IUGG 2007
PS010 New Insights into the Ocean and Its Circulation from ARGO and GODAE
July 9-10, Perugia, Italy

Optimal Spectral Decomposition for ARGO Data Analysis

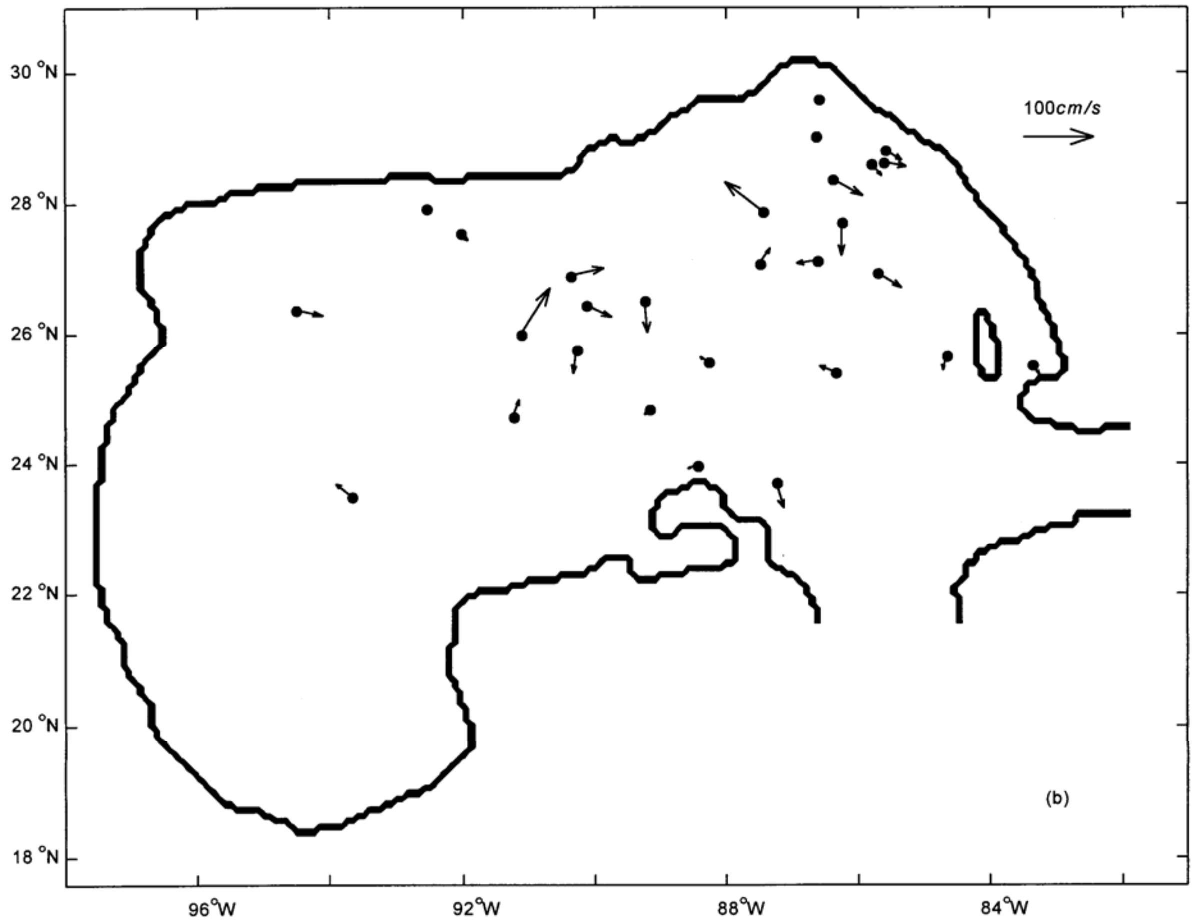
Peter C. Chu
Naval Postgraduate School

Charles Sun
NOAA/NODC

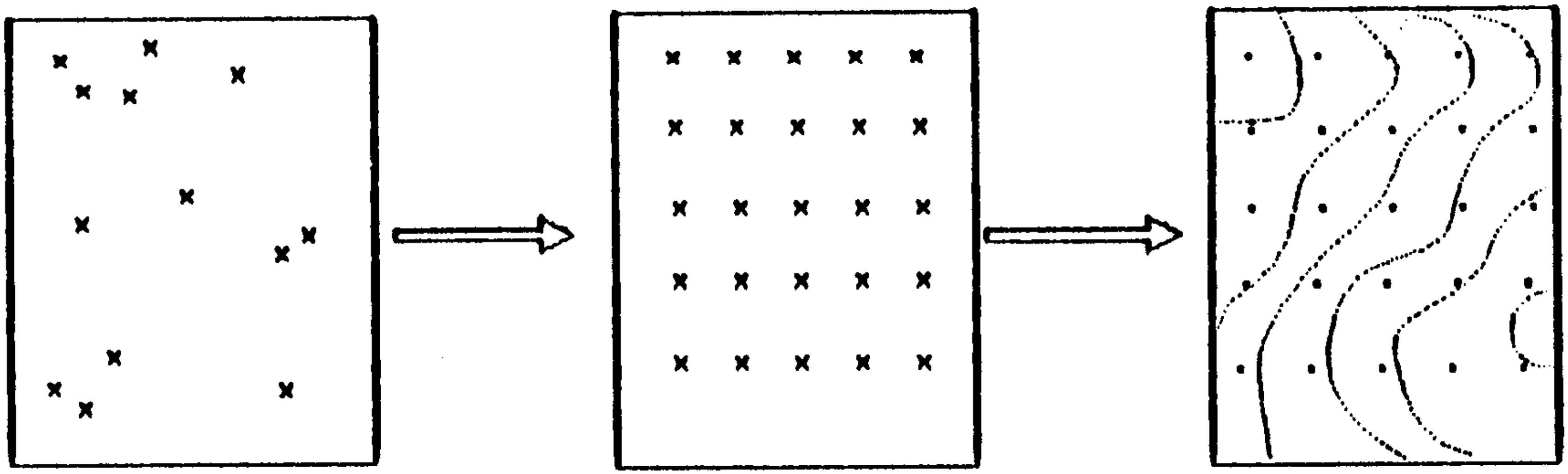
Oleg Melnichenko
University of Hawaii

Part-1
Theory and Methodology

Observational Data (Sparse and Noisy)



Most Popular Method for Ocean Data Analysis: Optimum Interpolation (OI)



Three Necessary Conditions For the OI Method

- (1) First guess field
- (2) Autocorrelation functions
- (3) Low noise-to-signal ratio

Ocean velocity data

- (1) First guess field (?)
- (2) Unknown autocorrelation function
- (3) High noise-to-signal ratio

It is not likely to use the OI method to process ocean velocity data.

Spectral Representation - a Possible Alternative Method

Theoretical Base: Fourier Series Expansion

$$c(\mathbf{x}, z_k, t) = A_0(z_k, t) + \sum_{m=1}^{M} A_m(z_k, t)\ \Psi_m(\mathbf{x}, z_k),$$

Flow Decomposition

$$u = \frac{\partial \Psi}{\partial y} + \frac{\partial^2 \Phi}{\partial x \partial z}, \qquad v = -\frac{\partial \Psi}{\partial x} + \frac{\partial^2 \Phi}{\partial y \partial z},$$

- $$\Delta \Psi = -\zeta$$

  $$\Delta \Phi = -w$$

Basis Functions (Closed Basin)

$$\triangle\Psi_k = -\lambda_k \Psi_k, \quad \Psi_k|_\Gamma = 0, \qquad k = 1, ..., \infty$$

$$\triangle\Phi_m = -\mu_m \Phi_m, \quad \frac{\partial\Phi_m}{\partial n}|_\Gamma = 0, \qquad m = 1, ..., \infty.$$

Eremeev et al. (1992 JGR)

Rectangular Domain with Flat Bottom

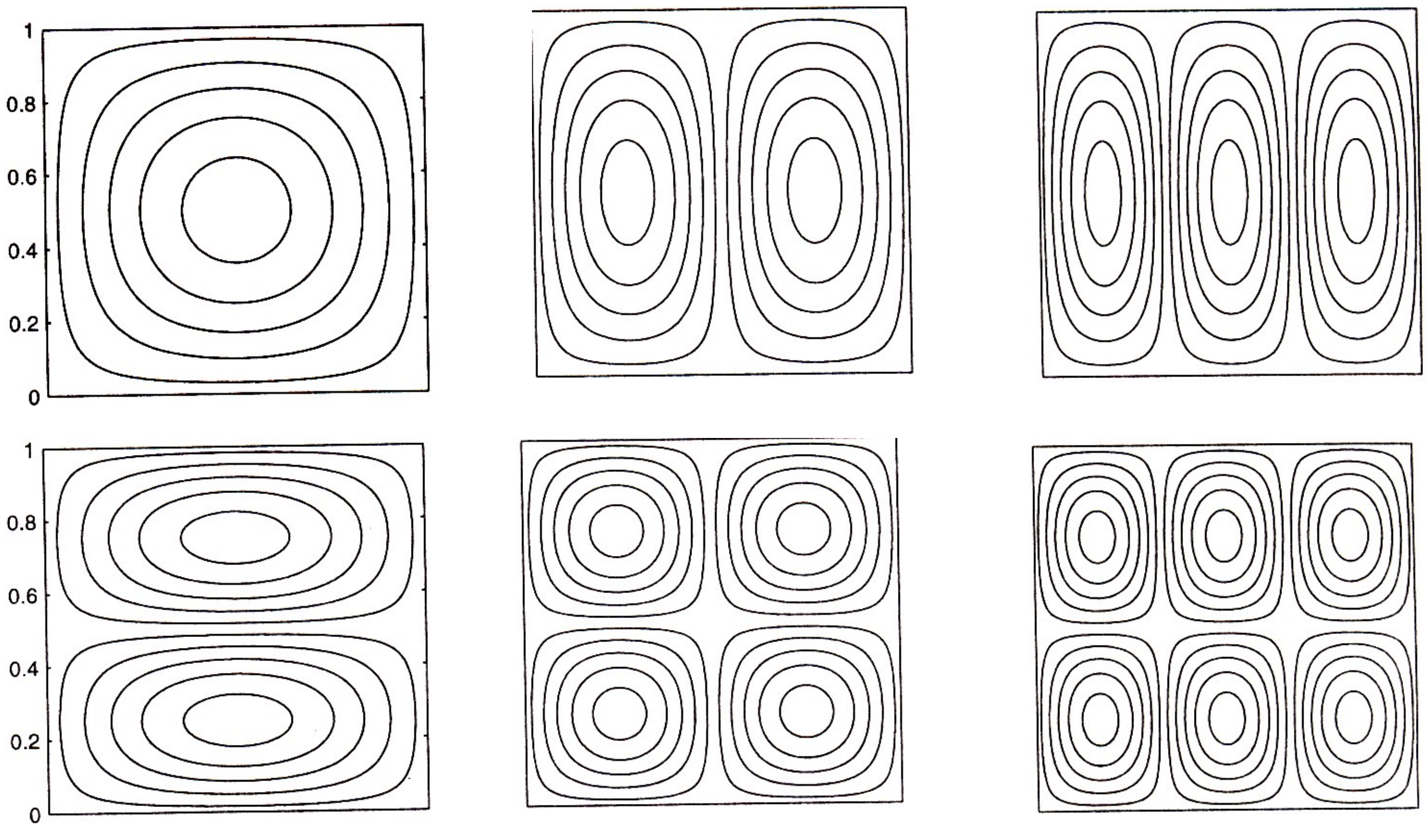
Sinusoidal Basis Functions

$$\Psi^{(i,j)} = \sin\frac{i\pi x}{L_x}\sin\frac{j\pi y}{L_y},$$

$$\Phi^{(i,j)} = \cos\frac{i\pi x}{L_x}\cos\frac{j\pi y}{L_y}$$

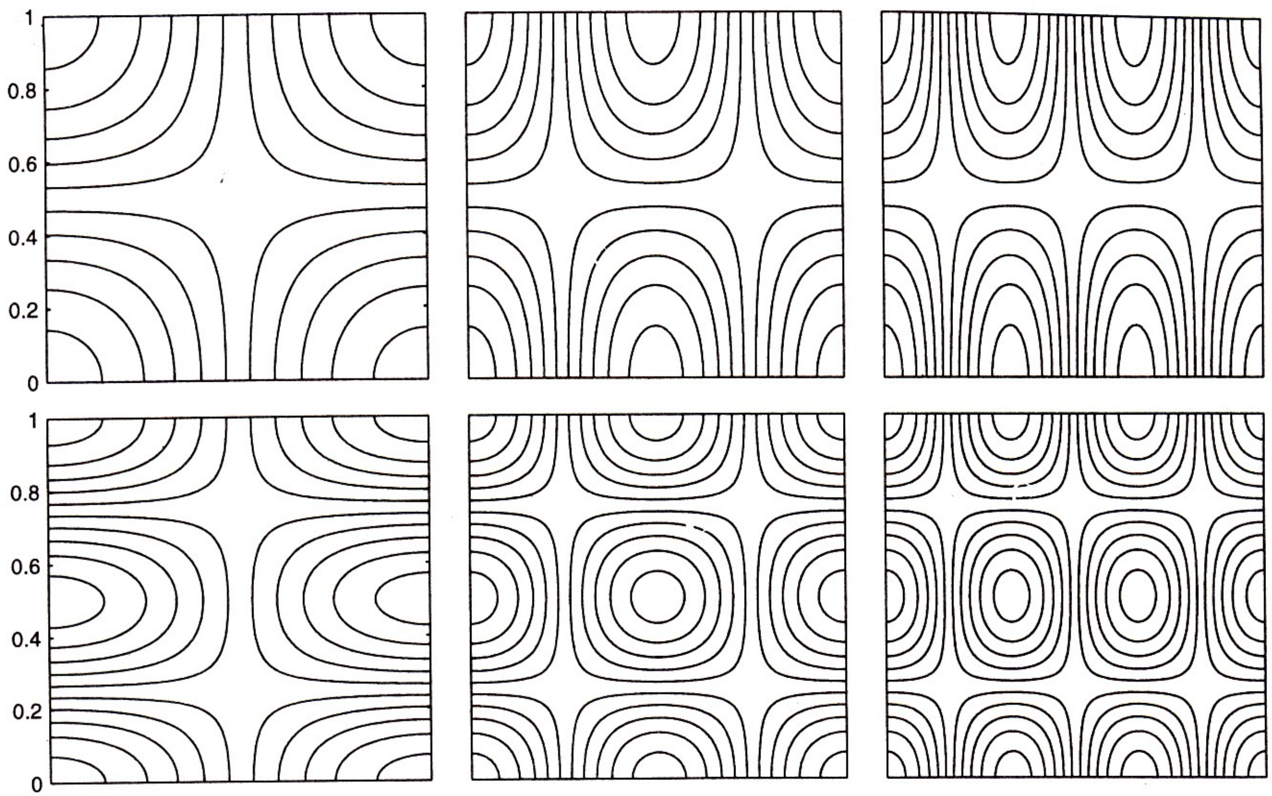
Basis Functions for Rectangular Domain

$$\Psi^{(i,j)} = \sin\frac{i\pi x}{L_x}\sin\frac{j\pi y}{L_y},$$



Basis Functions for Rectangular Domain

$$\Phi^{(i,j)} = \cos\frac{i\pi x}{L_x}\cos\frac{j\pi y}{L_y}$$



Basis Functions (Open Boundaries)

(Chu et al., 2003 a,b JTECH)

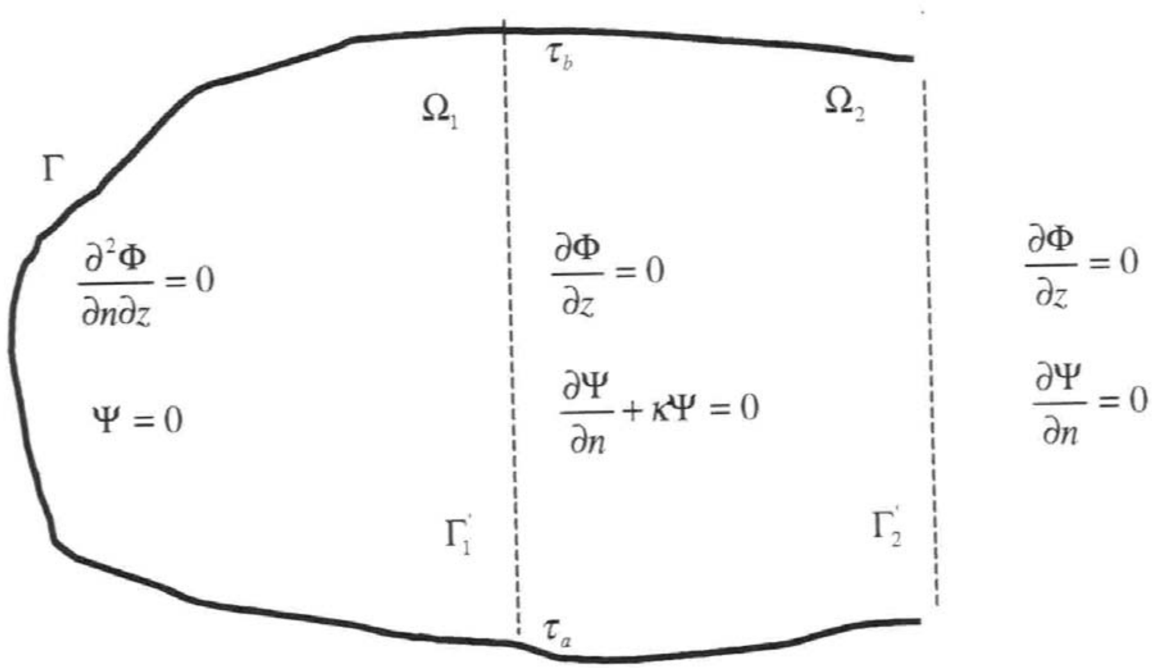
$$\triangle\Psi_k = -\lambda_k \Psi_k,$$

$$\triangle\Phi_m = -\mu_m \Phi_m,$$

$$\Psi_k|_\Gamma = 0, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0,$$

$$\left[\frac{\partial \Psi_k}{\partial n} + \kappa(\tau)\Psi_k\right]|_{\Gamma_1'} = 0, \quad \Phi_m|_{\Gamma_1'} = 0,$$

Boundary Conditions



Spectral Decomposition

$$\begin{aligned} u_{KM} &= \sum_{k=1}^{K} a_k(z,t^\circ)\frac{\partial \Psi_k(x,y,z,\kappa^\circ)}{\partial y} + \sum_{m=1}^{M} b_m(z,t^\circ)\frac{\partial \Phi_m(x,y,z)}{\partial x}, \\ v_{KM} &= -\sum_{k=1}^{K} a_k(z,t^\circ)\frac{\partial \Psi_k(x,y,z,\kappa^\circ)}{\partial x} + \sum_{m=1}^{M} b_m(z,t^\circ)\frac{\partial \Phi_m(x,y,z)}{\partial y} \end{aligned}$$

Benefit of Using OSD

- Ocean Topographic Configuration →

  Basis Functions (Pre-Determined)

Optimal Mode Truncation

$$J(a_{1,\dots,}a_K, b_{1,\dots,}b_M, \kappa, P) = \frac{1}{2}\left(\left\|u_p^{obs} - u_{KM}\right\|_P^2 + \left\|v_p^{obs} - v_{KM}\right\|_P^2\right) \to \min,$$

Vapnik (1983) Cost Function →Optimal Mode Truncation

$$J_{emp} = J(a_{1,\ldots,}a_{K,}b_{1,\ldots,}b_M, \kappa, P).$$

$$\text{Prob}\left\{\sup_{K,M,S}|\langle J(K,M,S)\rangle - J_{emp}(K,M,S)| \geq \mu\right\} \leq g(P,\mu)$$

$$\lim_{P\to\infty} g(P,\mu) = 0$$

Optimal Truncation

- Gulf of Mexico, Monterey Bay, Louisiana-Texas Shelf, Tropical Atlantic

- $K_{opt} = 40$, $M_{opt} = 30$

Determination of Spectral Coefficients (Ill-Posed Algebraic Equation)

$$\mathbf{A}\hat{\mathbf{a}} = \mathbf{Q}\mathbf{Y},$$

Rotation Method (Chu et al., 2004)

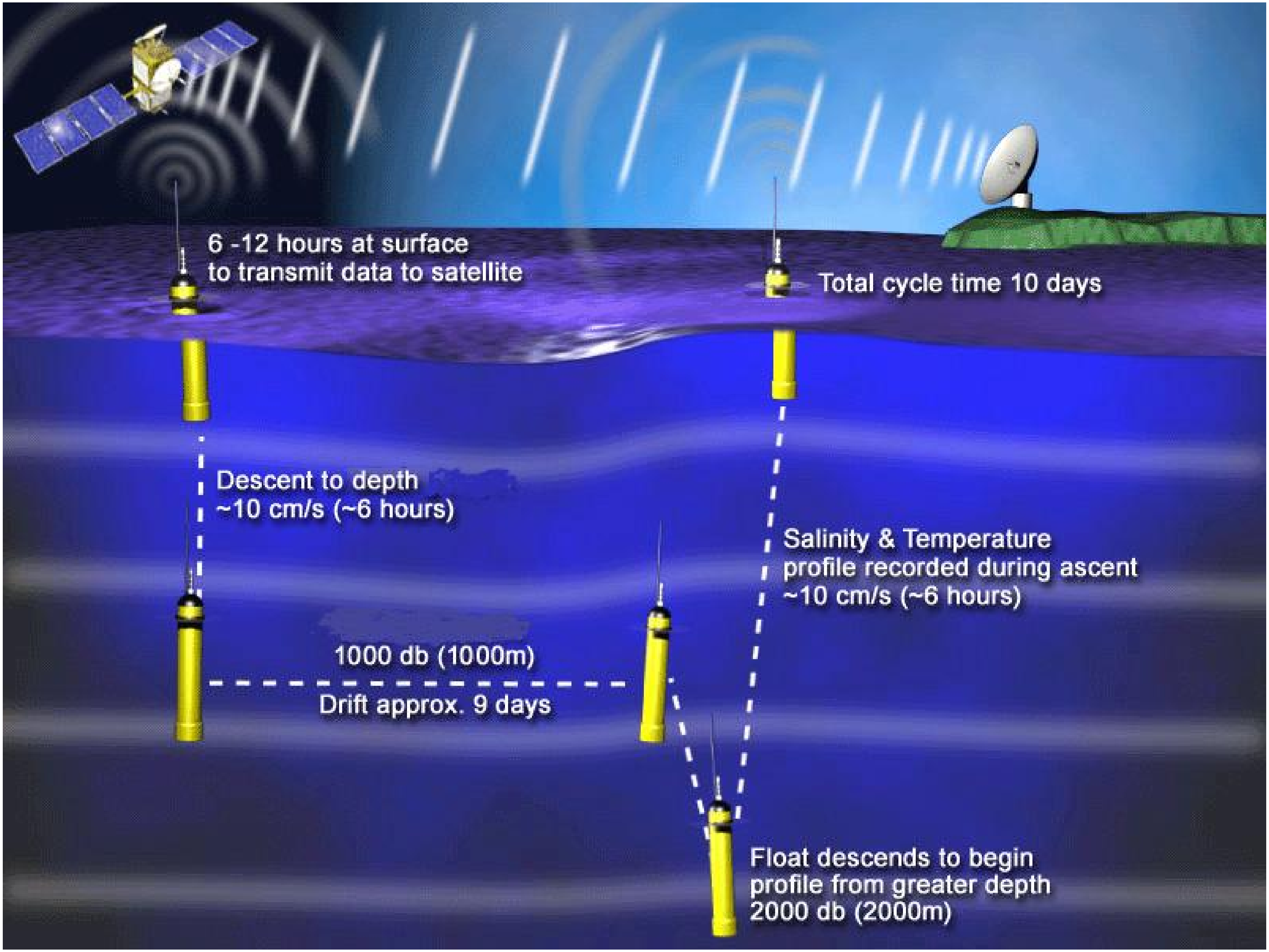
$$\mathbf{S}\mathbf{A}\hat{\mathbf{a}} = \mathbf{S}\mathbf{Q}\mathbf{Y},$$

$$J_1 = \|\mathbf{A}\|^2 - \frac{\|\mathbf{S}\mathbf{Q}\mathbf{Y}\|^2}{\|\mathbf{a}\|^2} \rightarrow \max,$$

Part-2
Applications

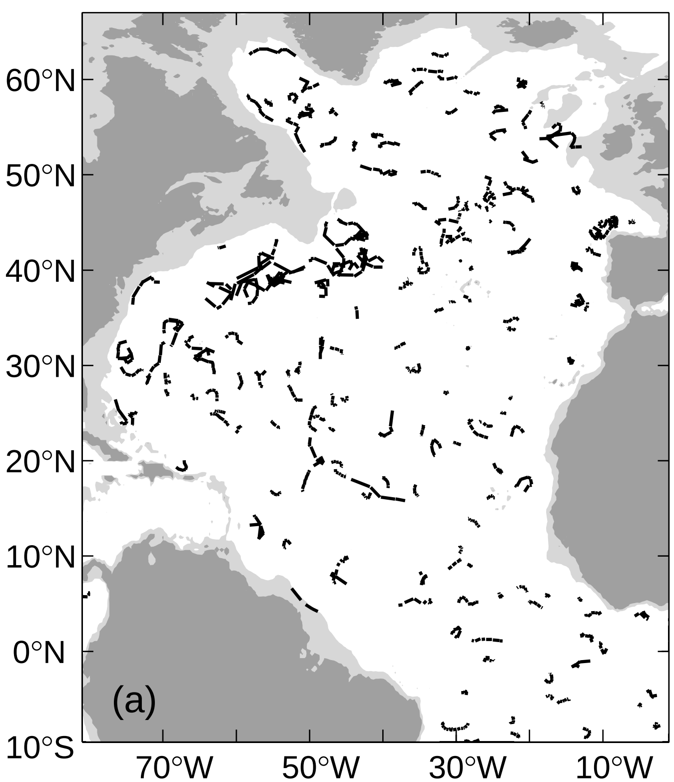
- Are mid-depth (~1000 m) ocean circulations steady?

- If not, what mechanisms cause the change? (Rossby wave propagation)

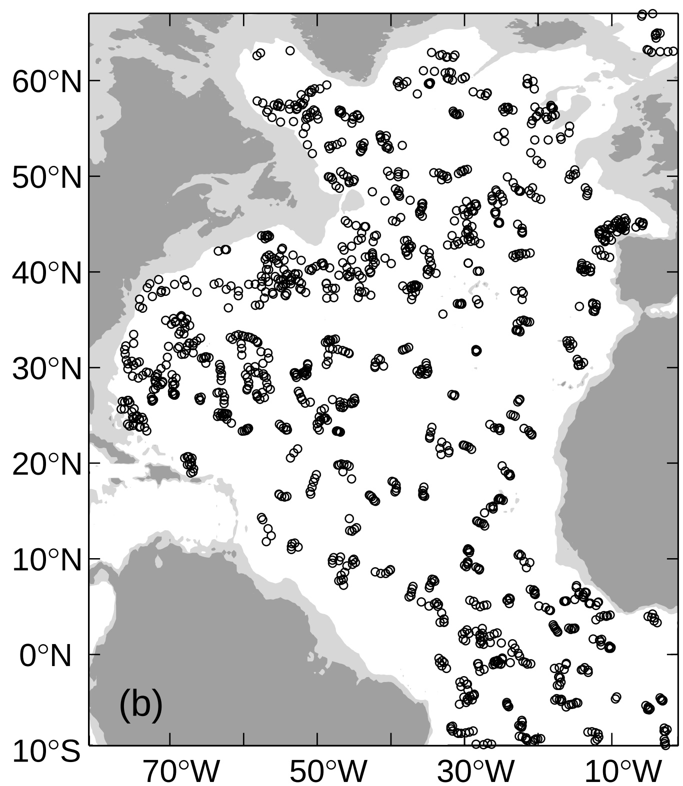

6 -12 hours at surface
to transmit data to satellite
Total cycle time 10 days
Descent to depth
~10 cm/s (~6 hours)
Salinity & Temperature
profile recorded during ascent
~10 cm/s (~6 hours)
1000 db (1000m)
Drift approx. 9 days
Float descends to begin
profile from greater depth
2000 db (2000m)

ARGO Observations (Oct-Nov 2004)

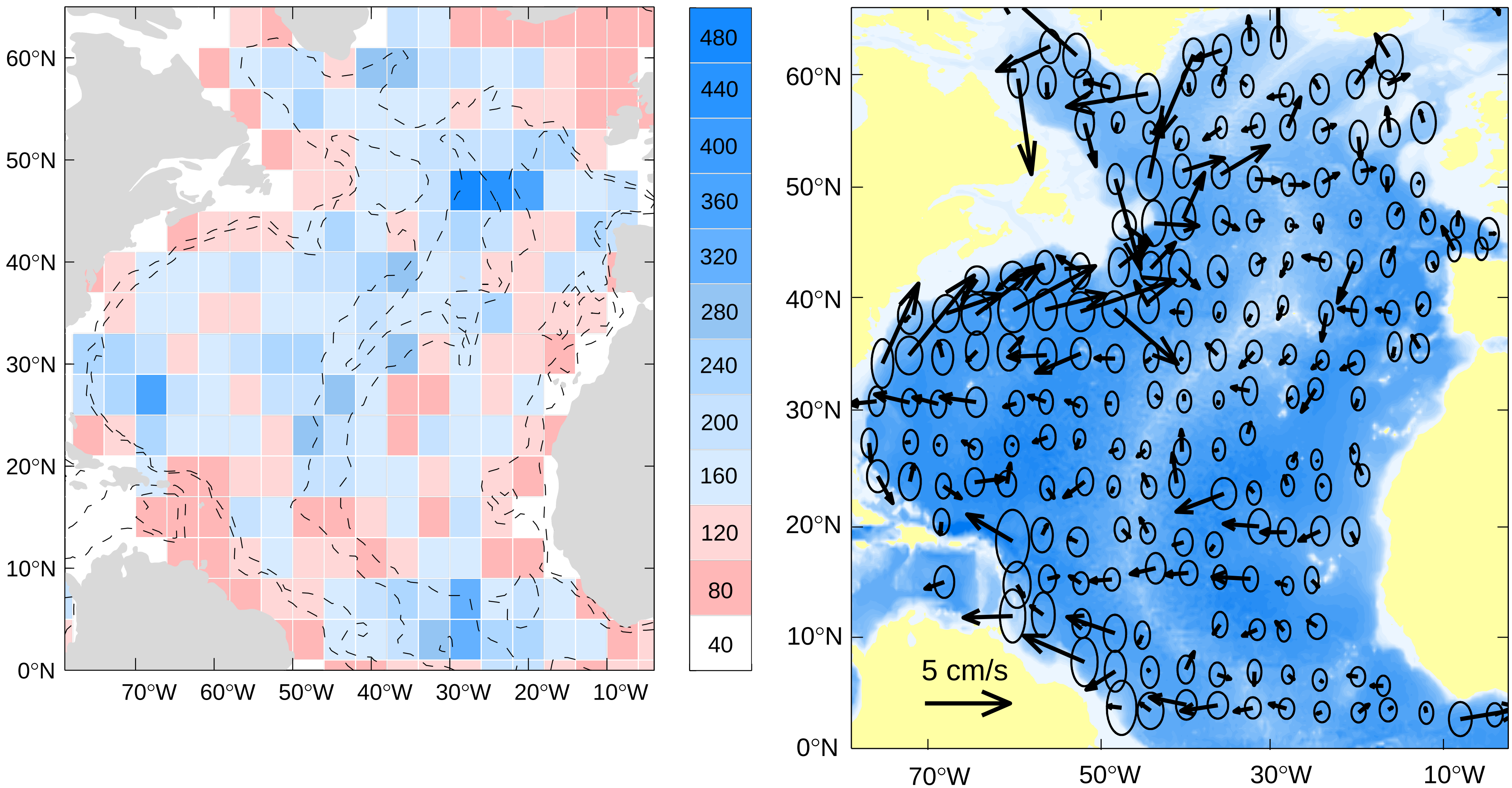
(a) Subsurface tracks



(b) Float positions where (T,S) were measured

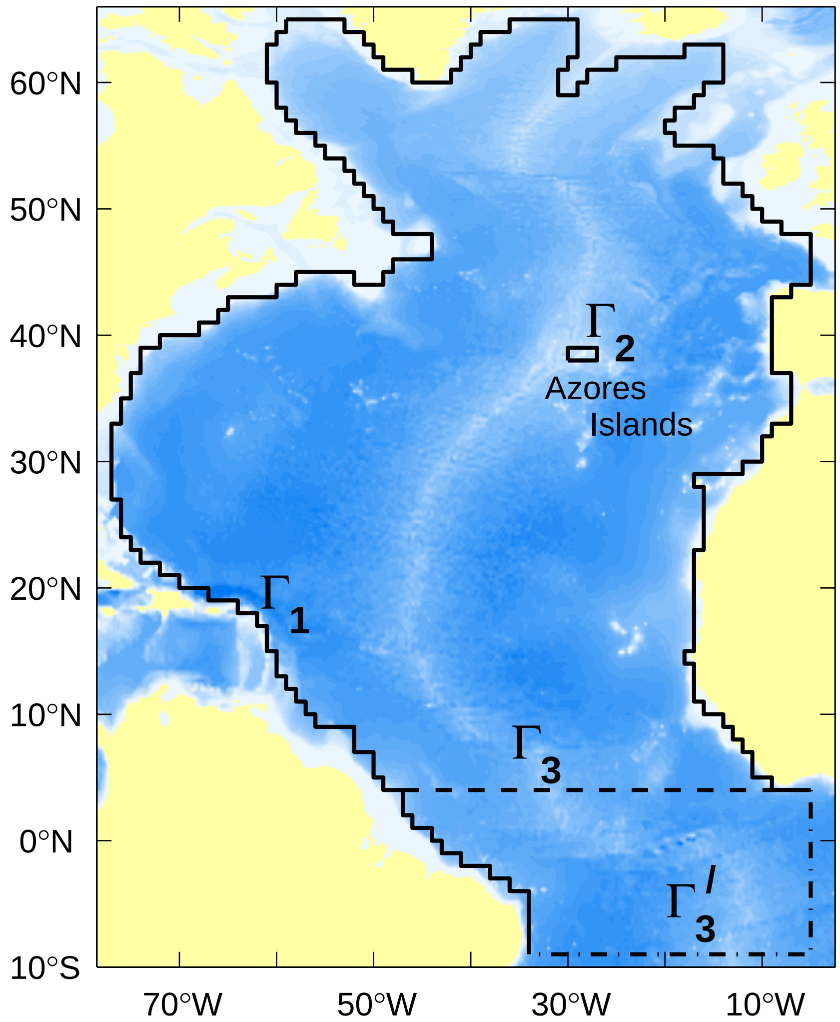


Circulations at 1000 m estimated from the original ARGO float tracks (bin method) April 2004 – April 2005

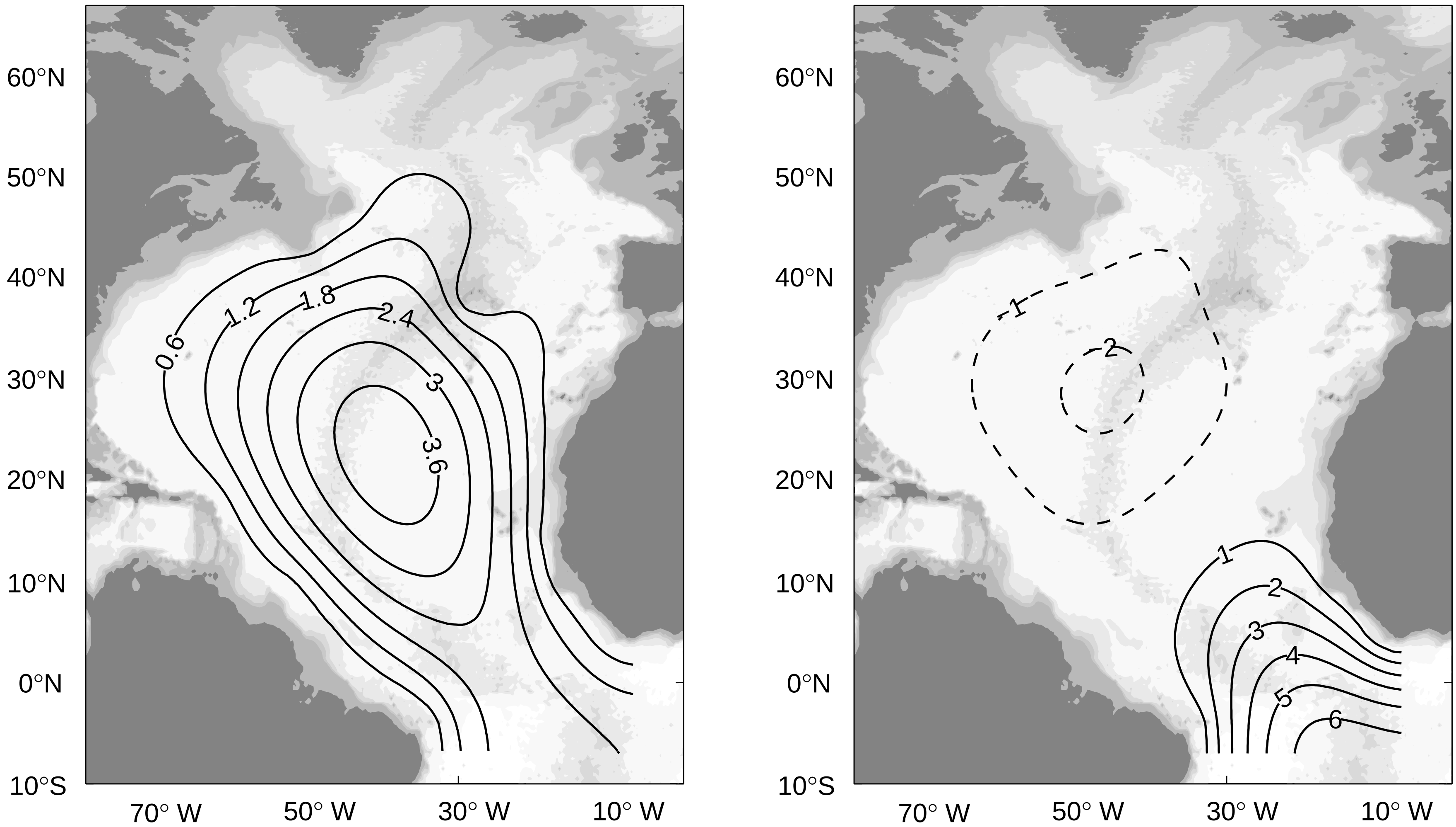


It is difficult to use such noisy data into ocean numerical models.

Boundary Configuration → Basis Functions for OSD

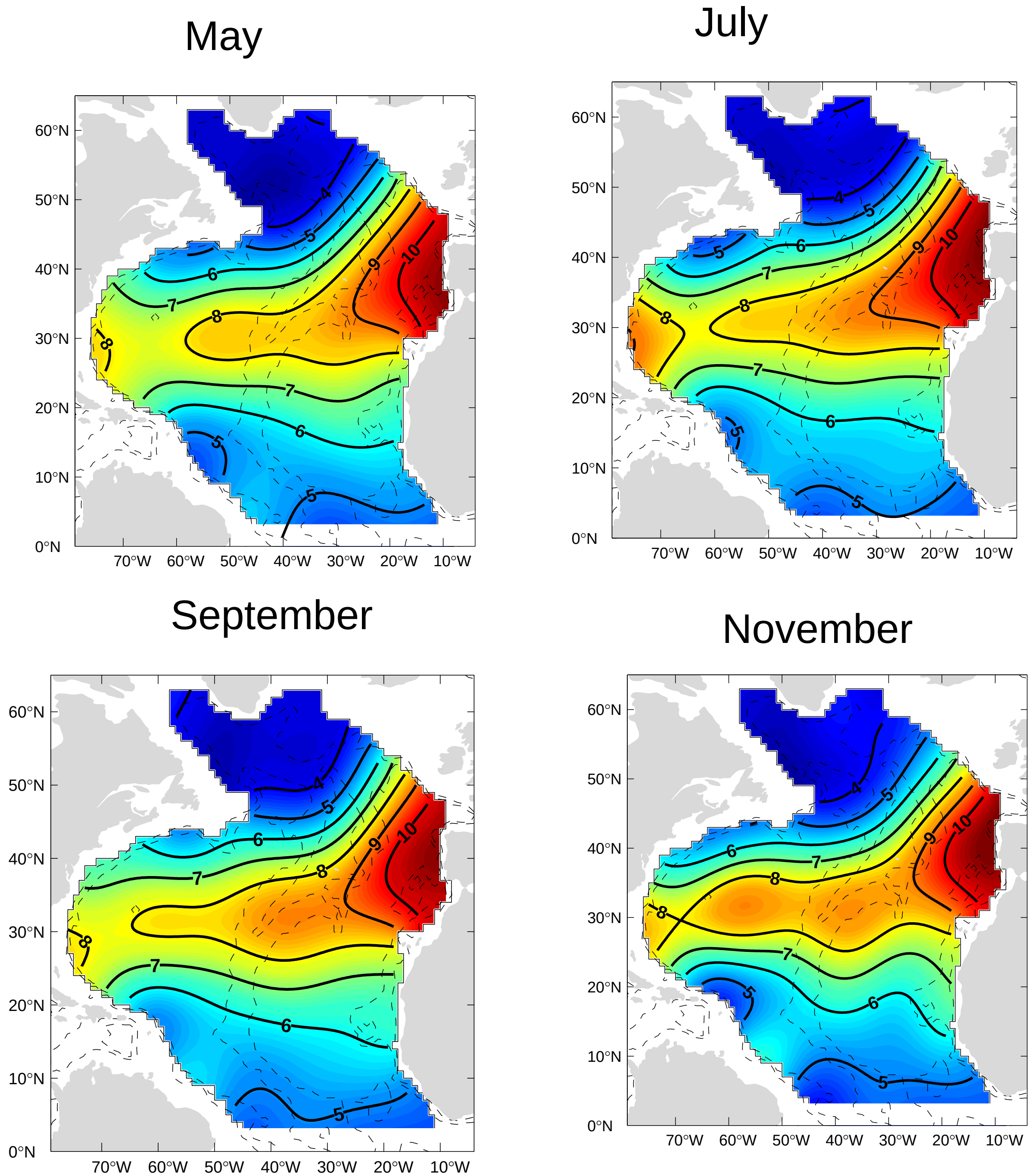


Basis Functions for Streamfunction
Mode-1 and Mode-2



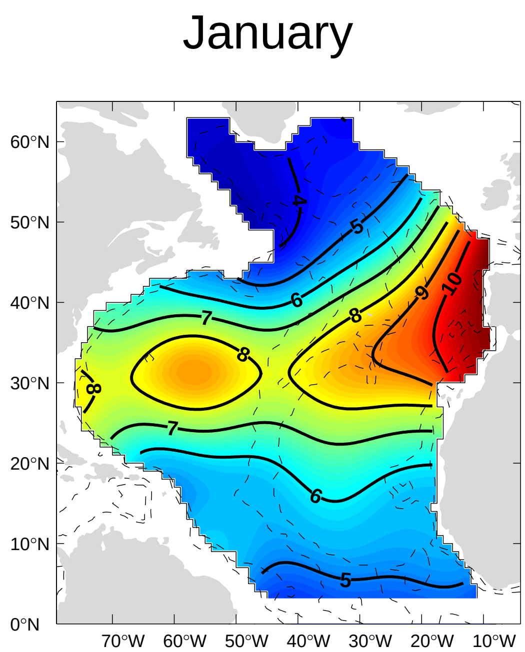
Temperature at 950 m depth

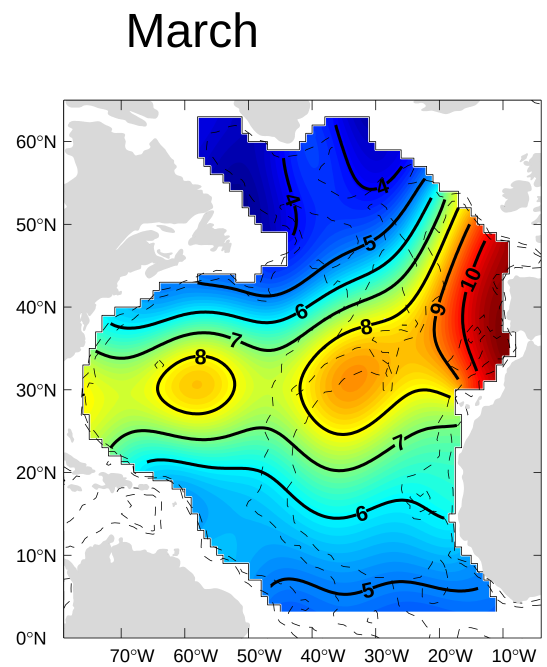
2004

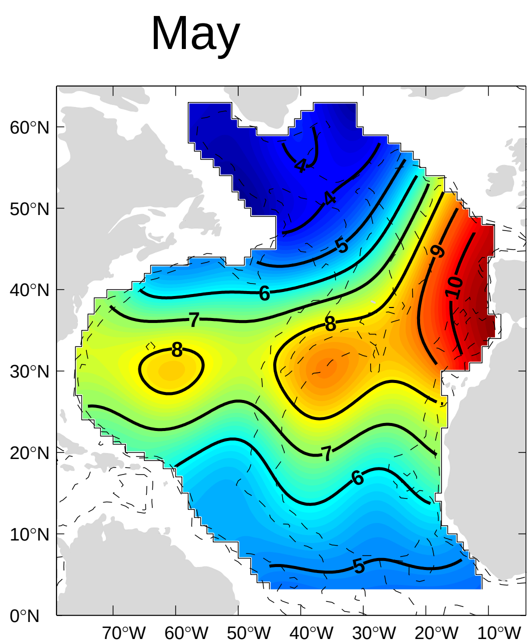


Temperature at 950 m depth

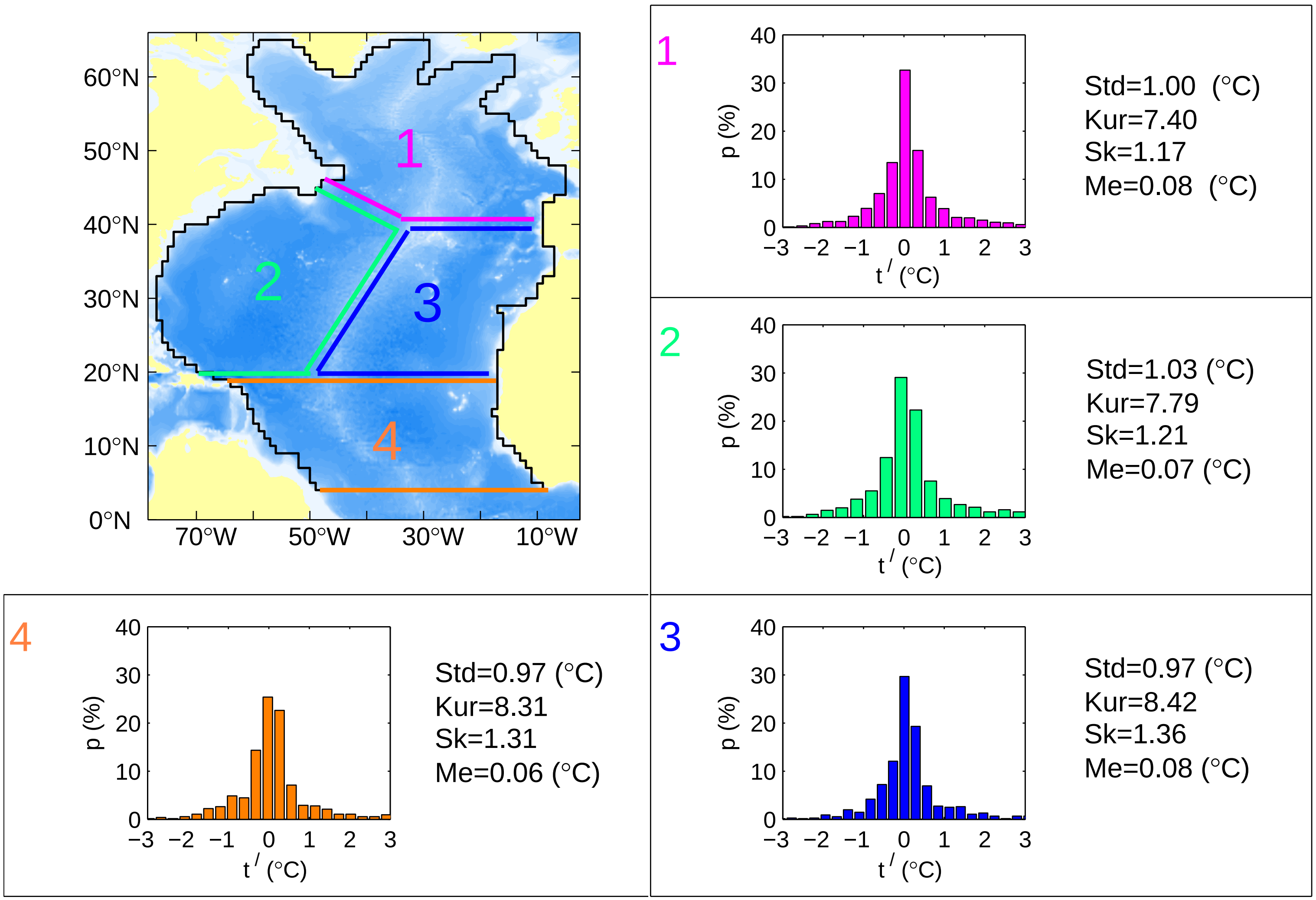
2005



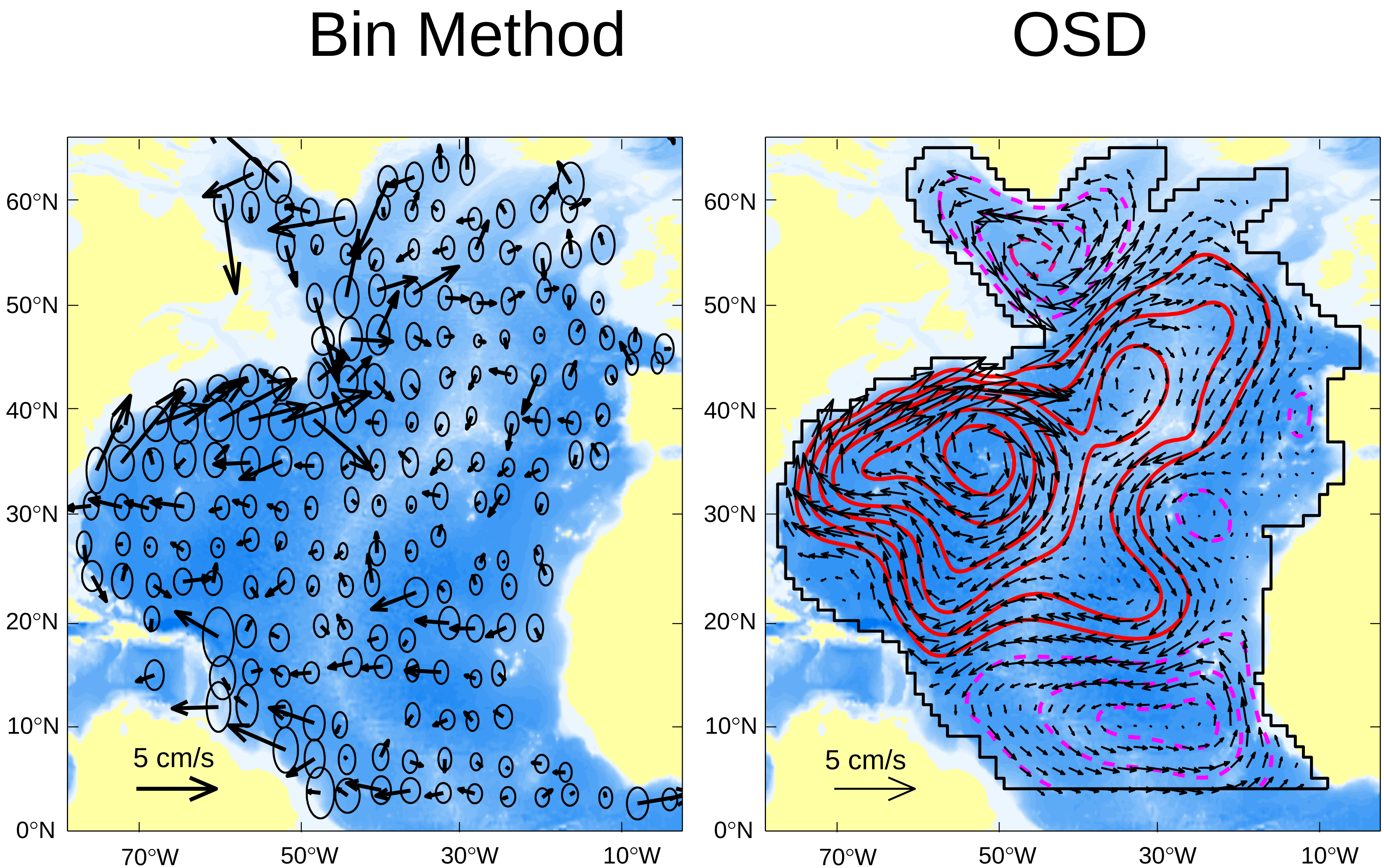




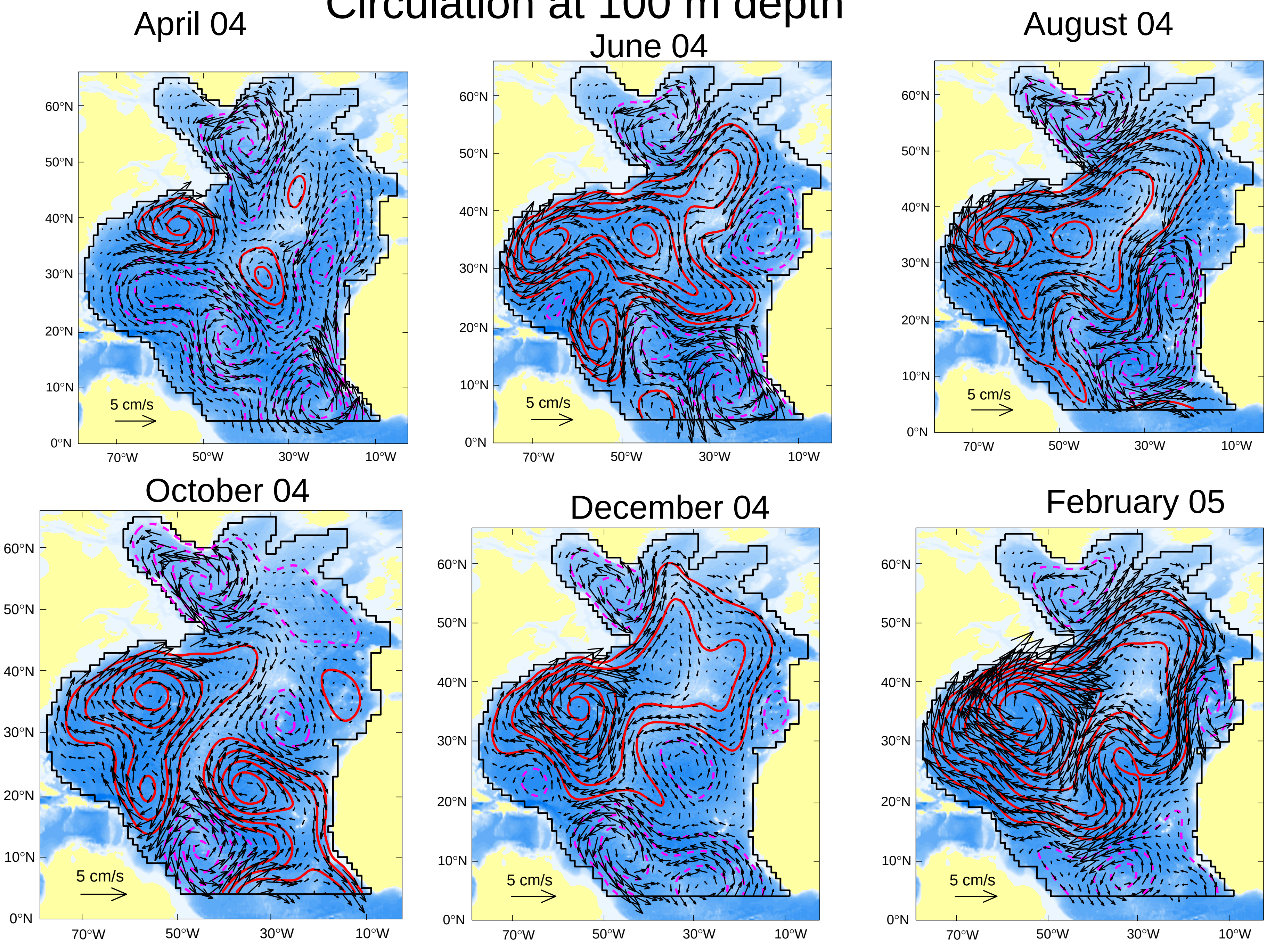
Statistical Features of Temperature at 950 m Depth



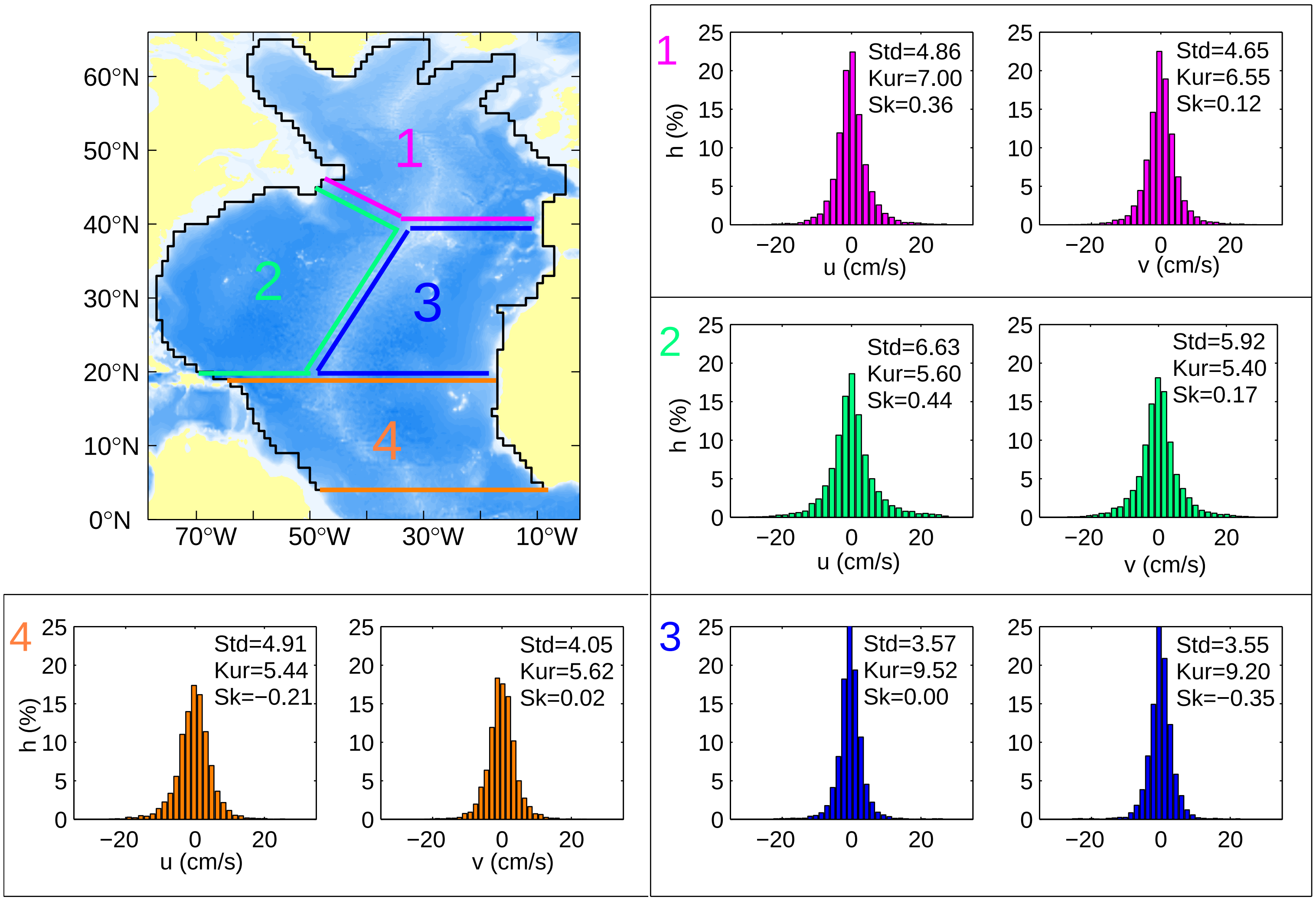
Circulations at 1000 m (March 04 to May 05)



Circulation at 100 m depth



Statistical Features of Velocity



Conclusions

- OSD is an effective method to analyze noisy and sparse data.
- OSD has successfully demonstrated the feasibility of optimal estimations of the Atlantic ocean temperatures and currents at 950 m and 1000 m, using the Argo data.
- We will apply OSD to reanalyze the Pacific ocean temperature and salinity fields, using the Global Temperature-Salinity Profile Program (GTSPP) data maintained at NODC.