Optimal Spectral Decomposition (OSD) for Ocean Data Analysis

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How can we effectively use observational ocean data to represent and to predict the ocean state?
Collaborators

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References


References


A Popular Method for Ocean Data Analysis: Optimum Interpolation (OI)
**OI – Equation**

Grid point $\rightarrow k$, Observational Point $\rightarrow j$

- $Q_k^f$ $\rightarrow$ First guess field (gridded)
- $Q_j^o$ $\rightarrow$ Observation
- $Q_j^f$ $\rightarrow$ First guess interpolated on the observational point

$$Q_k^a = Q_k^f + \sum_{j=1}^{N} \alpha_{kj}(Q_j^o - Q_j^f)$$

- $Q_k^a$ $\rightarrow$ Analyzed field at the grid point
OI – Weight Coefficients $\alpha_{kj}$

\[
\sum_{j=1}^{N} (\eta_{ij} + \delta_{ij} \lambda_{i}^{o}) \alpha_{kj} = \eta_{kj}
\]

$\eta_{ij}$, $\eta_{kj}$ → Autocorrelation functions

$\lambda_{i}^{o}$ → Signal-to-noise ratio
Three Requirements for the OI Method

• (1) First guess field
• (2) Autocorrelation functions
• (3) High signal-to-noise ratio
What happens if the three conditions are not satisfied?
Spectral Representation - a Possible Alternative Method

\[ c(x, z_k, t) = A_0(z_k, t) + \sum_{m=1}^{M} A_m(z_k, t) \Psi_m(x, z_k), \]

\( \Psi_m \rightarrow \text{Basis functions} \)

\( c \rightarrow \text{any ocean variable} \)
Flow Decomposition

\[ u = \frac{\partial \Psi}{\partial y} + \frac{\partial^2 \Phi}{\partial x \partial z}, \quad v = -\frac{\partial \Psi}{\partial x} + \frac{\partial^2 \Phi}{\partial y \partial z}, \]

\[ \Delta \Psi = -\zeta \]

\[ \Delta \Phi = -w \]
Basis Functions (Closed Basin)

\[ \Delta \Psi_k = -\lambda_k \Psi_k, \quad \Psi_k|_\Gamma = 0, \quad k = 1, \ldots, \infty \]

\[ \Delta \Phi_m = -\mu_m \Phi_m, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0, \quad m = 1, \ldots, \infty. \]
Basis Functions
(Open Boundaries)

\[ \Delta \Psi_k = -\lambda_k \Psi_k, \]

\[ \Delta \Phi_m = -\mu_m \Phi_m, \]

\[ \Psi_k \big|_{\Gamma} = 0, \quad \frac{\partial \Phi_m}{\partial n} \big|_{\Gamma} = 0, \]

\[ \left[ \frac{\partial \Psi_k}{\partial n} + \kappa(\tau) \Psi_k \right] \big|_{\Gamma_1'} = 0, \quad \Phi_m \big|_{\Gamma_1'} = 0, \]
Boundary Conditions

\[
\frac{\partial^2 \Phi}{\partial n \partial z} = 0 \\
\Psi = 0 \\
\frac{\partial \Phi}{\partial z} = 0 \\
\frac{\partial \Psi}{\partial n} + \kappa \Psi = 0 \\
\frac{\partial \Phi}{\partial n} = 0 \\
\frac{\partial \Psi}{\partial n} = 0
\]
Spectral Decomposition

\[ u_{KM} = \sum_{k=1}^{K} a_k(z, t^o) \frac{\partial \Psi_k(x, y, z, \kappa^o)}{\partial y} + \sum_{m=1}^{M} b_m(z, t^o) \frac{\partial \Phi_m(x, y, z)}{\partial x}, \]

\[ v_{KM} = -\sum_{k=1}^{K} a_k(z, t^o) \frac{\partial \Psi_k(x, y, z, \kappa^o)}{\partial x} + \sum_{m=1}^{M} b_m(z, t^o) \frac{\partial \Phi_m(x, y, z)}{\partial y}. \]

\[ T(x, t) = T_0(x) + \sum_{m=1}^{M} c_m(t) \Phi_m(x, t) \]

\[ S(x, t) = S_0(x) + \sum_{m=1}^{M} d_m(t) \Phi_m(x, t) \]
Benefits of Using OSD

• (1) Don’t need first guess field

• (2) Don’t need autocorrelation functions

• (3) Don’t require high signal-to-noise ratio

• (4) Basis functions are pre-determined before the data analysis.
Optimal Mode Truncation

\[ J(a_1,...,a_K, b_1,...,b_M, \kappa, P) = \frac{1}{2} \left( \| u_p^{obs} - u_{KM} \|_P^2 + \| v_p^{obs} - v_{KM} \|_P^2 \right) \rightarrow \min, \]
Vapnik (1983) Cost Function

\[ J_{emp} = J(a_1, \ldots, a_K, b_1, \ldots, b_M, \kappa, P). \]

\[
\text{Prob} \left\{ \sup_{K,M,S} |J(K, M, S) - J_{emp}(K, M, S)| \geq \mu \right\} \leq g(P, \mu)
\]

\[
\lim_{P \to \infty} g(P, \mu) = 0
\]
Optimal Truncation

- Gulf of Mexico, Monterey Bay, Louisiana-Texas Shelf, North Atlantic

- $K_{opt} = 40$, $M_{opt} = 30$
Determination of Spectral Coefficients (Ill-Posed Algebraic Equation)

\[ \hat{A} \hat{\alpha} = QY, \]

This is caused by the features of the matrix \( \mathbf{A} \).
Rotation Method (Chu et al., 2004)

\[ SA \hat{a} = SQY, \]

\[ J_1 = \|A\|^2 - \frac{\|SQY\|^2}{\|a\|^2} \rightarrow \text{max}, \]
Example-1

Temporal and spatial variability of Pacific Ocean
T (10 m) 1990-2008
T (100 m) 1990-2008
T (500 m) 1990-2008
Seasonal Anomaly versus WOA 94 (10 m)

- Monthly mean (1993-208)
  minus

WOA 94 Monthly Mean
Seasonal Anomaly versus WOA 94 (100 m)

- Monthly mean (1993-208) minus WOA 94 Monthly Mean
Seasonal Anomaly versus WOA 94 (250 m)

- Monthly mean (1993-2008)

minus

WOA 94 Monthly Mean
Seasonal Anomaly versus WOA 94 (500 m)

- Monthly mean (1993-2008)

minus

WOA 94 Monthly Mean
T: NINO-3 (5°S-5°N, 150°W-90°W)
Example-2  OSD for Analyzing ARGO Data

Baroclinic Rossby Waves in the tropical North Atlantic
Tropical North Atlantic (4° - 24°N) Important Transition Zone → Meridional Overturning Circulation (MOC) (Rahmstorf 2006)
MOC Variation →
Heat Transport Variation →
Climate Change
• Are mid-depth (~1000 m) ocean circulations steady?

• If not, what mechanisms cause the change? (Rossby wave propagation)
6-12 hours at surface to transmit data to satellite

Descent to depth
~10 cm/s (~6 hours)

1000 db (1000m)
Drift approx. 9 days

Salinity & Temperature profile recorded during ascent
~10 cm/s (~6 hours)

Total cycle time 10 days

Float descends to begin profile from greater depth
2000 db (2000m)
ARGO Observations (Oct-Nov 2004)

(a) Subsurface tracks       (b) Float positions where (T,S) were measured
Circulations at 1000 m estimated from the original ARGO float tracks (bin method)
April 2004 – April 2005

It is difficult to get physical insights and to use such noisy data into ocean numerical models.
Boundary Configuration → Basis Functions for OSD
Basis Functions for Streamfunction
Mode-1 and Mode-2
Circulations at 1000 m (March 04 to May 05)
Bin Method
OSD
Mid-Depth Circulations (1000 m)

Mar-May 04

May – Jul 04

Jul-Sep 04

Sep – Nov 04

Nov 04 – Jan 05

Jan-Mar 05

Mar – May 05
Temperature at 950 m (March 04 to May 05)

NOAA/WOA

OSD
Baroclinic Rossby Waves in Tropical North Atlantic
Fourier Expansion $\rightarrow$ Temporal Annual and Semi-annual

\[
\hat{\psi} \approx \overline{\psi}(x_\perp) + \psi_1(x_\perp,t) + \psi_2(x_\perp,t),
\]

\[
\psi_1(x_\perp,t) = \sum_{s=1}^{2} A_{\omega_1,s} \cos(\omega_1 t + \theta_{\omega_1,s}) Z_s(x_\perp) + \sum_{k=1}^{K_{opt}} B_{\omega_1,k} \cos(\omega_1 t + \vartheta_{\omega_1,k}) \Psi_k(x_\perp),
\]

\[
\psi_2(x_\perp,t) = \sum_{s=1}^{2} A_{\omega_2,s} \cos(\omega_2 t + \theta_{\omega_2,s}) Z_s(x_\perp) + \sum_{k=1}^{K_{opt}} B_{\omega_2,k} \cos(\omega_2 t + \vartheta_{\omega_2,k}) \Psi_k(x_\perp),
\]

\[T_0 = 12 \text{ months}; \quad \omega_1 = \frac{2\pi}{T_0}; \quad \omega_2 = \frac{4\pi}{T_0}\]
Fourier Expansion $\rightarrow$ Temporal Annual and Semi-annual

$$\hat{T}(x_\perp, z, t) \approx \overline{T}(x_\perp, z) + T_1(x_\perp, z, t) + T_2(x_\perp, z, t),$$

$$T_1(x_\perp, z, t) = \sum_{m=1}^{M_{\text{opt}}} C_{\omega_1, m}(z) \cos[\omega_1 t + \chi_{\omega_1, m}(z)] \Xi_m(x_\perp, z),$$

$$T_2(x_\perp, z, t) = \sum_{m=1}^{M_{\text{opt}}} C_{\omega_2, m}(z) \cos[\omega_2 t + \chi_{\omega_2, m}(z)] \Xi_m(x_\perp, z),$$

$$T_0 = 12 \text{ months}; \quad \omega_1 = \frac{2\pi}{T_0}; \quad \omega_2 = \frac{4\pi}{T_0}.$$
Optimization

\[ J_s = \left( \int_{t_0}^{t_0+T_o} \left[ a_s(t) - \sum_{\omega=\omega_1,\omega_2} A_{\omega,s} \cos(\omega t + \theta_{\omega,s}) \right] \, dt \right)^2 \rightarrow \min \]

\[ I_k = \left( \int_{t_0}^{t_0+T_o} \left[ b_k(t) - \sum_{\omega=\omega_1,\omega_2} B_{\omega,s} \cos(\omega t + \theta_{\omega,s}) \right] \, dt \right)^2 \rightarrow \min \]
Annual Component

![Map of Annual Component](image.png)
Semi-annual Component
Time – Longitude Diagrams of Meridional Velocity

Along 11°N

(a) Annual

(b) Semi-Annual
Time – Longitude Diagrams of temperature Along 11°N

(a) (b) (c) (d)

Annual Semi-Annual Annual Semi-Annual

550 m 950 m
Annual Currents (1000 m)

May-Jun 2004

Jul-Aug 2004

Sep-Oct 2004

Nov-Dec 2004
Characteristics of Annual Rossby Waves

<table>
<thead>
<tr>
<th>Latitude</th>
<th>March, 04 – May, 05 float data</th>
<th>March, 04 – May, 06 float data</th>
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<tr>
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<td>$c_p$ (cm/s)</td>
<td>$L_1$ (km)</td>
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<tr>
<td>5°N</td>
<td>12</td>
<td>1200</td>
</tr>
<tr>
<td>8°N</td>
<td>16</td>
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<td>11°N</td>
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<td>2200</td>
</tr>
<tr>
<td>13°N</td>
<td>11</td>
<td>2100</td>
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</table>
Annual Monthly Temperature Anomaly (°C) at 950 m Depth → Annual Rossby Waves (7-10 cm/s)

Jun 04

Aug 04

Oct 04

Dec 04
Annual Monthly Temperature Anomaly (°C) at 250 m Depth
Equatorially Forced Coastal Kelvin waves (27-30 cm/s)
Zonal cross-sections of the annual component of the temperature anomaly (°C)

6°N in Jun 04 →

11°N in Oct, 04 →

16°N in Oct 04 →
Baroclinic Modes

(a) Baroclinic Modes

(b) Baroclinic Modes with Combination
Annual Component in the Western Sub-Basin

Mean wind KE

Mean KE for mid-depth currents

Correlation between Winds and currents

Correlation between wind Stress curl and streamfunction (solid: no-lag, dashed: 3 mon lag
Annual Component in the Eastern Sub-Basin

Mean wind KE

Mean KE for mid-depth currents

Correlation between Winds and currents

Correlation between wind Stress curl and streamfunction (solid: no-lag, dashed: 3 mon lag

Zonal: circle

Meridional: square
Semi-annual currents at 1000 m depth (2004)

(a) 5/15
(b) 5/30
(c) 6/14
(d) 6/29
(e) 7/13
Semi-annual monthly temperature anomaly at 950m depth

(a) Jun 04  
(b) Aug 04  
(c) Oct 04  
(d) Dec 04

(a) 6/4
(b) 7/4
(c) 8/4
(d) 9/4
Semi-annual temperature anomaly at 550m depth (2004)

(a) 5/15

(b) 6/29
Semiannual Component in the Western Sub-Basin

(a) wind KE
(b) current KE
(c) corr wind stress and currents
(d) corr between semi-annual currents and mean wind
(e) corr between semiannual currents and annual wind stress.
Semiannual Component in the Eastern Sub-Basin

(a) wind KE
(b) current KE
(c) corr wind stress and currents
(d) corr between semi-annual currents and mean wind
(e) corr between semiannual currents and annual wind stress
Results

• The annual and semi-annual unstable standing Rossby waves are detected in both the western and eastern sub-basins.

• The wind-driven Ekman pumping seems to be responsible for the standing wave generation in both the sub-basins.
Example-3
OSD for Analyzing Combined Current Meter and Surface Drifting Buoy Data
Ocean Velocity Observation

• 31 near-surface (10-14 m) current meter moorings during LATEX from April 1992 to November 1994

• Drifting buoys deployed at the first segment of the Surface Current and Lagrangian-drift Program (SCULP-I) from October 1993 to July 1994.
Moorings and Buoys
LTCS current reversal detected from SCULP-I drift trajectories.
Reconstructed and observed circulations at Station-24.
Probability of TLCS Current Reversal for Given Period (T)

- $n_0 \sim$ 0-current reversal
- $n_1 \sim$ 1-current reversal
- $n_2 \sim$ 2-current reversals
- $m \sim$ all realizations

$$P_0(T) = \frac{n_0}{m}, P_1(T) = \frac{n_1}{m}, P_2(T) = \frac{n_2}{m},$$
Fitting the Poison Distribution

\[ P_k(T) = \frac{1}{k!} (\mu T)^k \exp(-\mu T) \]

\[ k=0, 1, 2 \]

\( \mu \) is the mean number of reversal for a single time interval

\( \mu \sim 0.08 \)
Dependence of $P_0$, $P_1$, $P_2$ on $T$

For observational periods larger than 20 days, the probability for no current reversal is less than 0.2.

For 15 day observational period, the probability for 1-reversal reaches 0.5

Data – Solid Curve
Poison Distribution Fitting – Dashed Curve
Time Interval between Successive Current Reversals (not a Rare Event)

\[ p(\tau) = \mu \exp(-\mu \tau) \]
LTCS current reversal detected from the reconstructed velocity data

December 30, 1993

January 3, 1994

January 6, 1994
EOF Analysis of the Reconstructed Velocity Filed

<table>
<thead>
<tr>
<th>EOF</th>
<th></th>
<th>01/21/93-05/21/93</th>
<th>12/19/93-04/17/94</th>
<th>10/05/94-11/29/94</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80.2</td>
<td>77.1</td>
<td>74.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.1</td>
<td>9.5</td>
<td>9.3</td>
<td></td>
</tr>
<tr>
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<td>3.9</td>
<td>5.6</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
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<td>1.4</td>
<td>3.3</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
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<td>1.1</td>
<td>1.4</td>
<td>2.3</td>
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<tr>
<td>6</td>
<td>0.7</td>
<td>1.1</td>
<td>0.8</td>
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</tr>
</tbody>
</table>
Mean and First EOF Mode

\[ \tilde{u}(x, y, t) = \bar{u}(x, y) + A_1(t)u_1(x, y), \]
Mean Circulation

1. First Period  
   (01/21-05/21/93)

2. Second Period  
   (12/19/93-04/17/94)

3. Third Period  
   (10/05-11/29/94)
EOF1

1. First Period
   (01/21-05/21/93)

2. Second Period
   12/19/93-04/17/94)

3. Third Period
   (10/05-11/29/94)
- Calculated $A_1(t)$ Using Current Meter Mooring (solid) and SCULP-1 Drifters (dashed)
• 8 total reversals observed

\[ \eta = \frac{A_1^2}{\sum_{n=2}^{6} A_n^2} \]

• \( U_{als} \sim \) alongshore wind
Morlet Wave

- $A_1(t)$

- $U_{als}$

$\Phi(t) = \pi^{-4} \exp(i m t - t^2 / 2), \; m = 6$
Surface Wind Data

• 7 buoys of the National Data Buoy Center (NDBC) and industry (C-MAN) around LATEX area
• Regression between
• $A_1(t)$ and Surface Winds

• Solid Curve (reconstructed)
• Dashed Curve (predicted using winds)

$$A_1(t) = \alpha [U(t) - \bar{U}] + \beta [V(t) - \bar{V}] + \gamma$$
Results

• Alongshore wind forcing is the major factor causing the synoptic current reversal.

• Other factors, such as the Mississippi-Atchafalaya River discharge and offshore eddies of Loop Current origin, may affect the reversal threshold, but can not cause the synoptic current reversal.
Part-4
OSD for Analyzing CODAR Data
CODAR
Monterey Bay
Place for comments: left - radar derived currents for 17:00 UT December 1, 1999
right – reconstructed velocity field.
Conclusions

• OSD is a useful tool for processing real-time velocity data with short duration and limited-area sampling especially the ARGO data.

• OSD has wide application in ocean data assimilation.