Weibull Distribution for the Global Surface Current Speeds Obtained from Satellite Altimetry

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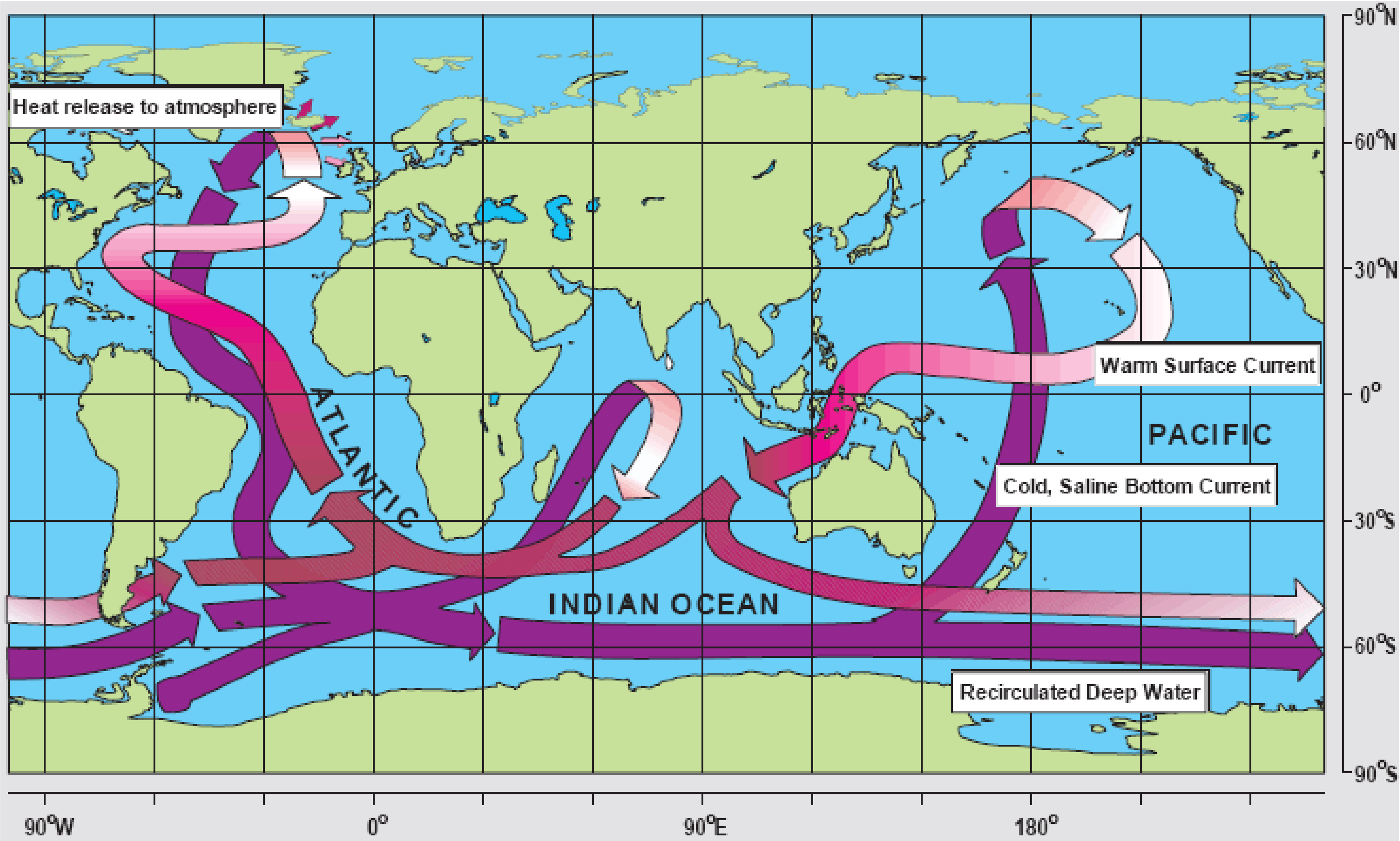
Reference

- Chu, P. C., 2008: Probability distribution function of the upper equatorial Pacific current speeds. Geophysical Research Letters, doi:10.1029/2008GL033669

Thermohaline Circulation



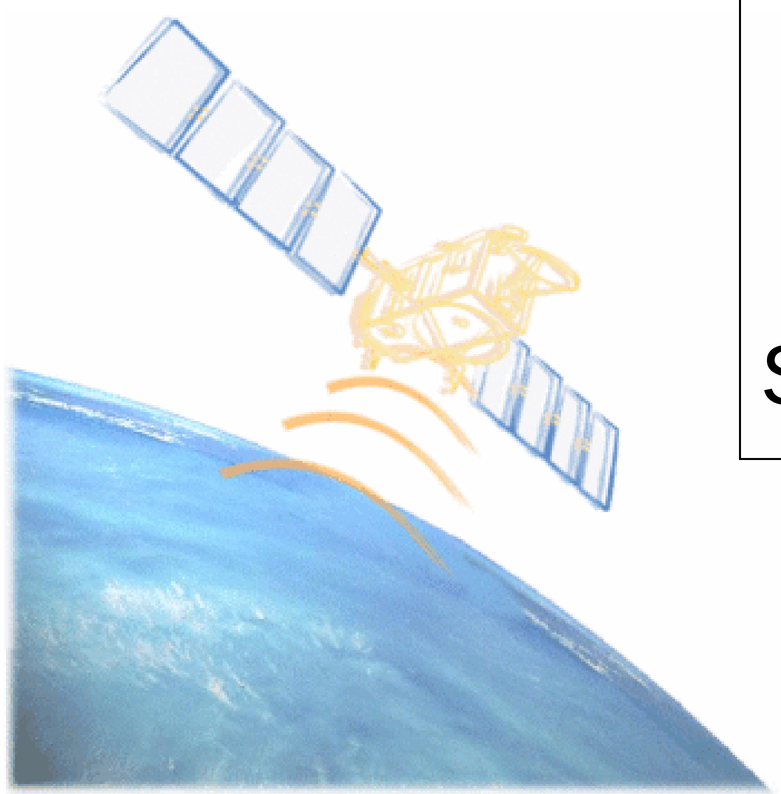
Poleward Heat Transport → Global Climate Change

- *Nonlinear dependence on the current speed (w) and temperature (T)*
- *Space or time average flux not generally equal to the flux of the averaged filed*

$$\langle wT \rangle \neq \langle w \rangle \langle T \rangle$$

- *Urgent needs to know the probability distribution function (PDF) of w and T*

Ocean Surface Velocity



Satellite Altimeters

(JASON-1, GFO, ENVISAT)

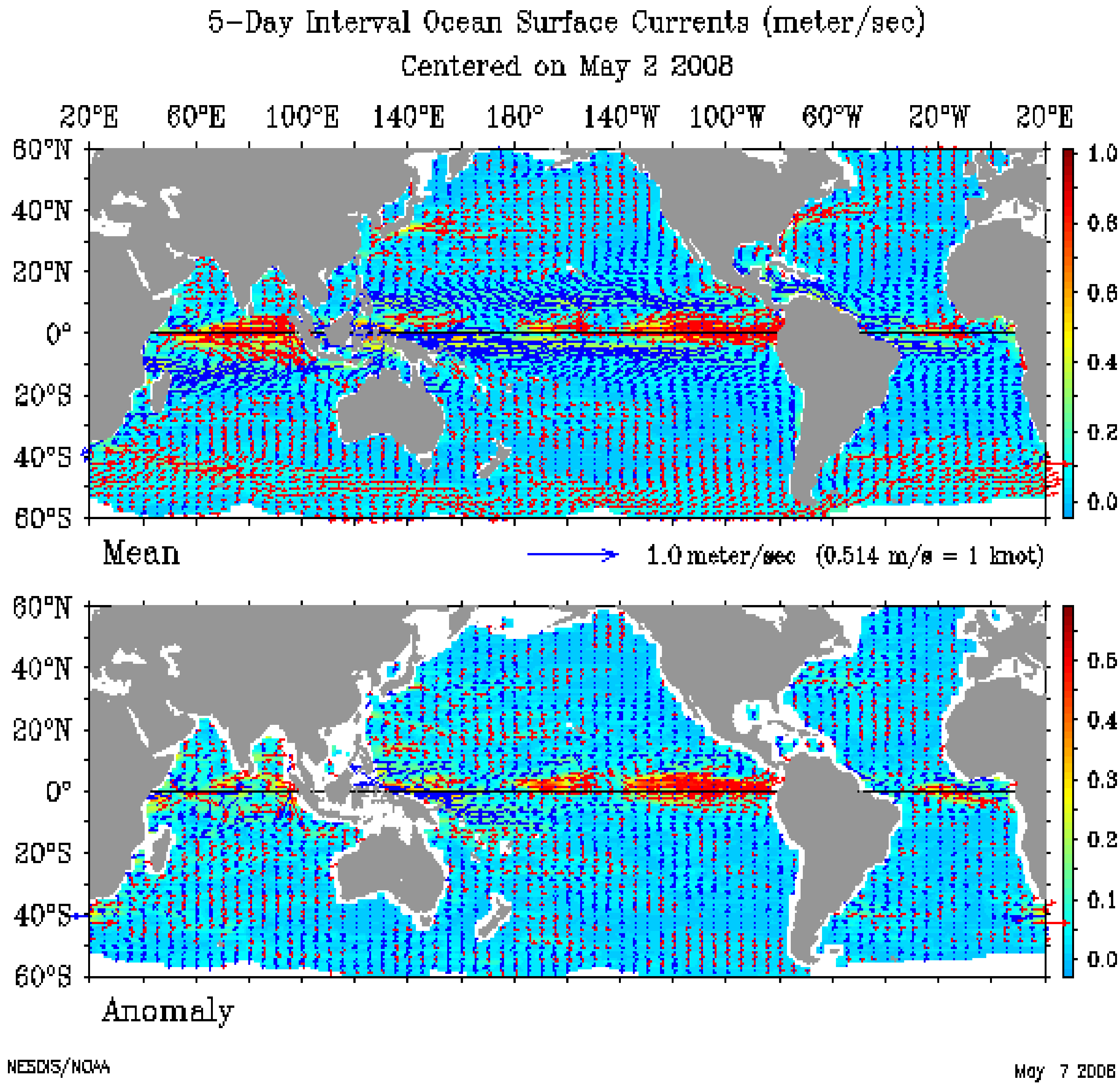
Scatterometer (QSCAT)

Ocean Surface Current Analyses – Realtime (OSCAR) Data

(1) Ocean Surface currents data available for whole world' oceans at www.oscar.noaa.gov

(2) Ocean Currents are computed from Sea Surface Height (SSH) data which is derived from satellite based altimeters JASON-1, GFO, Envisat and wind data which is derived from QUICKSCAT satellite

(3) Data continuously available every 5 days



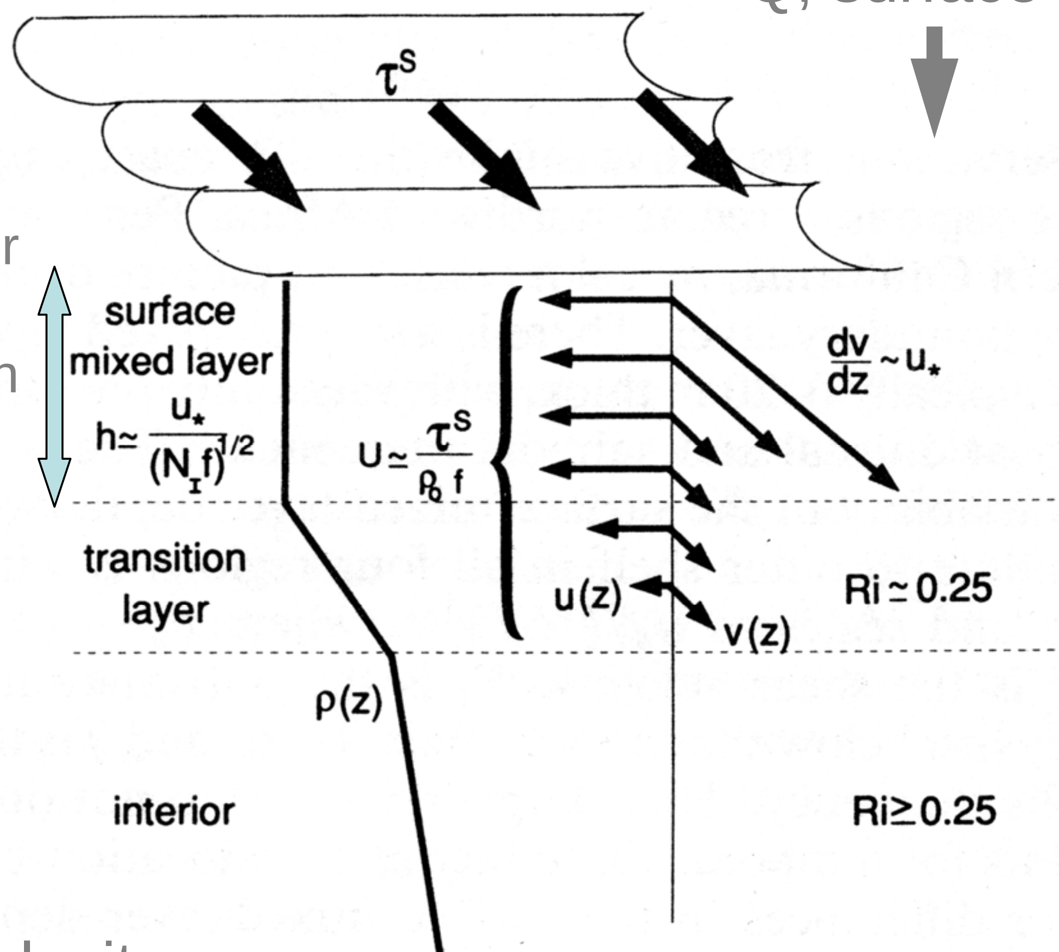
5-Day Interval Ocean Surface Currents (meter/sec)
Centered on May 2 2008
20°E 60°E 100°E 140°E 180° 140°W 100°W 60°W 20°W 20°E
60°N 40°N 20°N 0° 20°S 40°S 60°S
1.0 0.8 0.6 0.4 0.2 0.0
Mean
1.0 meter/sec (0.514 m/s = 1 knot)
0.5 0.4 0.3 0.2 0.1 0.0
Anomaly
NESDIS/NOAA
May 7 2008

Stochastic Dynamics for the Ocean Surface Currents

What does the oceanic surface boundary layer look like?

Q, surface heat flux

"slab-like" layer velocity 90° to right of wind (in northern hemisphere), well-mixed



u_*, shear velocity

FIG. 20. Schematic summarizing some of the characteristics of the surface boundary layer in a coastal upwelling region: $u_* = (\tau^S/\rho_0)^{1/2}$ is the shear velocity and U is the cross-shelf transport in the surface mixed layer plus the transition layer.

Lentz, 1992

Vertically Averaged Horizontal Velocity (u, v) within the Mixed Layer -- Slab Model --

$$\frac{\partial u}{\partial t} = \frac{1}{h}\Lambda_u - \frac{K}{h^2}u \qquad (1)$$

$$\frac{\partial v}{\partial t} = \frac{1}{h}\Lambda_v - \frac{K}{h^2}v \qquad (2)$$

h → mixed layer depth

K → eddy viscosity

(τ_x, τ_y) → Surface wind stress

$$\Lambda_u \equiv fV_E + \frac{\tau_x}{\rho}, \quad \Lambda_v \equiv -fU_E + \frac{\tau_y}{\rho}$$

Ekman Transport (U_E, V_E)

$$(U_E, V_E) = \int_{-h}^{0} (\tilde{u} - u_g, \tilde{v} - v_g) dz,$$

$(\tilde{u}, \tilde{v}) \rightarrow$ Vertically Varying Velocity

$(u_g, v_g) \rightarrow$ geostrophic velocity

$(u_g, v_g) = 0 \rightarrow$ Eqs(1) (2) $\rightarrow$ Wind-forced Slab model

Ensemble Mean and Stochastic Fluctuations of the Forcing

- (1) Ensemble Mean → Ekman Transport is determined by the surface wind stress →

$$\langle \Lambda_u \rangle = 0, \quad \langle \Lambda_v \rangle = 0$$

- (2) Fluctuations (Σ → strength)

$$\Lambda_u(t) = \langle \Lambda_u \rangle + \dot{W}_1(t) h\Sigma, \quad \Lambda_v(t) = \langle \Lambda_v \rangle + \dot{W}_2(t) h\Sigma \quad (3)$$

$$\langle \dot{W}_i(t_1) \dot{W}_j(t_2) \rangle = \delta_{ij} \delta(t_1 - t_2)$$

Stochastic Dynamic System

Eq(3) → Eqs(1) (2)

$$\frac{\partial u}{\partial t} = -\frac{K}{h^2} u + \dot{W}_1(t)\Sigma \qquad (4)$$

$$\frac{\partial v}{\partial t} = -\frac{K}{h^2} v + \dot{W}_2(t)\Sigma \qquad (5)$$

Fokker-Planck Equation for PDF of (*u*, *v*)

$$\frac{\partial p}{\partial t} = \left(\frac{\Sigma^2}{2}\right)\left(\frac{\partial^2 p}{\partial u^2} + \frac{\partial^2 p}{\partial v^2}\right) + \frac{\partial}{\partial u}\left[(\frac{K}{h^2}u)p\right] + \frac{\partial}{\partial v}\left[(\frac{K}{h^2}v)p\right] \qquad (6)$$

Polar Coordinate → $u = w\cos\varphi, \quad v = w\sin\varphi$

w → Current Speed

For Constant K → PDF of w → the Rayleigh Distribution

(Special Case of the Weibull Distribution)

$$p(w) = \frac{2w}{a^2}\exp\left[-\left(\frac{w}{a}\right)^2\right], \quad a \equiv \frac{\Sigma h}{\sqrt{K}}$$

For Non-Constant K → Weibull Distribution

$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{2}\right]$$

$$\text{mean}(w) = a\Gamma\left(1+\frac{1}{b}\right)$$

$$\text{std}(w) = a\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^{2}\left(1+\frac{1}{b}\right)\right]^{1/2}$$

Γ → Gamma Function

Characteristics of the Weibull Distribution

$$b \simeq \left[\frac{\mathrm{mean}(w)}{\mathrm{std}(w)}\right]^{1.086}, \quad a = \frac{\mathrm{mean}(w)}{\Gamma(1+1/b)}.$$

$$\mathrm{skew}(w) = \frac{\Gamma\left(1+\frac{3}{b}\right) - 3\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) + 2\Gamma^3\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^{3/2}}$$

$$\mathrm{kurt}(w) = \frac{\Gamma\left(1+\frac{4}{b}\right) - 4\Gamma\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{3}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^{2}} + \frac{6\Gamma^2\left(1+\frac{1}{b}\right)\Gamma\left(1+\frac{2}{b}\right) - 3\Gamma^4\left(1+\frac{1}{b}\right)}{\left[\Gamma\left(1+\frac{2}{b}\right) - \Gamma^2\left(1+\frac{1}{b}\right)\right]^{2}} - 3$$

Kurt $\leftrightarrow$ Skew

- Since

$$\text{Skew} = f_1(b), \quad \text{Kurt} = f_2(b)$$

$\rightarrow$

$$\text{Kurt} = F(\text{Skew})$$

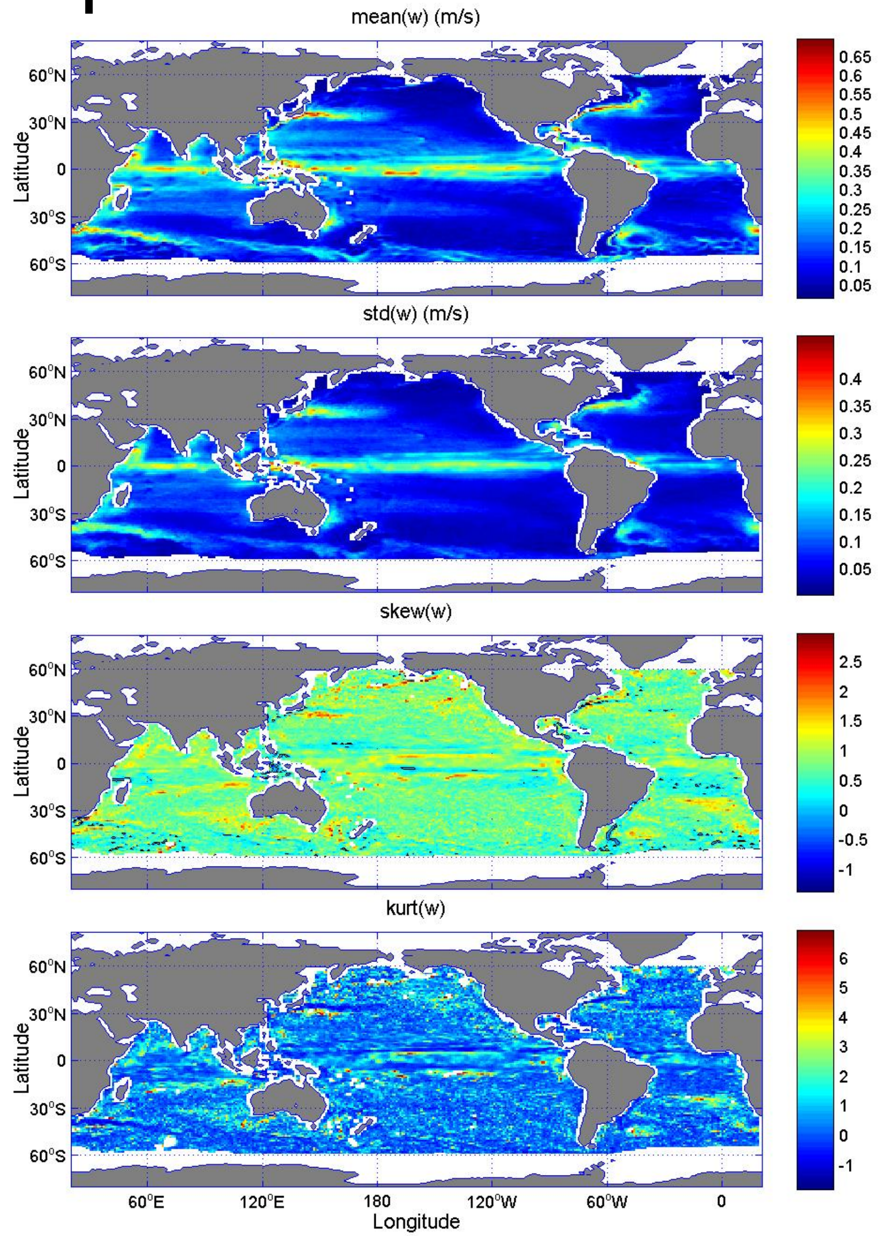
Statistical Characteristics of Global Surface Current Speed

Mean

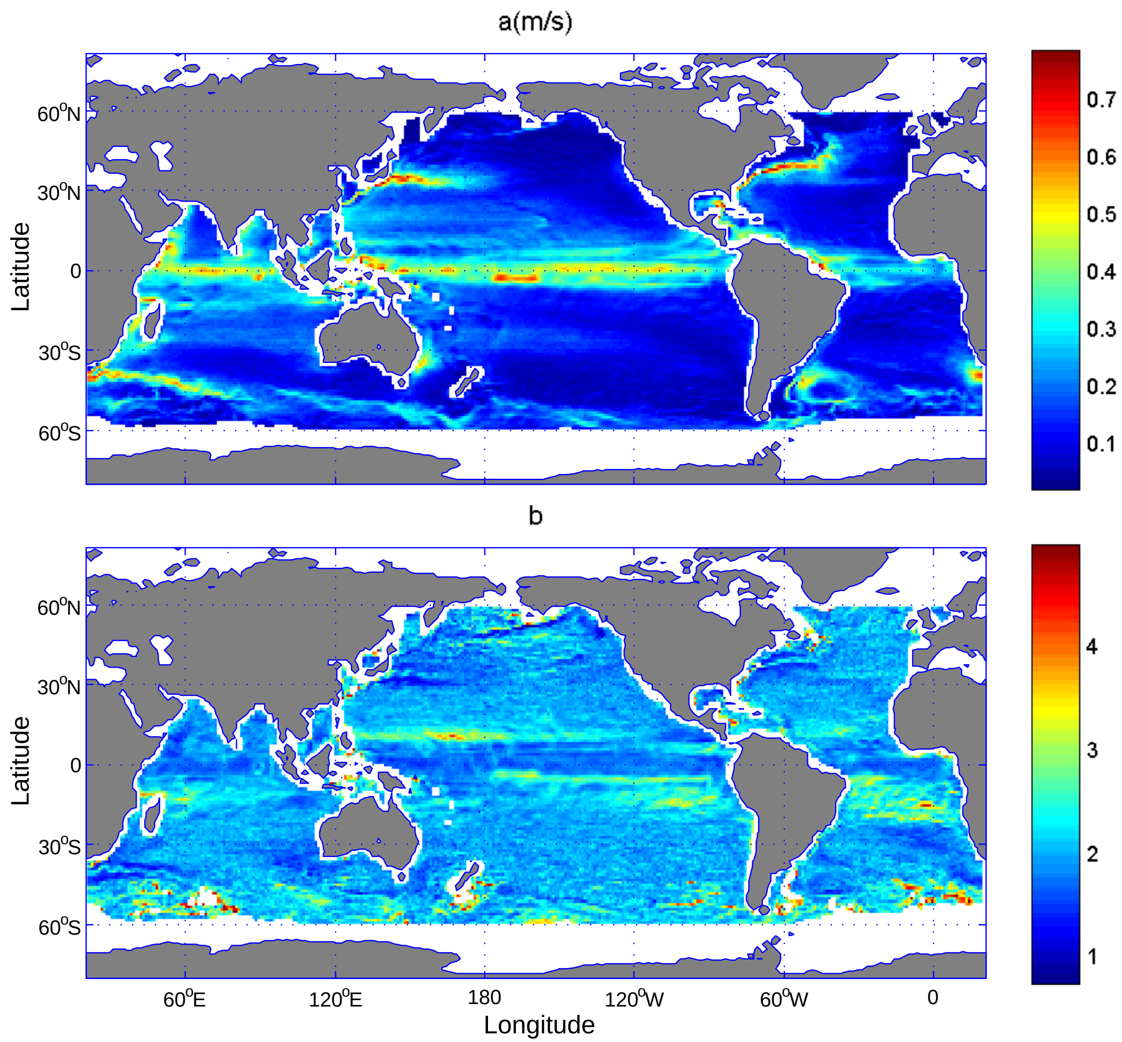
Standard Deviation

Skewness

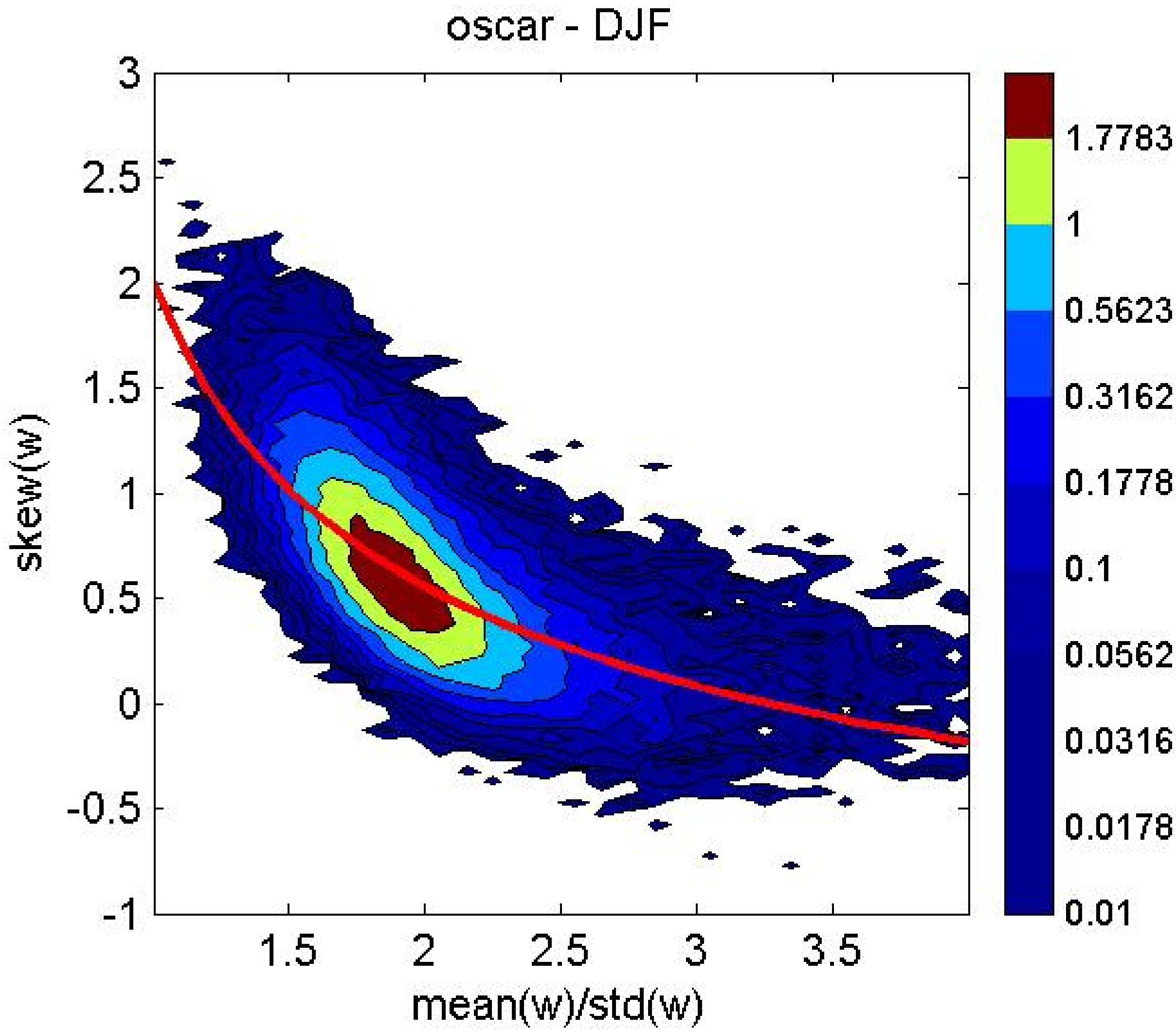
Kurtosis



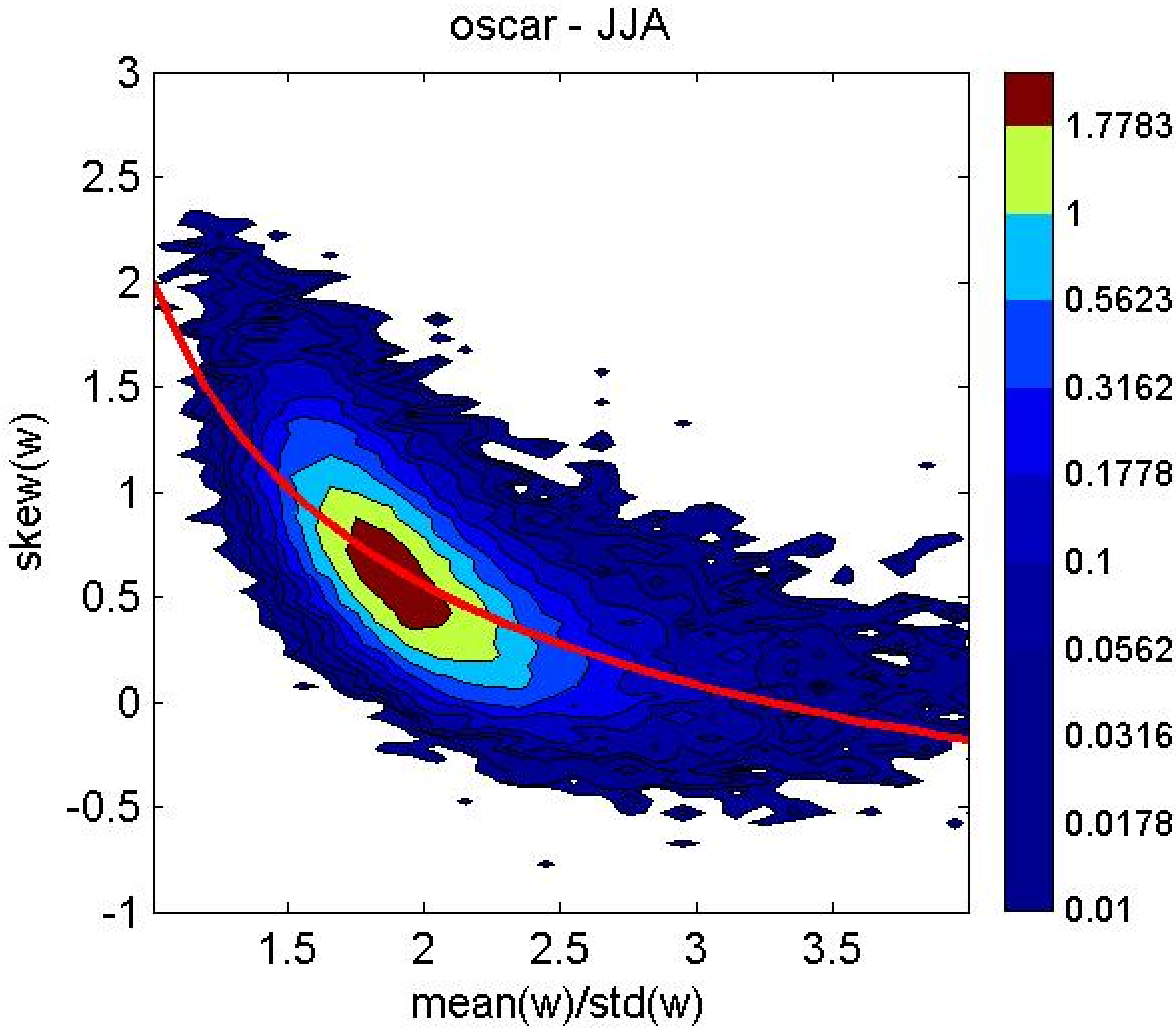
Weibull Parameters (a, b)



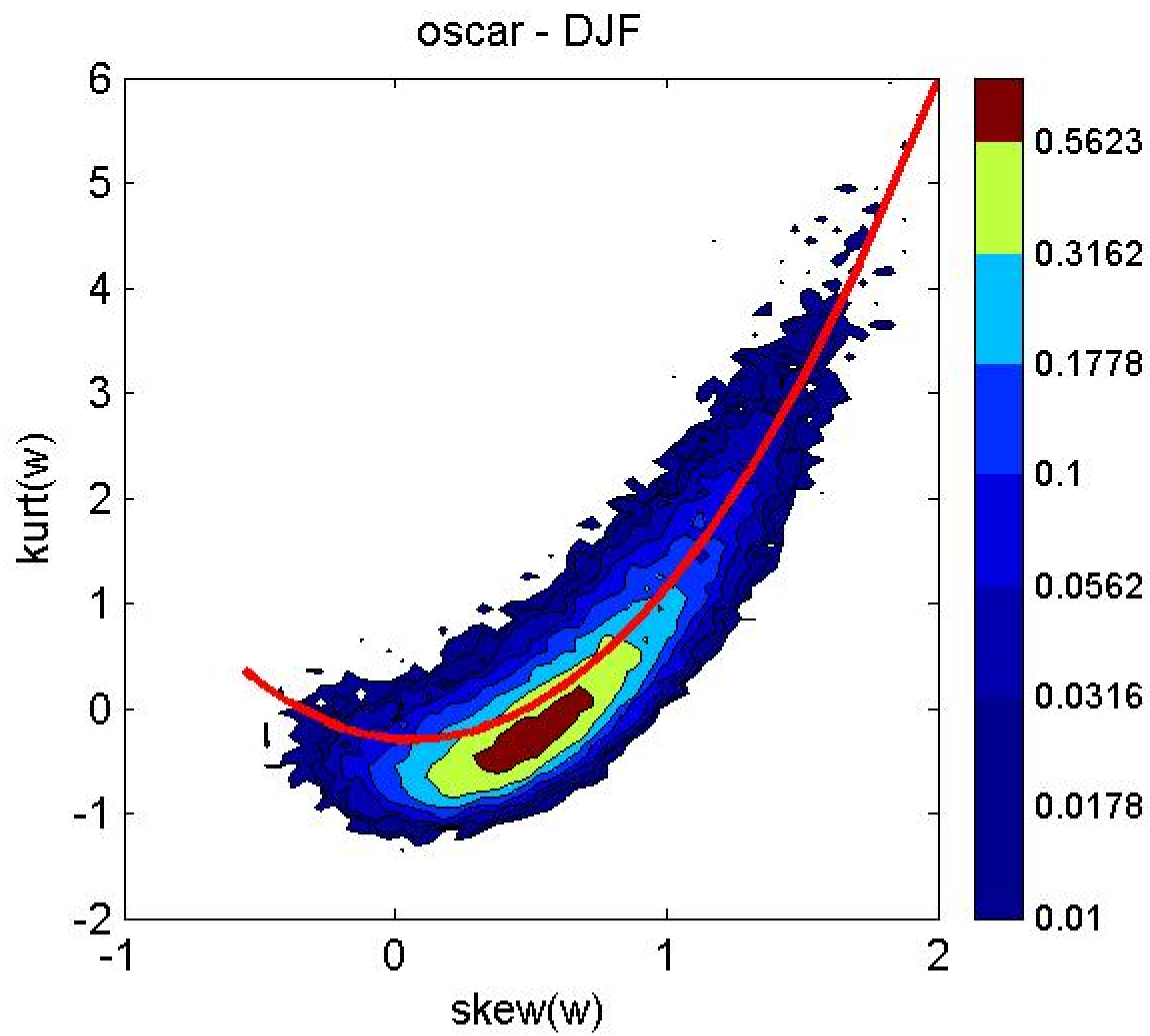
Kernel Density Estimates of Joint PDFs of *skewness* and ‘*b*’ (DJF)



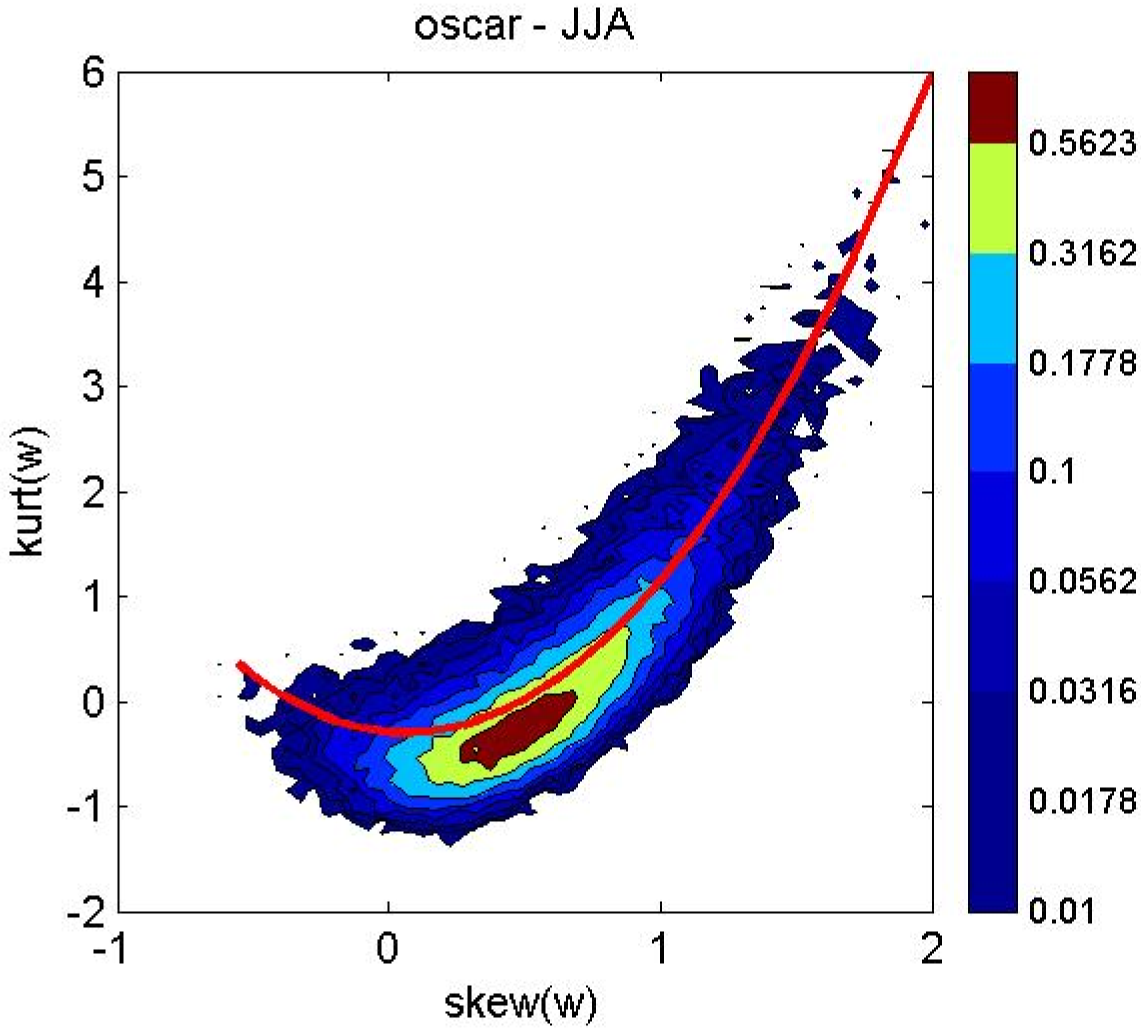
Kernel Density Estimates of Joint PDFs of *skewness* and ‘*b*’ (JJA)



Kernel Density Estimates of Joint PDFs of kurtosis and *skewness* (DJF)



Kernel Density Estimates of Joint PDFs of kurtosis and *skewness* (JJA)



Conclusions

- The ***Weibull distribution*** provides a reasonable empirical approximation to the PDF of the surface current speeds (*w*), which presents the possibility of improving the representation of the horizontal fluxes that are at the heart of the coupled physical–biogeochemical dynamics of the marine system and climate system.