Wave-Turbulence Mixing for Upper Ocean Multifractal Thermal Structure

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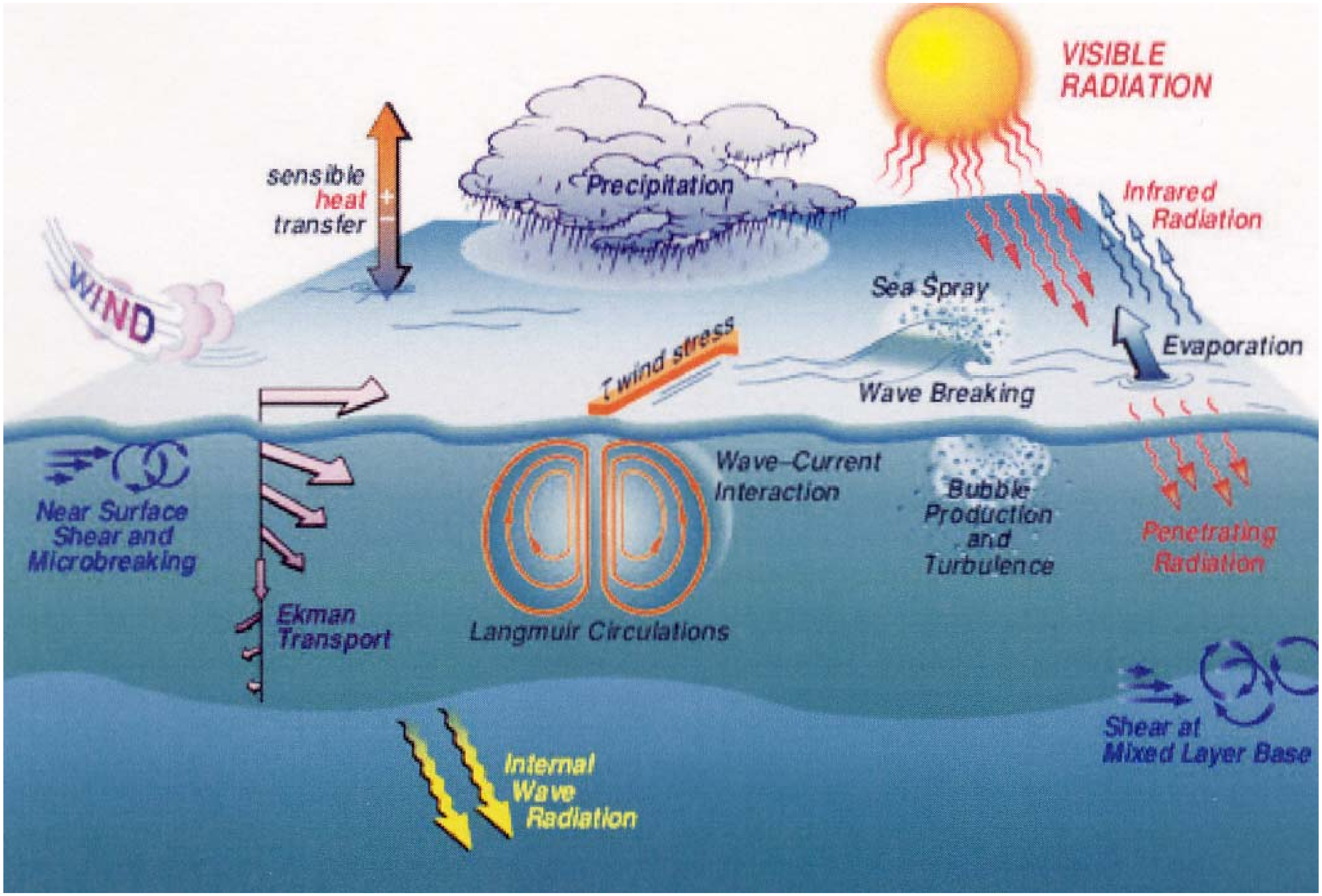
Upper Ocean Dynamics

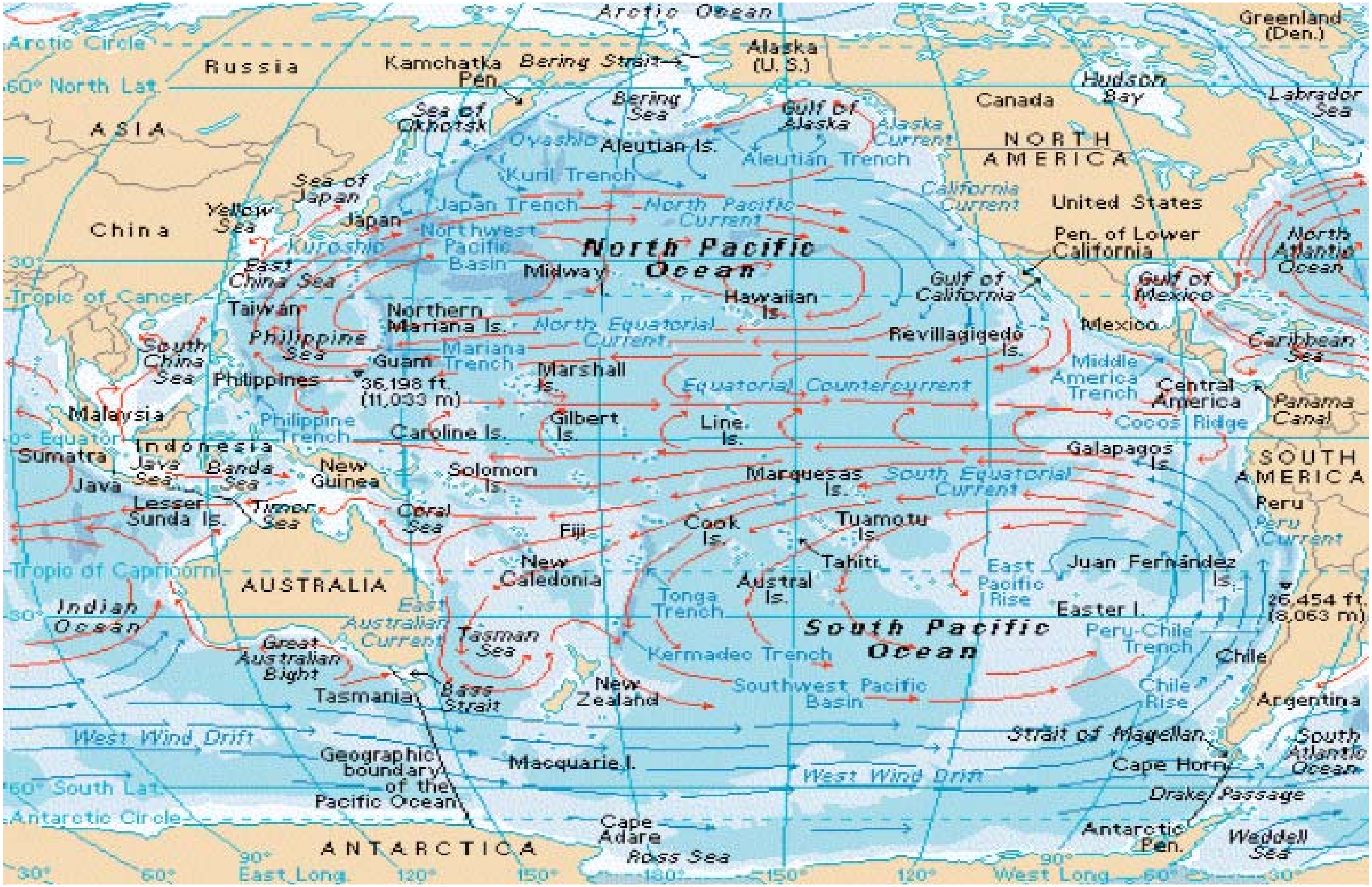
from http://www.hpl.umces.edu/ocean/sml_main.htm



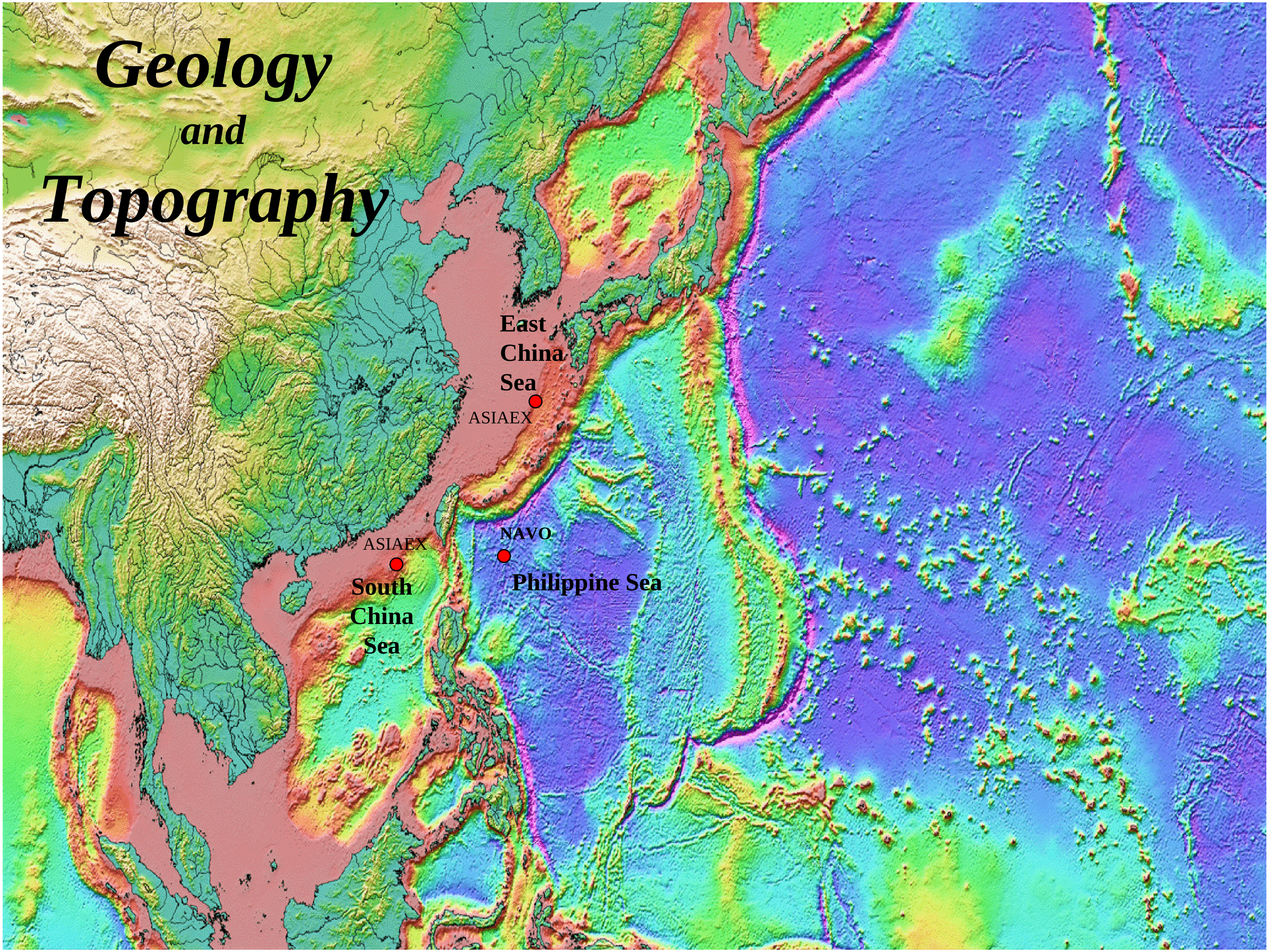
What is the upper ocean thermal structure with internal wave propagation?

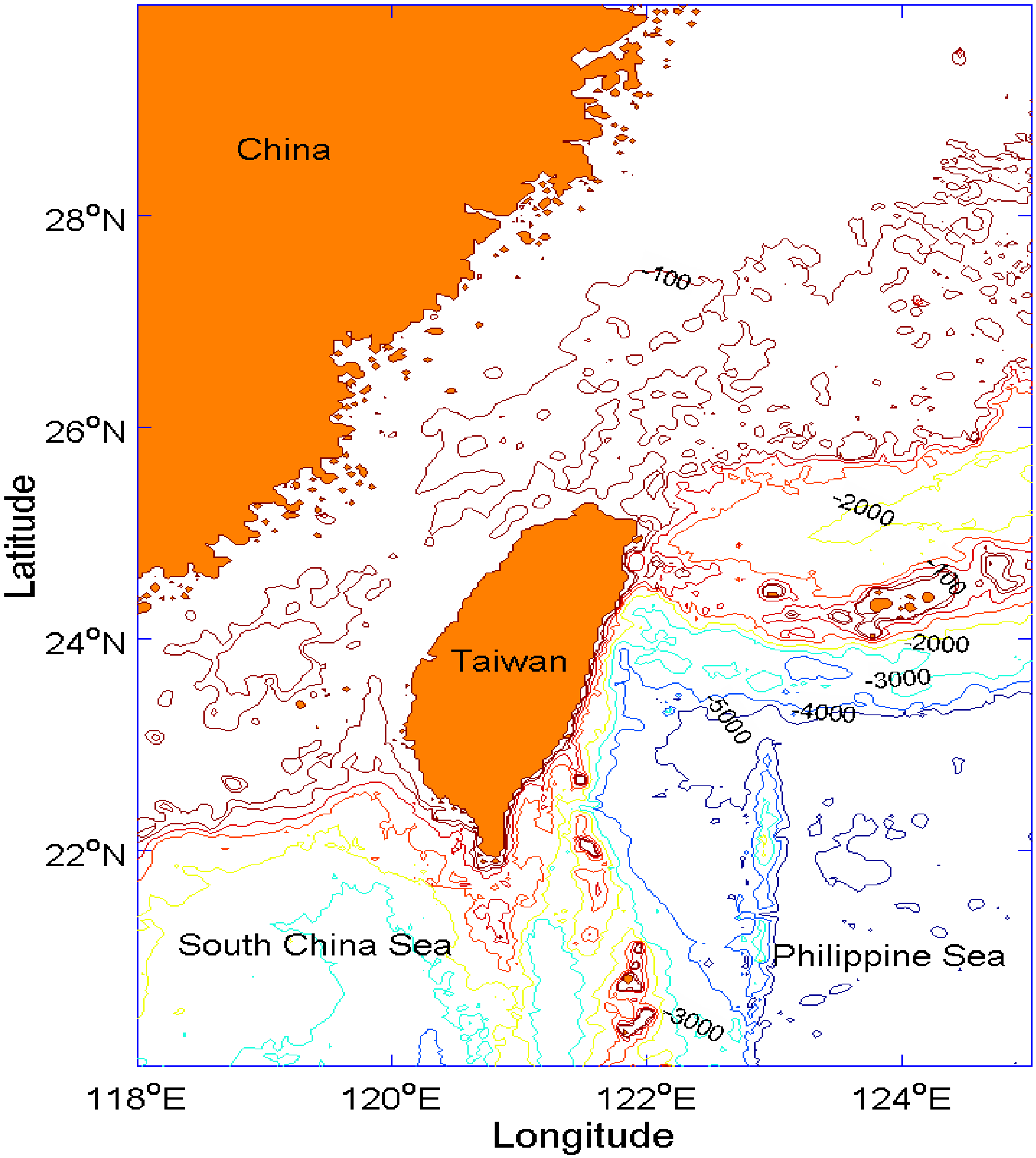
An Observational study in the western ***Philippine Sea*** is taken as an example for illustration.

Philippine Sea in World Oceans

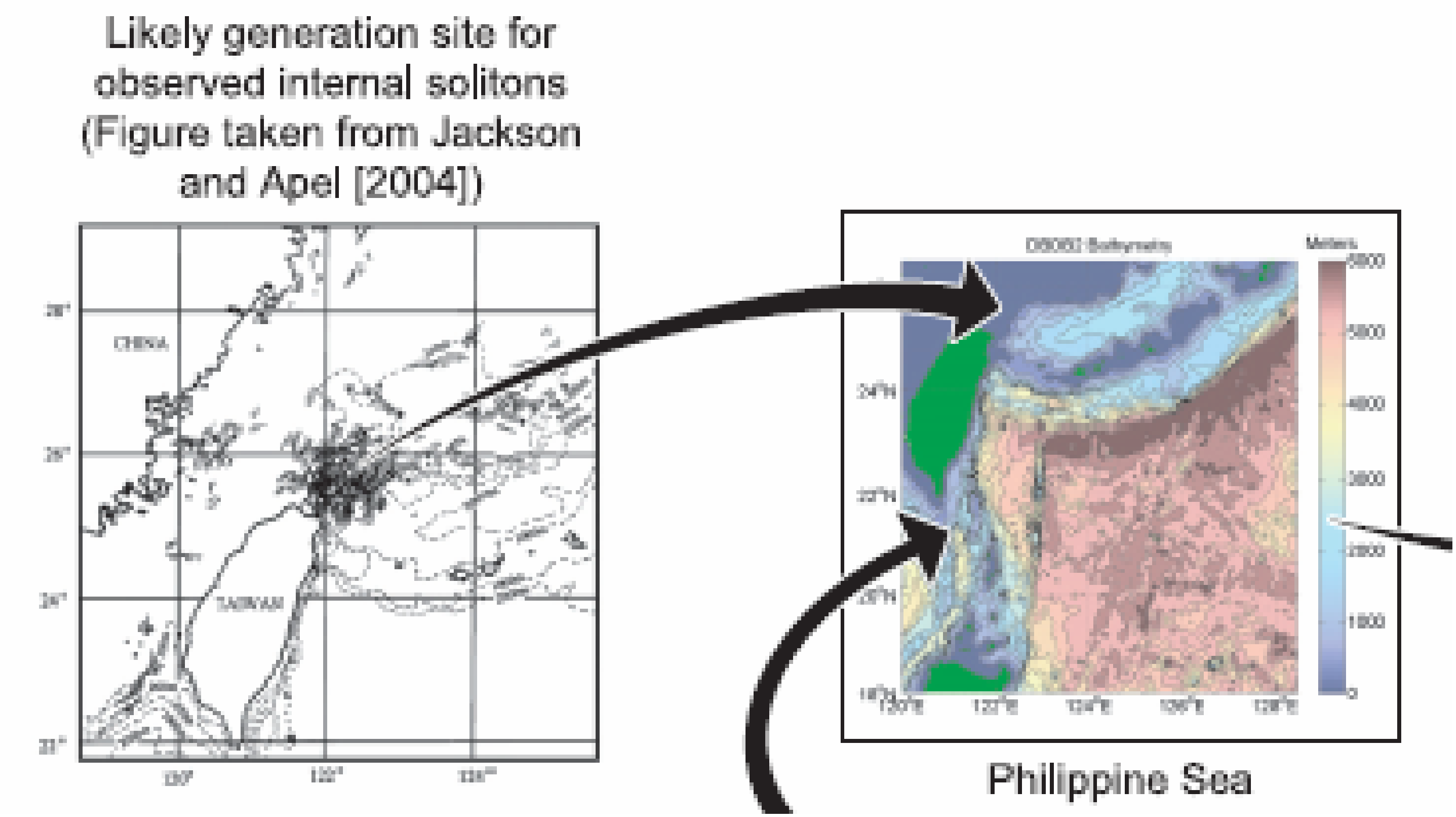


Geology and Topography




China
Taiwan
South China Sea
Philippine Sea
-100
-100
-2000
-2000
-3000
-4000
-5000
-3000
28°N
26°N
24°N
22°N
118°E
120°E
122°E
124°E
Latitude
Longitude

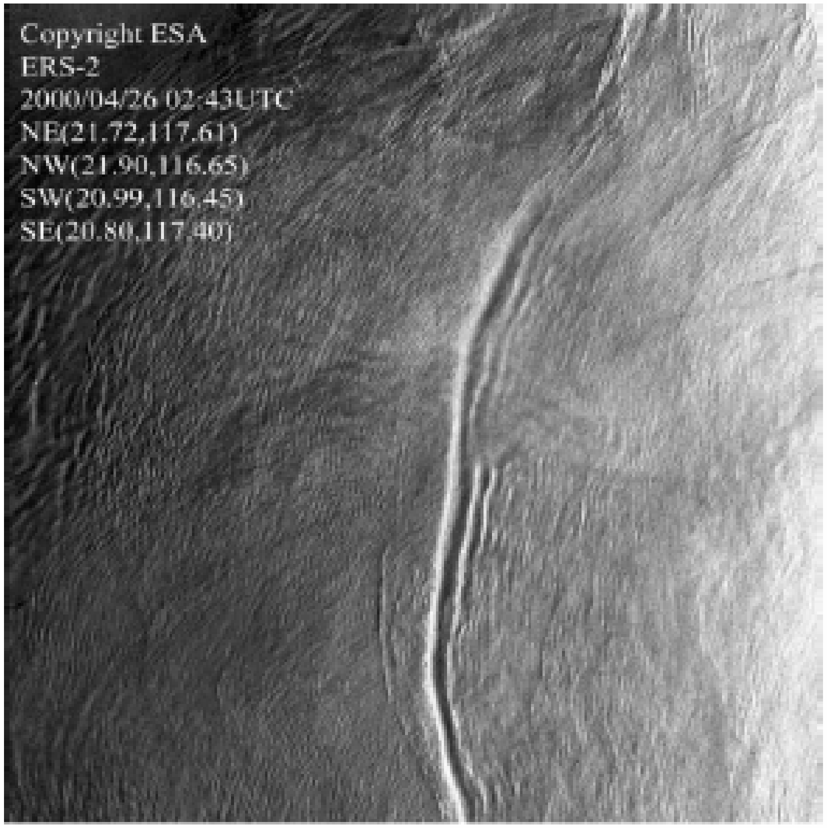
Internal Waves and Solitons near Taiwan


Likely generation site for observed internal solitons (Figure taken from Jackson and Apel [2004])

(Liu et al., 1998)

MODIS (bands 1,3,4) 250-m resolution visible image over the East China Sea
August 3, 2003 at 0235 UTC (Alpers et al. 2004)

Internal Waves/Solitons in the South China Sea (Liu and Hsu, 2007) width ~ 0.8 km



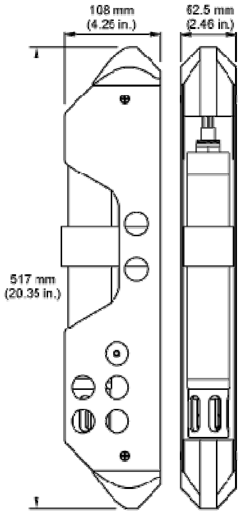
ESA 4/26/2000

RADARSAT 4/22/2000

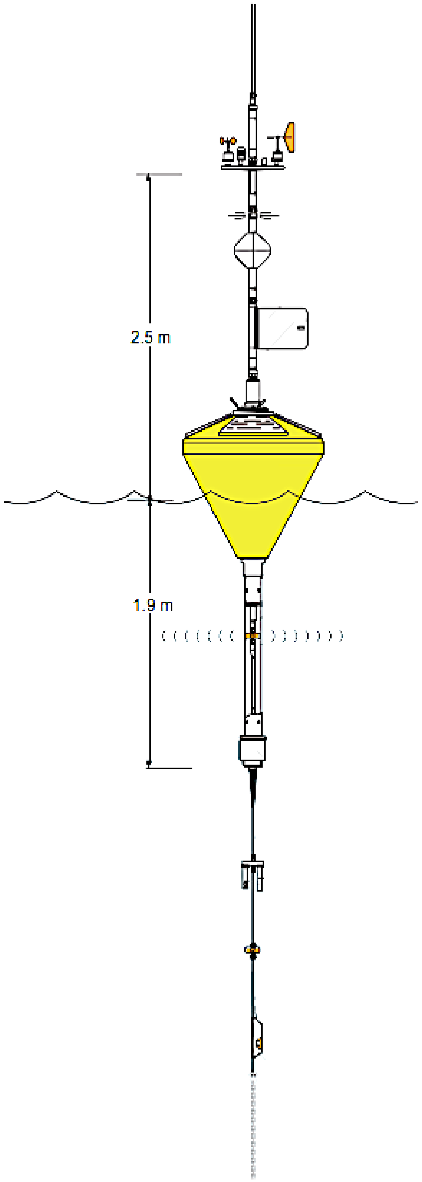
Data Observation

- Coastal Monitoring Buoy (CMB)
  - U.S. Naval Oceanographic Office
  - July 28 - August 7, 2005
  - Ocean data 1,3,5,18, and 20 m

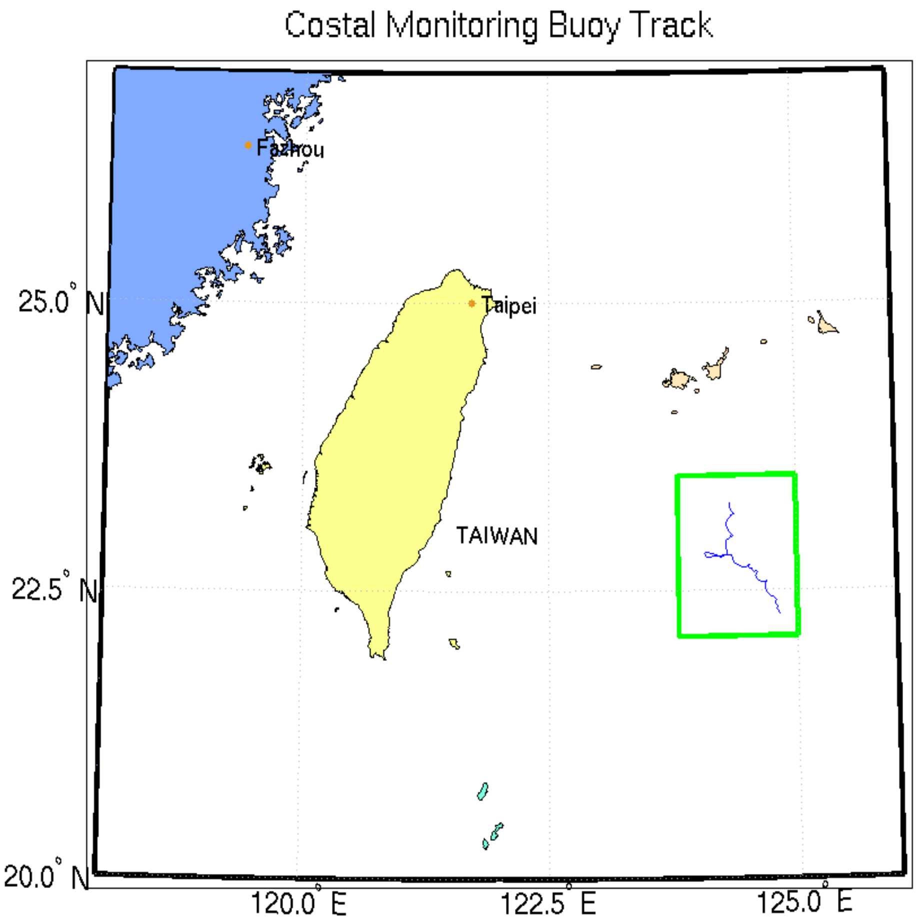
    Surface atmospheric data
  - Record intervals - 10 min
- Thermistors
  - SBE 39
  - Attached at 15 depths from 25 to 140 m.
  - Records intervals - 15 s.

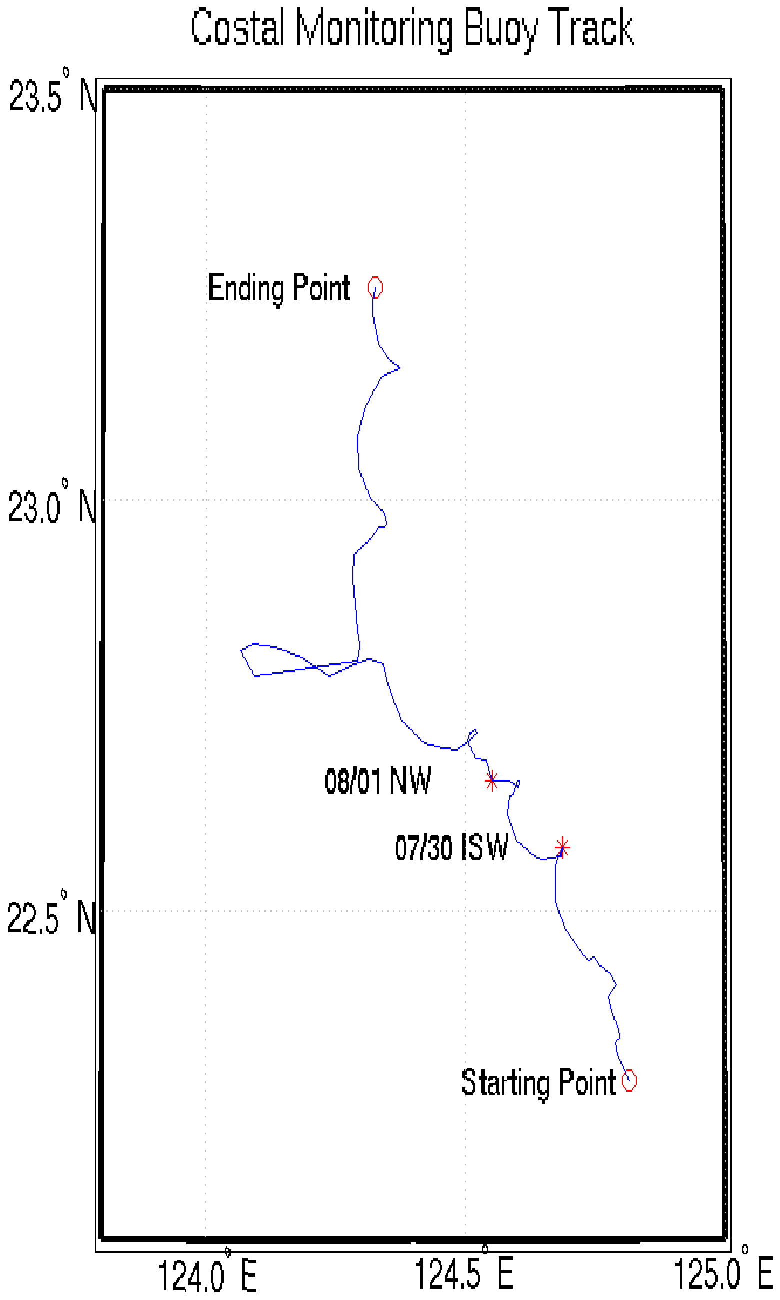


39 with Fairing
(shown with external thermistor)

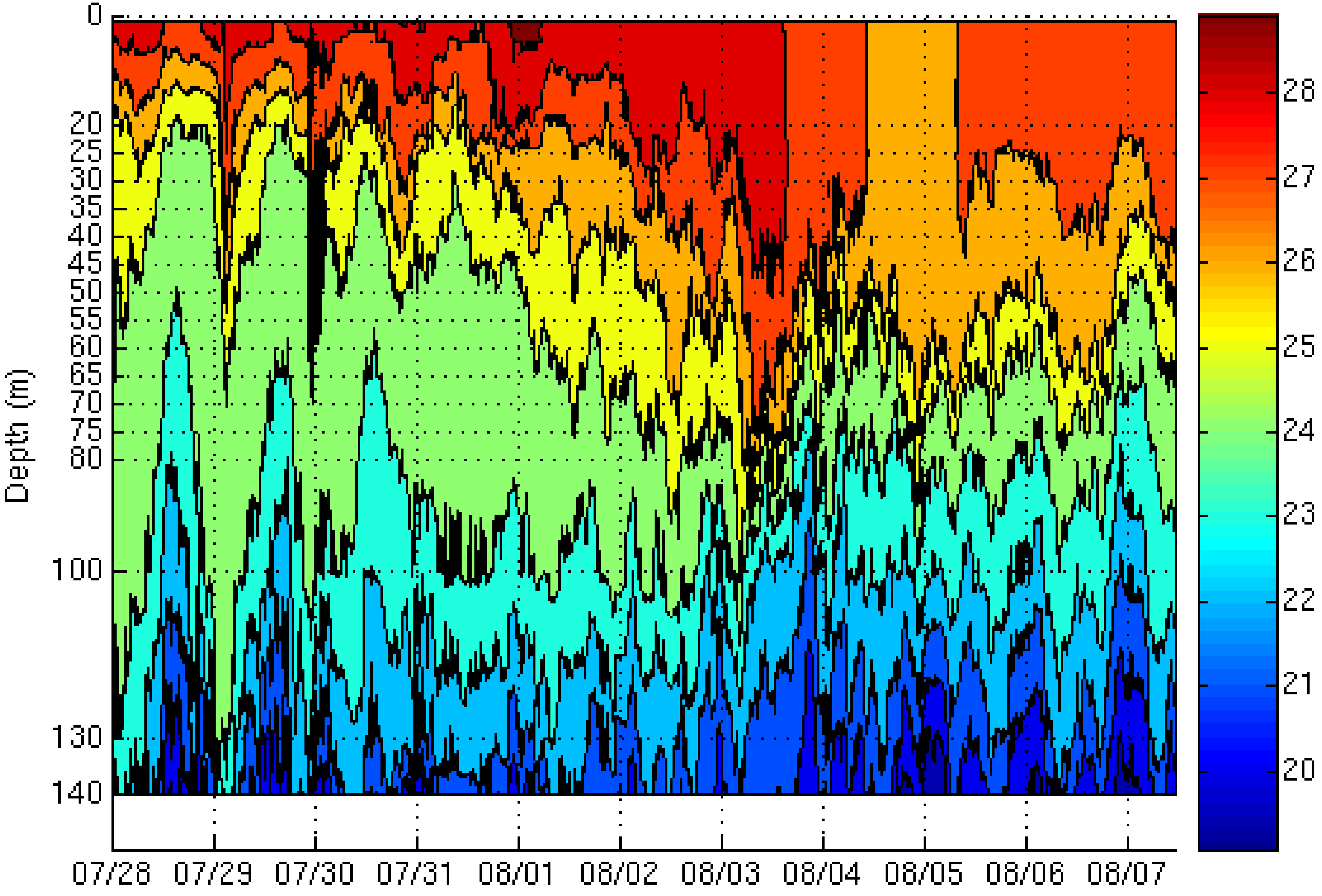


- Latitude - 22°17´N - 23°15´ N
- Longitude - 124°14´E - 124°49´ E
- Distance - 229.14 Km
- Velocity - 3.82m/ 15s





Temperature : Contoured at 1 ^{o} C
Beginning 28-Jul-2005 09:08:00



High-Order Structure Function

$$T_i = T(x_i), \quad x_i = il, \quad i = 0,1,\ldots,\Lambda, \quad L = \Lambda l \,,$$

$$|\Delta T(x_i, rl)| = |T(x_{i+r}) - T(x_i)|, \quad i = 0,1,\ldots,\Lambda - r$$

$$S(r,q) \equiv \left\langle |\Delta T(x,rl)|^q \right\rangle = \frac{1}{\Lambda - r}\sum_{i=0}^{\Lambda - r} |\Delta T(x_i, rl)|^q \,.$$

Here, r is the lag, q is the order of the structure function.
S(*r*, 1) is the commonly used structure function.

$$S(1,1) = \frac{1}{\Lambda - 1}\sum_{i=0}^{\Lambda - 1} \left| T(x_{i+1}) - T(x_i) \right|$$

S(1,1) is the mean gradient .

Scale-Invariance

$$\left|\Delta T(x_i, l)\right| = r^{-H}\left|\Delta T(x_i, rl)\right|,$$

H is the scaling exponent, or called the Hurst exponent. In 1941, Kolmogorov suggested that the velocity increment in high-Reynolds number turbulent flows should scale with the mean (time-averaged) energy dissipation and the separation length scale.

The Hurst exponent *H* is equal to 1/3.

- Simple self-similarity

$$\left\langle \left|\Delta T(x_i, l)\right|^q \right\rangle = r^{-qH} \left\langle \left|\Delta T(x_i, rl)\right|^q \right\rangle .$$

- Multifractal behavior

$$\left\langle \left|\Delta T(x_i, l)\right|^q \right\rangle = r^{-\zeta(q)} \left\langle \left|\Delta T(x_i, rl)\right|^q \right\rangle .$$

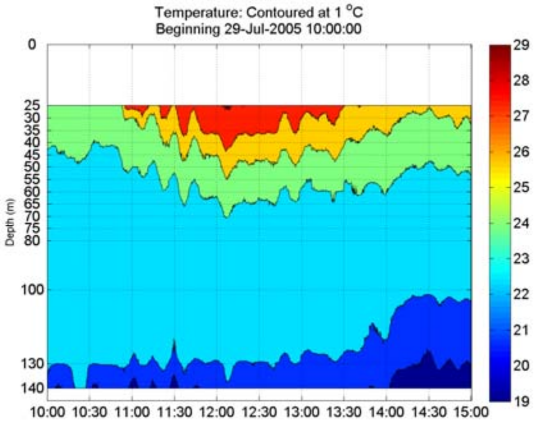
$$\zeta(q) \neq qH .$$

$$S(r,q) \sim r^{\zeta(q)}$$

Three Types

(a) Internal Wave -turbulence (IW-T)

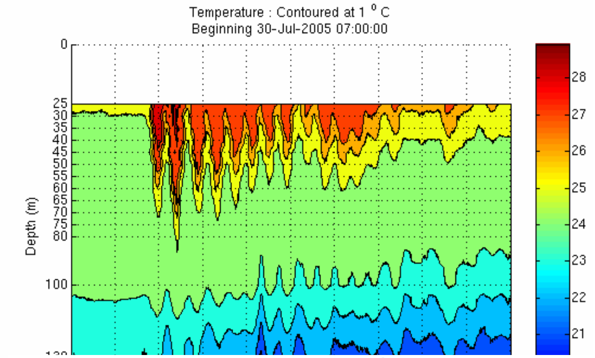
(1000-1500 GMT July 29)



Temperature: Contoured at 1 °C
Beginning 29-Jul-2005 10:00:00

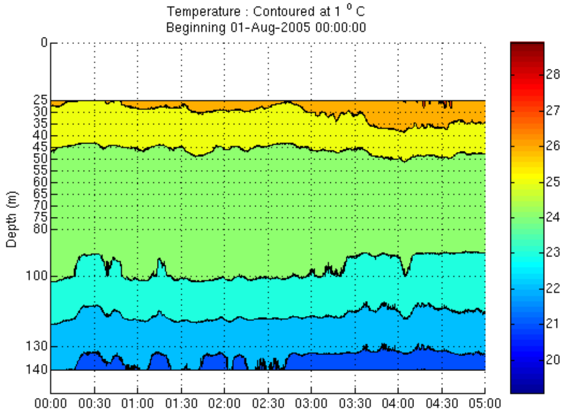
(b) Internal Soliton - turbulence (IS-T)

(0700-1200 GMT July 30)



Temperature : Contoured at 1 ° C
Beginning 30-Jul-2005 07:00:00

(c) Turbulence-dominated (T) (0000- 0500 GMT August 1)



Temperature : Contoured at 1 ° C
Beginning 01-Aug-2005 00:00:00

Isopycnal Displacement

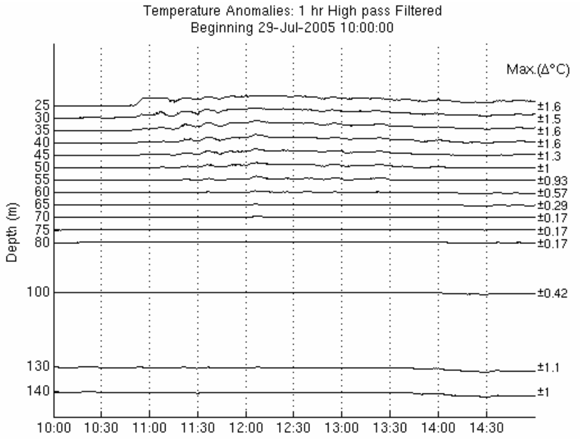
(Desaubles and Gregg, 1981,JPO)

$$T'(t,z) = T(t,z) - \overline{T}(z),$$

$$\eta(t,z) = -\frac{T'(t,z)}{d\overline{T}/dz},$$

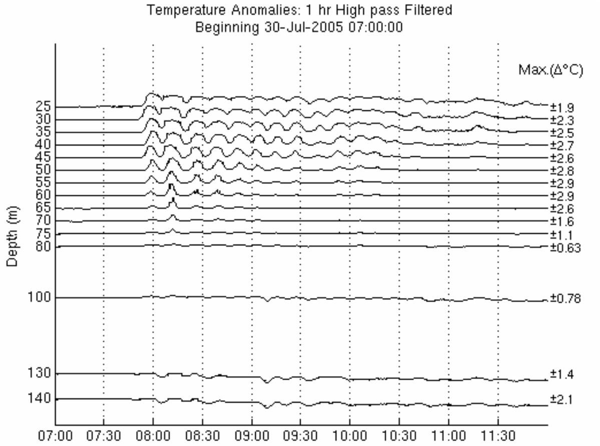
(a) IW-T type

(1000-1500 GMT July 29)



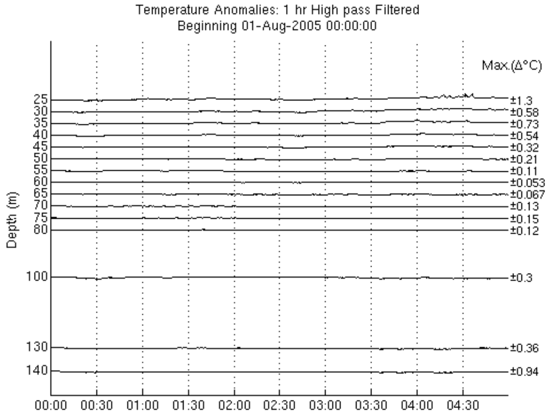
(b) IS-T type

(0700-1200 GMT July 30)



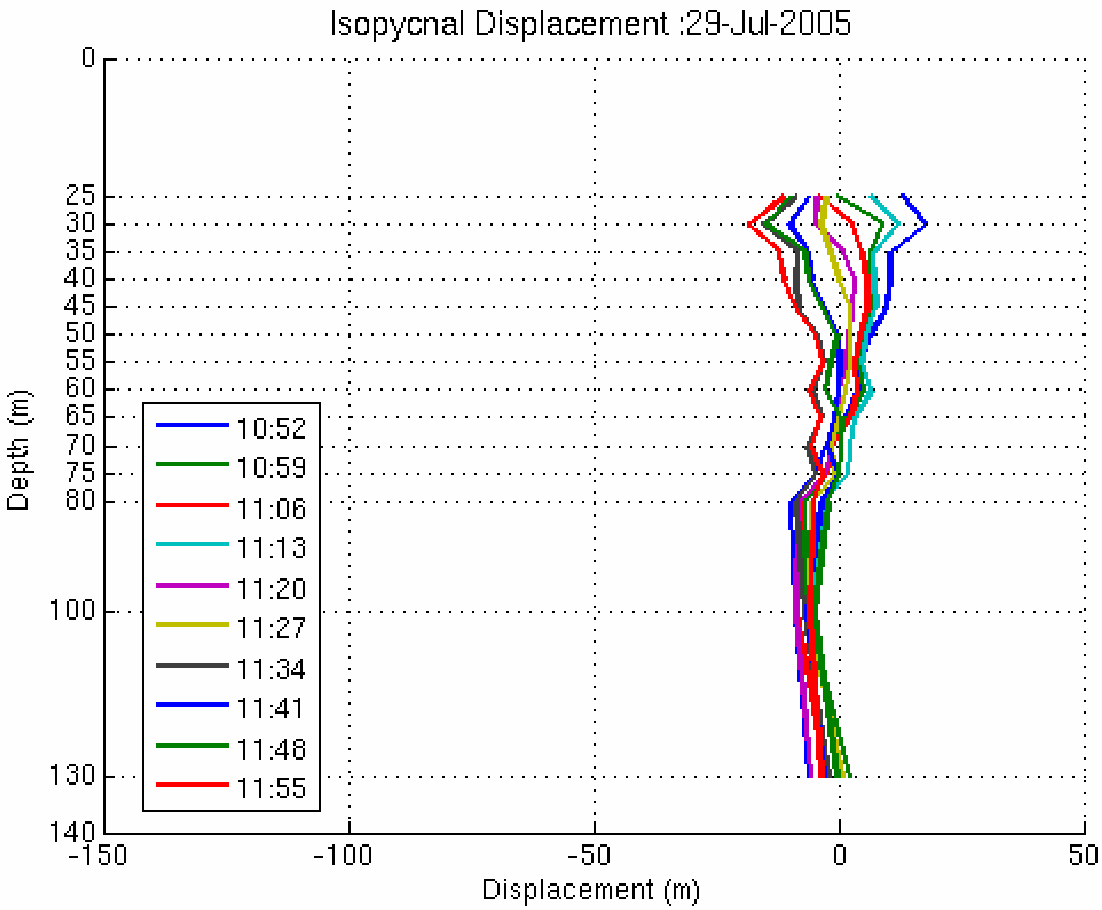
(c) T- type

(0000- 0500 GMT August 1)



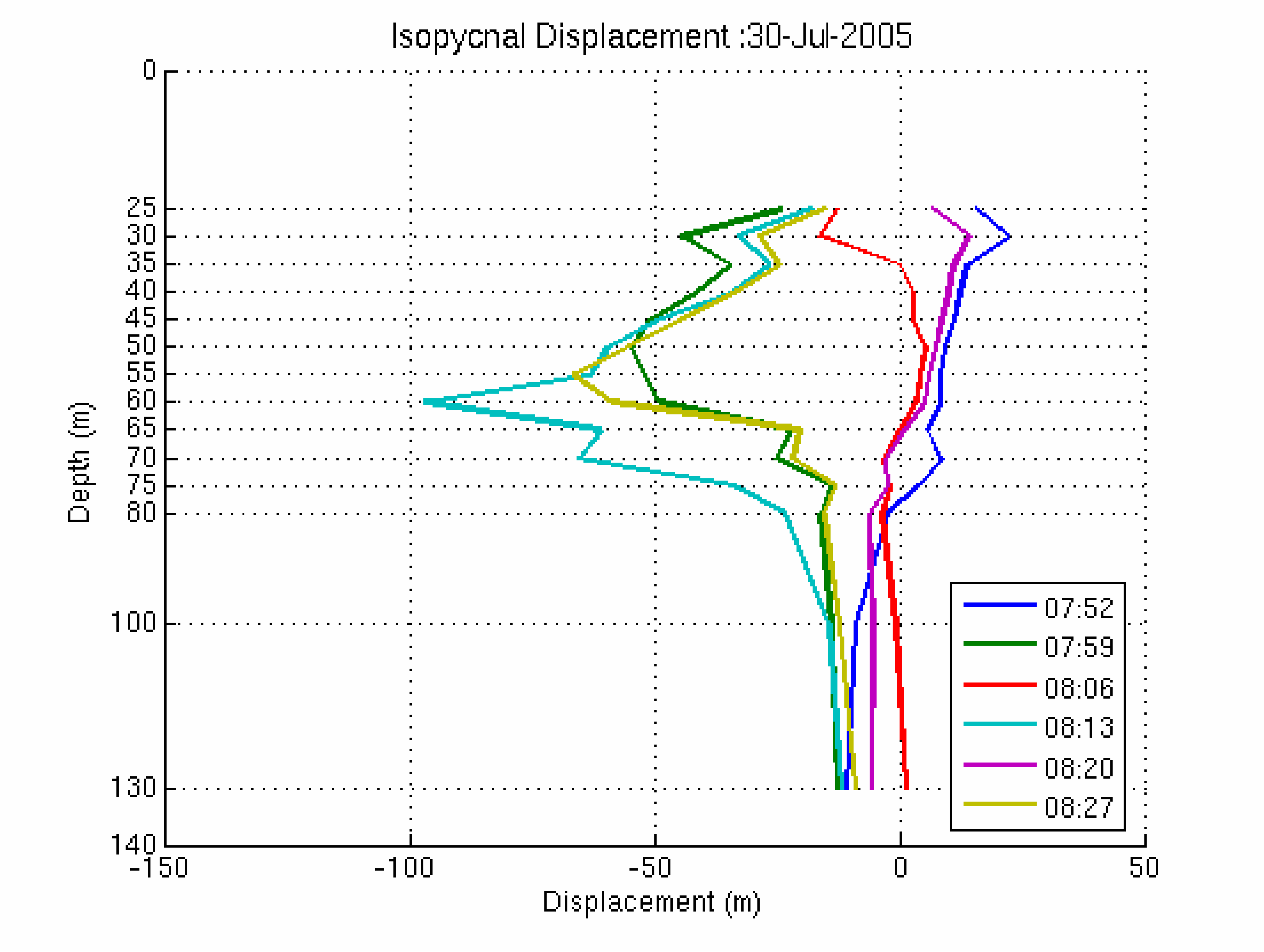
Isopycnal Displacement

IW-T (10-15 GMT July 29)



Isopycnal Displacement

IS-T (07-12 GMT July 30)

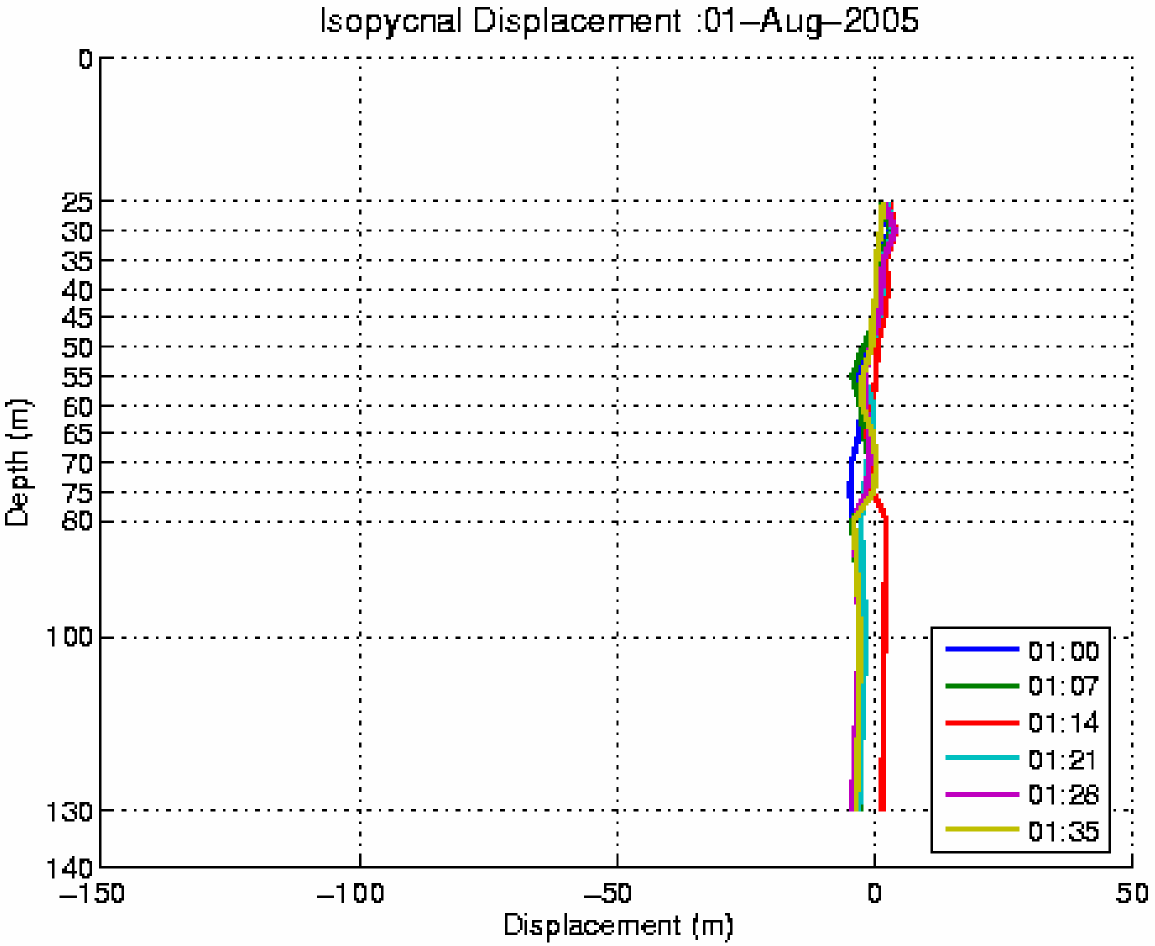


Frequency
is around

4 CPH

Isopycnal Displacement

turbulence-Dominated (00-05 GMT Aug 1)

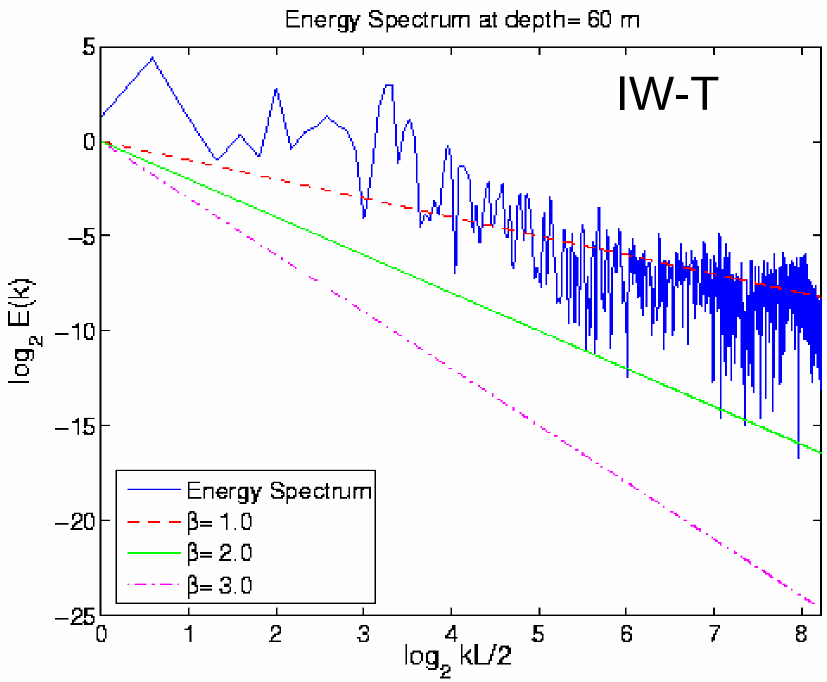


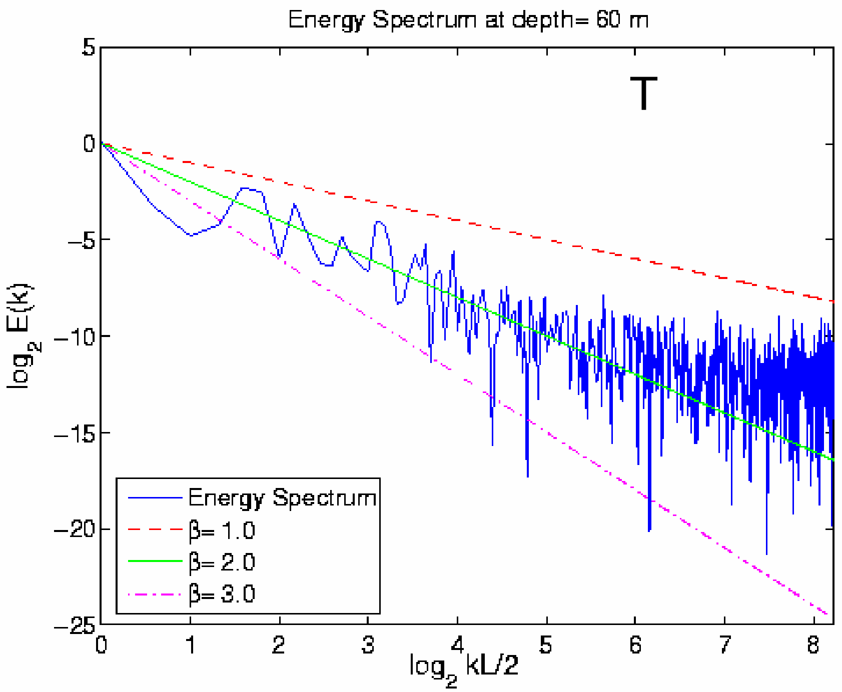
Power Spectrum

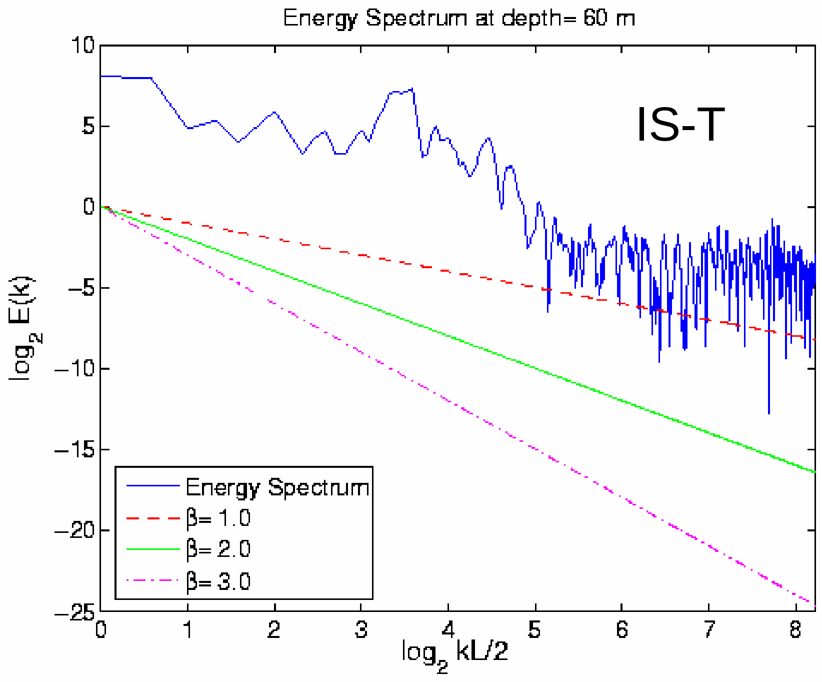
$$E_j = E(k_j), \quad k_j = j/L, \quad j = 1, 2, \ldots, \Lambda/2, \quad L = \Lambda l \qquad \Lambda = 1{,}200$$

$$E(k) \propto k^{-\beta},$$

$\beta < 1,$	Stationary
$1 < \beta < 3,$	Nonstationary with stationary increments
$\beta > 3,$	Nonstationary with nonstationary increments



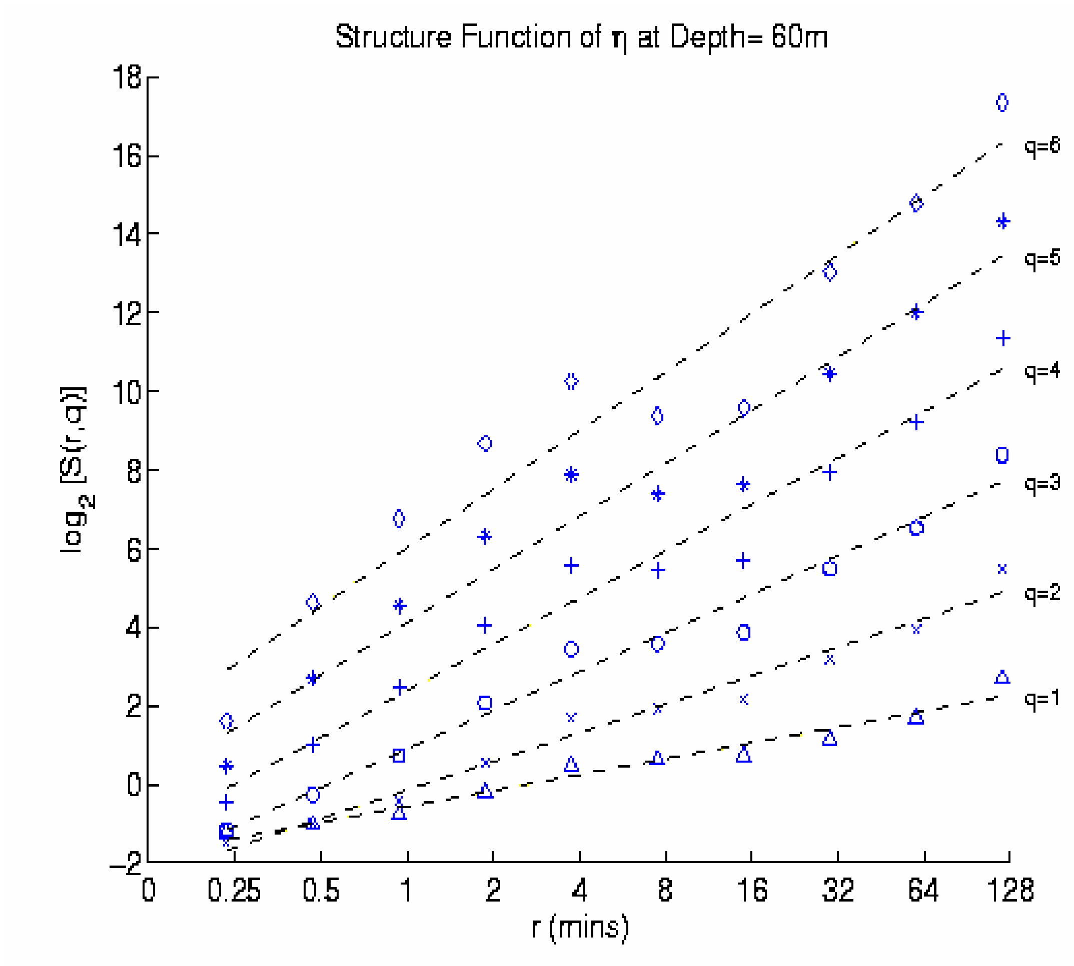




IW-T and T have similar multi scaling characteristics with β around 0.4 (stationary) for low wavenumbers and nearly 5/3 (non-stationary with stationary increment) for high wave numbers.

60 m depth

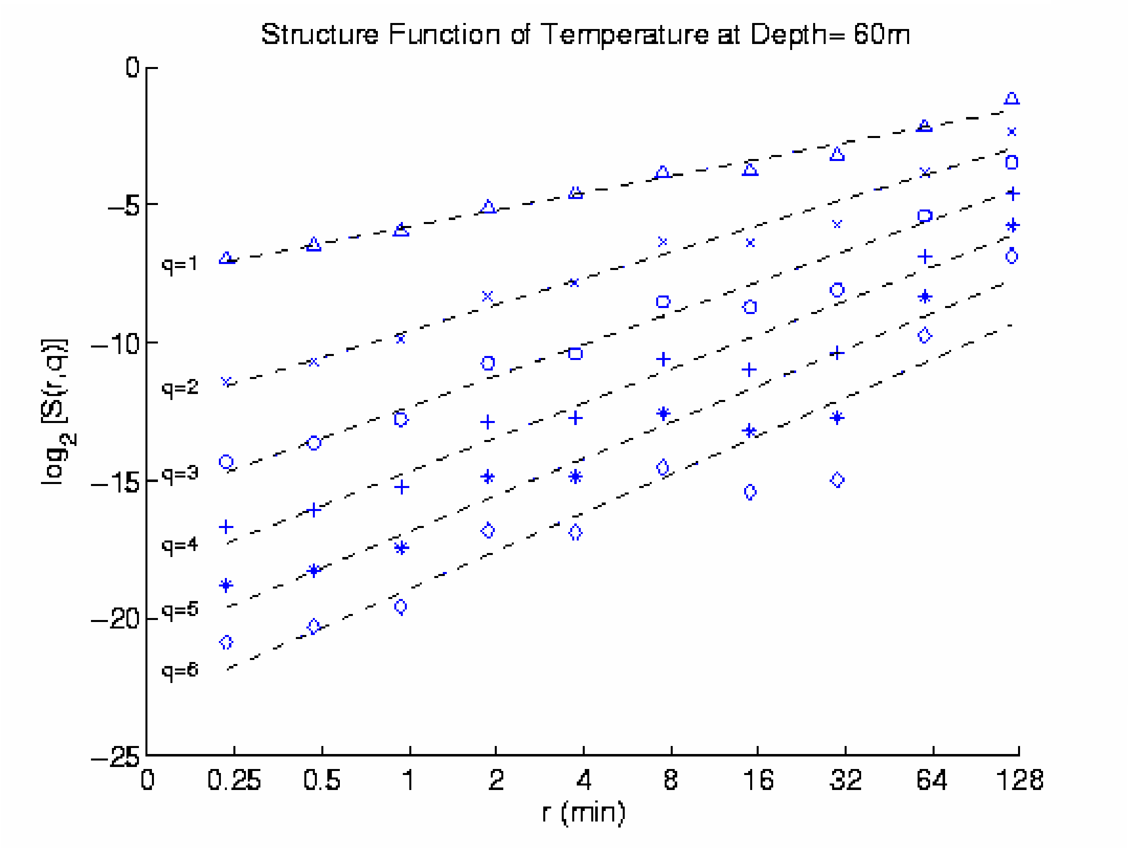
Structure Function (Power Law)
IW-T type



$$S(r,q) \propto r^{\zeta(q)},$$

$$\zeta(q) = H(q)q,$$

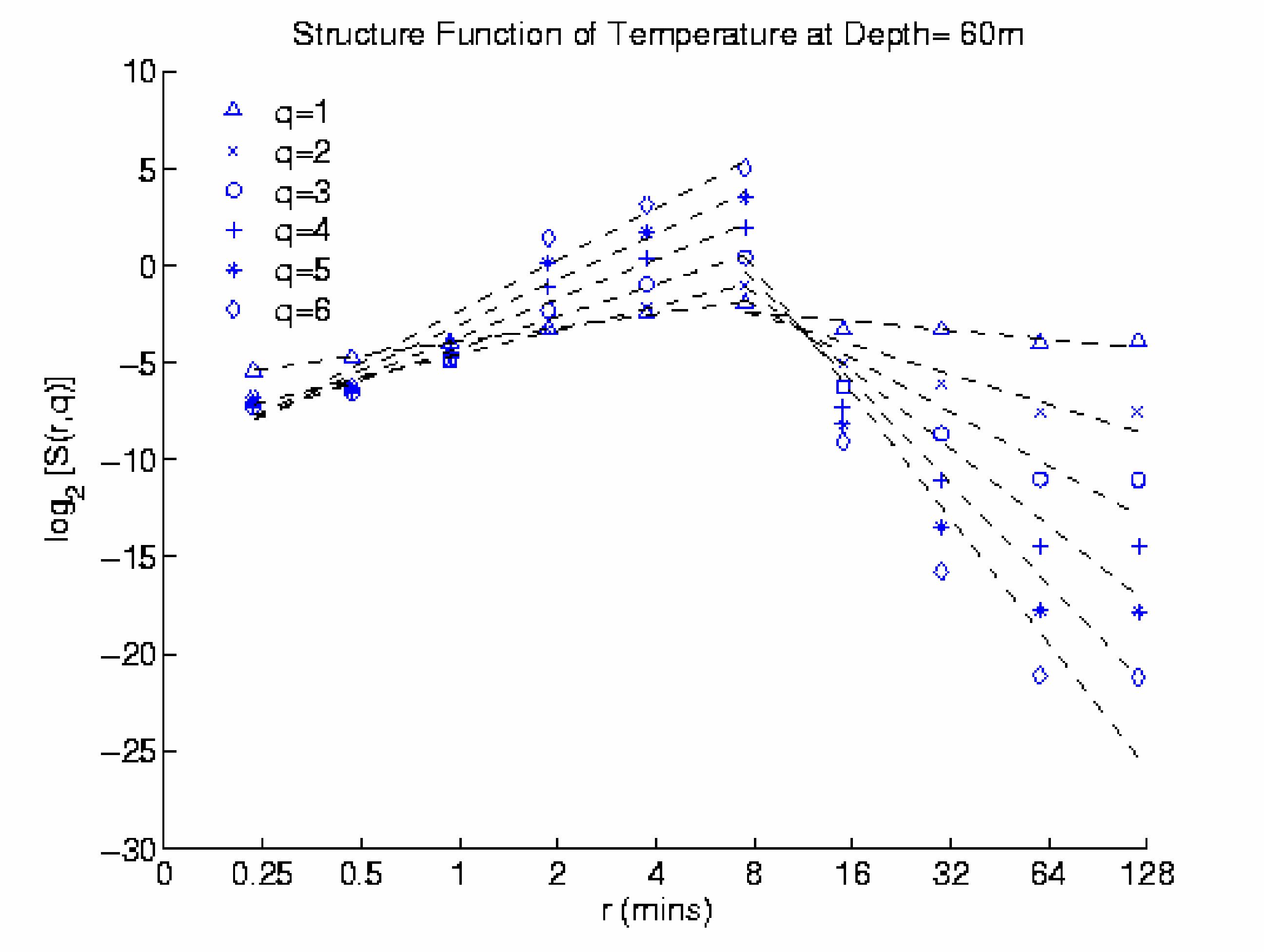
Structure Function (Power Law)
T type



$$S(r,q) \propto r^{\zeta(q)},$$

$$\zeta(q) = H(q)q,$$

Structure Function
IS-T type



Power law breaks at 8 min, near half period (4 CPH) of the internal solitons

Possible Reason for Preservation of the Power Law in IW-T Type

Using the Hamitonian formulation, Lvov and Tabak (2001) modified the Garrett-Munk spectrum into

$$E(k,m) = \frac{2fNE}{\pi}\frac{(m/m^*)A(m/m^*)}{N^2k^2 + f^2m^2},$$

$$m^* \equiv \gamma(\omega^2 - f^2)^{-\delta/2},\ A(\lambda) \equiv \frac{t-1}{(1+\lambda)^t}$$

which represents both internal waves and wave turbulence.

Possible Reason for Break of the Power Law in IS-T Type

- The internal solitary waves are a class of nonsinusoidal, nonlinear, more-or-less isolated waves of complex shape that maintain their coherence. Their energy spectrum is totally different from the internal wave spectrum.

Conclusions

- (1) Three types of thermal variability (IW-T, IS-T, and T) are identified.

- (2) Multifractal structures are found in the upper layer of the western Philippine Sea.

- (3) Power law preserves in structure function with multifractal characteristics for the IW-T and T types, but not for the IS-T type.

- (4) The internal waves increase the power of the structure function especially for high moments.

- (5) The internal solitons destroy the multifractal characteristics of the structure function at the lag of 8 min, which is nearly half period of the IS (with frequency of 4 CPH).