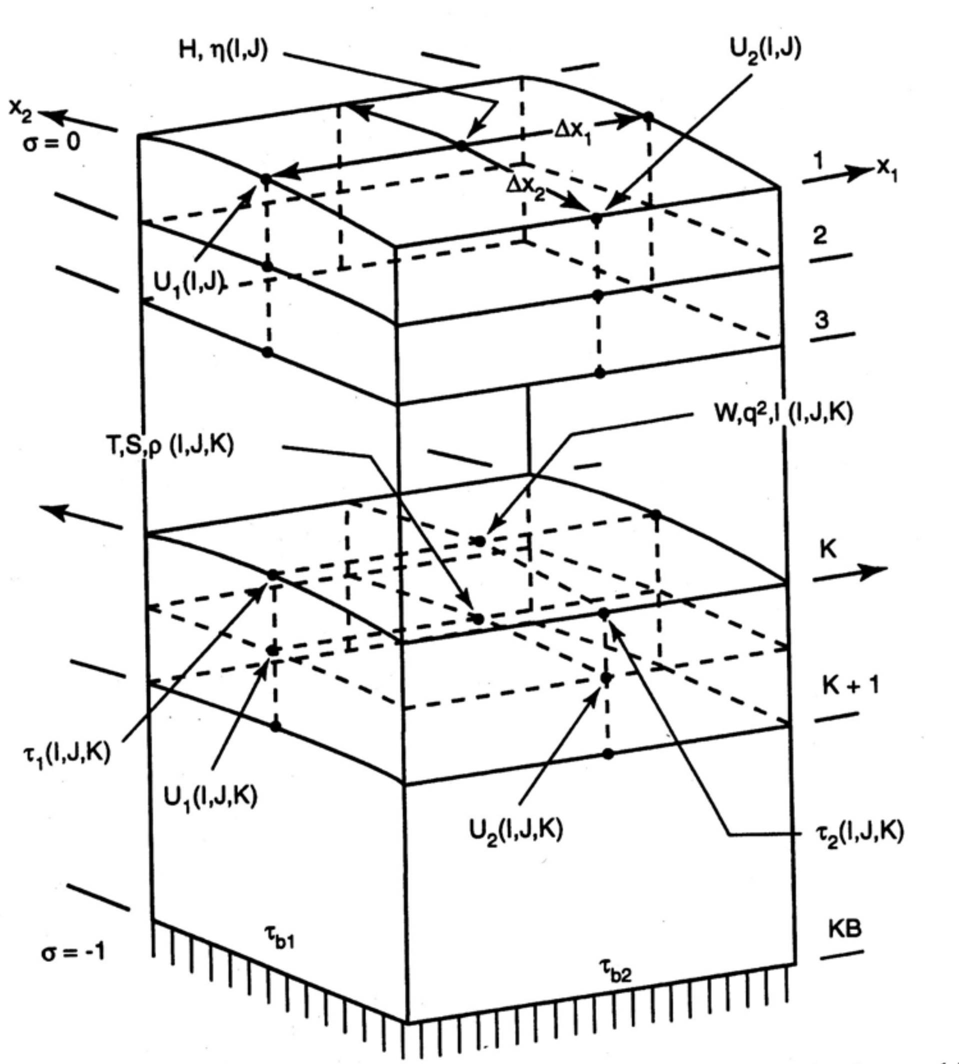
$$\mathbf{W}_i = \mathbf{P}^f(t_i)\mathbf{H}^T[\mathbf{R}_i + \mathbf{H}_i\mathbf{P}^f(t_i)\mathbf{H}^T]^{-1}$$

→Depends on forecast error covariance matrix

$\mathbf{P}^f(t_i)$

R (known)

Data Assimilation → Horizontal Planes



Problem 1

- Ocean observational (*T*, *S*) data assimilation is usually conducted in horizontal levels. Due to nonlinearity of the Equation of State

$$\rho(S,T,p)=\frac{\rho(S,T,0)}{1-p/K(S,T,p)}$$

such a treatment may lead to artificial static instability.

Problem 2

- Ocean observational (*T*, *S*) profile data has different sizes in vertical. The number of observational data may vary with horizontal level, i.e., more data points are assimilated in some levels than others. Due to nonlinearity of the Equation of State, such a treatment may lead to artificial static instability.

Density Difference

$$\rho_k^a(n) - \rho_k^a(n+1) = \Delta\rho$$

$n \rightarrow z$-level $\rightarrow z_n$

Here, n increases downward.

NODC Static Stability Criterion

- Density inversion → Depth-decrease of density of two consecutive z-levels

$$\Delta\rho \quad < \; 0.03 \;\; \text{kg m}^{-3} \;\; (-30 \text{ m} \leq z < 0)$$

$$< \; 0.02 \;\; \text{kg m}^{-3} \;\; (-400 \text{ m} \; < \; z < -30 \text{ m})$$

$$< \; 0 \;\; \text{kg m}^{-3} \;\; (z < -400 \text{ m})$$

Observational (T, S) profiles are usually through static stability check before the data assimilation

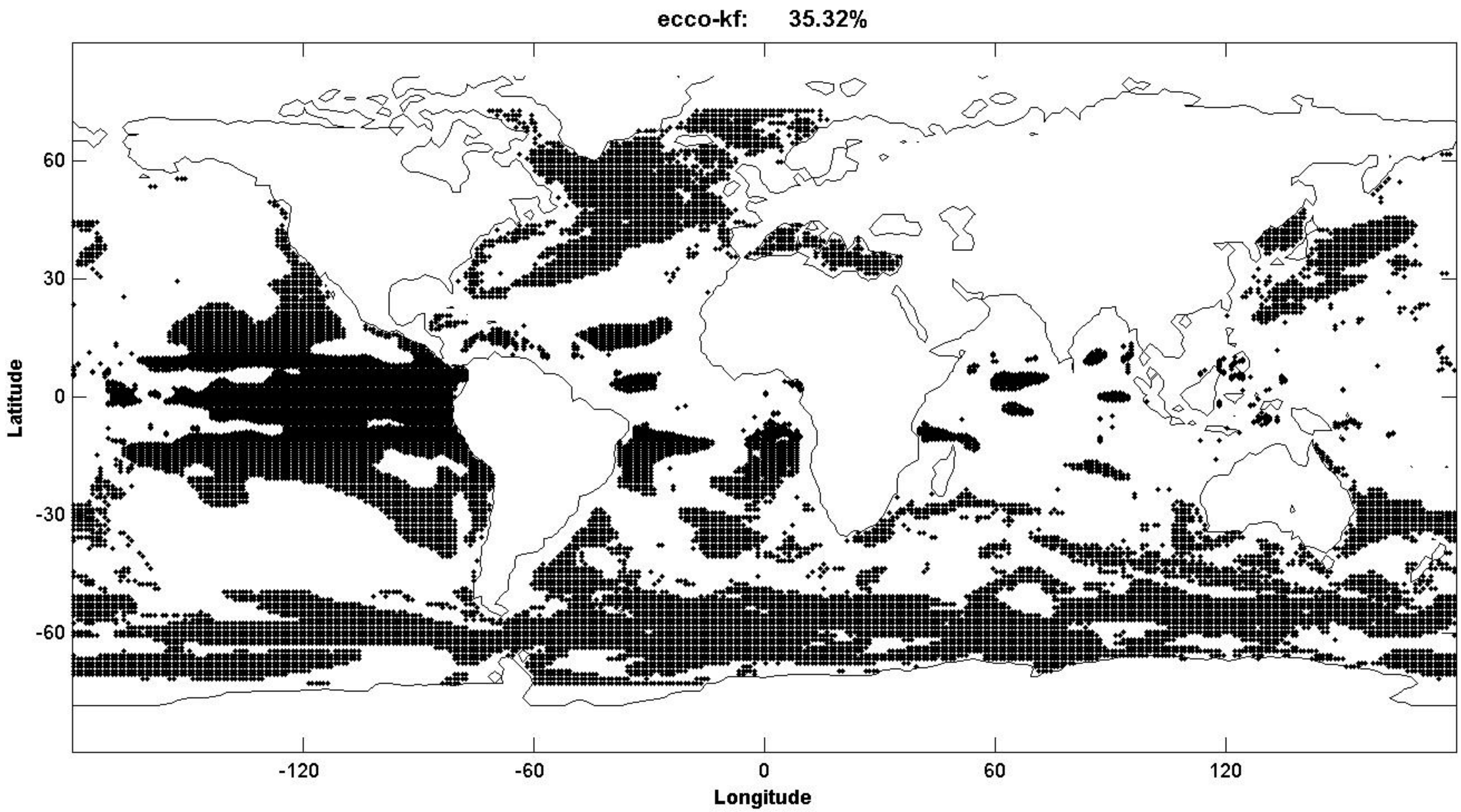
Example
JPL Estimating the Circulation and Climate of the Ocean (JPL-ECCO)
10 day

MIT General Circulation Model
+ Kalman Filter

JPL-ECCO 2007 (10-day)

Centered on Dec 31, 2008

Unstable Profiles → 35.32%



Way out

- Vertical structure of the (T, S) fields should be preserved during the data analysis procedure.

Way Out

- (1) Using vertical structure functions such as in the Navy's Global Digital Environmental Model (GDEM), which is comparable to NOAA-WOA.
- (2) Using the OSD method

OSD
Spectral Representation

$$c(\mathbf{x}, z_k, t) = A_0(z_k, t) + \sum_{m=1}^{M} A_m(z_k, t)\ \Psi_m(\mathbf{x}, z_k),$$

Spatial Variability is represented by the basis functions

→ Vertical structure is preserved

References

- Chu, P.C., L.M. Ivanov, T.P. Korzhova, T.M. Margolina, and O.M. Melnichenko, 2003a: Analysis of sparse and noisy ocean current data using flow decomposition. Part 1: Theory. Journal of Atmospheric and Oceanic Technology, 20 (4), 478-491.

- Chu, P.C., L.M. Ivanov, T.P. Korzhova, T.M. Margolina, and O.M. Melnichenko, 2003b: Analysis of sparse and noisy ocean current data using flow decomposition. Part 2: Application to Eulerian and Lagrangian data. Journal of Atmospheric and Oceanic Technology, 20 (4), 492-512.

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- Chu, P.C., L.M. Ivanov, O.M. Melnichenko, and N.C. Wells, 2007: On long baroclinic Rossby Waves in the tropical North Atlantic observed from profiling floats. Journal of Geophysical Research – Oceans, in press.

- These papers can be downloaded from:
- http://faculty.nps.edu/pcchu

Basis Functions (Closed Basin)

$$\triangle\Psi_k = -\lambda_k \Psi_k, \quad \Psi_k|_\Gamma = 0, \qquad k = 1, ..., \infty$$

$$\triangle\Phi_m = -\mu_m \Phi_m, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0, \qquad m = 1, ..., \infty.$$

Basis Functions (Open Boundaries)

$$\triangle \Psi_k = -\lambda_k \Psi_k,$$

$$\triangle \Phi_m = -\mu_m \Phi_m,$$

$$\Psi_k|_\Gamma = 0, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0,$$

$$\left[\frac{\partial \Psi_k}{\partial n} + \kappa(\tau)\Psi_k\right]|_{\Gamma_1'} = 0, \quad \Phi_m|_{\Gamma_1'} = 0,$$

Temporally varying 3D global gridded synoptic temperature, salinity, and velocity (STSV) dataset using OSD

- There is NO DENSITY INVERSION in STSV data.

Monthly Temperature (10 m) in the Pacific Ocean since 1990

Monthly Temperature (1000 m) in the Pacific Ocean since 1990

Monthly Temperature (10 m) in the Atlantic Ocean since 1990

Monthly Temperature (1000 m) in the Atlantic Ocean since 1990

Conclusions

- (1) False static instability in ocean climate (T, S) datasets with data analysis and assimilation on z-levels due to nonlinearity of the equation of state.
- (2) Assimilation of (T, S) data should keep the vertical structure such as the treatments in building GDEM and the OSD method.
- (3) It is urgent to reprocess the (T, S) datasets with false static instability.
- (4) Numerical model results need to be re-evaluated if the (T, S) datasets with false static instability were used.