Observation-Model Compatibility in Coastal Data Assimilation

-Filtering & Optimal Spectral Decomposition -

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How can we effectively use observational ocean data to represent and to model/predict the ocean state?
Outline

• (1) Model-Data Compatibility
• (2) Filtering Observational Data
• (3) Optimal Spectral Decomposition

  (Chu et al., 2003 a, b JTECH)

  – ARGO Data: Baroclinic Rossby Waves in Tropical Atlantic (Chu et al. JGR 2007)
  – Surface Drifting Buoy Data: Synoptic Current Reversals on the Texas-Louisiana Continental Shelf
    (Chu et al. 2005 JPO)
Part-1

Model-Data Compatibility
Difference between modeled and observed data

• Model
  – Regular in (t, x, y, z)
  – Representing mean value of a grid cell

• Observation
  – Irregular in (t, x, y, z) usually noisy and sparse
  – Representing value at the observational point
Example: RAFOS Floats (NPS#92) in Monterey Bay (Collins’ website)
NCOM Model Data (Hong et al. 2005)
Advection-Diffusion Equation

\[
\frac{\partial \Phi}{\partial t} + \nabla \cdot (V \Phi) = \nabla \cdot (\kappa \nabla \Phi) + S.
\]

\[
\tilde{\Phi}_{i,j,k}^{n+1} - \tilde{\Phi}_{i,j,k}^n = \frac{\langle F \rangle_{i+1/2,j,k}^{n+1} - \langle F \rangle_{i-1/2,j,k}^{n+1}}{\Delta x} + \frac{\langle G \rangle_{i,j+1/2,k}^{n+1} - \langle G \rangle_{i,j-1/2,k}^{n+1}}{\Delta y} + \frac{\langle H \rangle_{i,j,k+1/2}^{n+1} - \langle H \rangle_{i,j,k-1/2}^{n+1}}{\Delta z} + \hat{S}_{i,j,k},
\]
Characteristic Line

\[ F(x, t_0 + \Delta t) \]

\[ F(x - c^* \Delta t, t_0) \]
Modeled-Observational Data Difference at the same location

- (1) Observation → along the red curve
- (2) Model → spatial mean (upper blue line)
- (3) Temporal mean of observation ↔ Model
NOAA Buoy Data Center ↔ WAM

significant wave height

WAM-4 model
(Galanis et al., 2006)

Near California Coast
WAM -4

• (1) Integrating on 30 frequencies and 24 directions.
• (2) First integration frequency $\rightarrow 0.0417$ Hz
• (3) Time step $\rightarrow 300$ seconds
• (4) Spatial grid $\rightarrow 0.5^\circ \times 0.5^\circ$
• (5) Wind input (10 m) $\rightarrow$ NCEP/GFS $0.5^\circ \times 0.5^\circ$
Observational and WAM Modeled Data
Part-2  Data Filtering

Kolmogorov-Zurbenko (KZ) Filter
KZ Filter

• Original Data

• First Iteration

\[ x_i^1 = \frac{1}{2q + 1} \sum_{j=-q}^{q} x_{i+j}^0 \]

• Second Iteration

\[ x_i^2 = \frac{1}{2q + 1} \sum_{j=-q}^{q} x_{i+j}^1 \]

• Number of Iteration (N)

\[ (2q + 1) \cdot \sqrt{N} \leq P \]

• P → Time Steps
• Appropriate selection of the parameters \((N, P, q)\) leads to smoothed time series of observational and modeled data.
Observations vs Forecasts

Filtered data ( > 1 day)

Daily variability has been removed.
The systematic error has not been affected.
Data Assimilation Window (12 hrs)

Assimilating SWH for 12 hrs and running the model for 24 hrs

Assimilation $\rightarrow$ Kalman Filter
Model with data assimilation (Kalman Filter) and no KZ (Buoy- D)

Data (Significant Wave Height) input → Every hours
Model with data assimilation (Kalman Filter) and KZ (Buoy-D)
# Impact of Data Assimilation and Filtering

WAM-no assimilation and KZ filtering
WAM2 – assimilation and no KZ filtering
WAM3 – Assimilation and KZ-filtering

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<th></th>
<th>Buoy B</th>
<th></th>
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Part-3

Optimal Spectral Decomposition
Spectral Decomposition

\[ u_{KM} = \sum_{k=1}^{K} a_k(z,t^o) \frac{\partial \Psi_k(x,y,z,\kappa^o)}{\partial y} + \sum_{m=1}^{M} b_m(z,t^o) \frac{\partial \Phi_m(x,y,z)}{\partial x}, \]

\[ v_{KM} = -\sum_{k=1}^{K} a_k(z,t^o) \frac{\partial \Psi_k(x,y,z,\kappa^o)}{\partial x} + \sum_{m=1}^{M} b_m(z,t^o) \frac{\partial \Phi_m(x,y,z)}{\partial y}. \]
Basis Functions (Open Boundaries)

(Chu et al., 2003 a,b JTECH)

\[ \Delta \Psi_k = -\lambda_k \Psi_k, \]

\[ \Delta \Phi_m = -\mu_m \Phi_m, \]

\[ \Psi_k|_\Gamma = 0, \quad \frac{\partial \Phi_m}{\partial n}|_\Gamma = 0, \]

\[ \left[ \frac{\partial \Psi_k}{\partial n} + \kappa(\tau) \Psi_k \right]|_{\Gamma'_1} = 0, \quad \Phi_m|_{\Gamma'_1} = 0, \]
Boundary Conditions

\[
\frac{\partial^2 \Phi}{\partial n \partial z} = 0 \\
\Psi = 0 \\
\frac{\partial \Phi}{\partial n} + \kappa \Psi = 0 \\
\frac{\partial \Phi}{\partial z} = 0 \\
\frac{\partial \Psi}{\partial n} = 0
\]
Benefit of Using OSD

• Ocean Topographic Configuration ➔

  Basis Functions (Pre-Determined)
Vapnik (1983) Cost Function

→ Optimal Mode Truncation

\[ J(a_1, \ldots, a_K, b_1, \ldots, b_M, \kappa, P) = \frac{1}{2} \left( \left\| u_p^{obs} - u_{KM} \right\|_P^2 + \left\| v_p^{obs} - v_{KM} \right\|_P^2 \right) \rightarrow \min, \]

\[ J_{emp} = J(a_1, \ldots, a_K, b_1, \ldots, b_M, \kappa, P). \]

\[ \text{Prob} \left\{ \sup_{K,M,S} \left| \langle J(K, M, S) \rangle - J_{emp}(K, M, S) \right| \geq \mu \right\} \leq g(P, \mu) \]

\[ \lim_{P \to \infty} g(P, \mu) = 0 \]
Optimal Truncation

- Gulf of Mexico, Monterey Bay, Louisiana-Texas Shelf, Tropical Atlantic

- $K_{opt} = 40$, $M_{opt} = 30$
Determination of Spectral Coefficients
(Ill-Posed Algebraic Equation)

\[ A \hat{\alpha} = QY, \]
Rotation Method (Chu et al., 2004)

\[ S A \hat{a} = SQY, \]

\[ J_1 = \|A\|^2 \cdot \frac{\|SQY\|^2}{\|a\|^2} \rightarrow \text{max}, \]
Near-realtime ocean surface currents derived from satellite altimeter and scatterometer data
NOAA OSCAR Data: http://www.oscar.noaa.gov/

Original Data 2007 Jan 14

![Map of ocean currents](image)
OSD on OSCAR Data

OSD smooth data 2007 Jan 24
6-12 hours at surface to transmit data to satellite

Descent to depth
~10 cm/s (~6 hours)

1000 db (1000m)
Drift approx. 9 days

Total cycle time 10 days

Salinity & Temperature profile recorded during ascent
~10 cm/s (~6 hours)

Float descends to begin profile from greater depth
2000 db (2000m)
ARGO Observations (Oct-Nov 2004)

(a) Subsurface tracks       (b) Float positions where (T,S) were measured
It is difficult to use such noisy data into ocean numerical models.
Boundary Configuration → Basis Functions for OSD
Basis Functions for Streamfunction
Mode-1 and Mode-2
Circulations at 1000 m (March 04 to May 05)
Bin Method
OSD
Annual Component
Semi-annual Component
Time –Longitude Diagrams of Meridional Velocity
Along 11°N

(a) Annual
(b) Semi-Annual
Time –Longitude Diagrams of temperature Along 11°N

(a) (b) (c) (d)

Annual Semi-Annual Annual Semi-Annual

550 m 950 m
OSD for Analyzing Combined Current Meter and Surface Drifting Buoy Data
Ocean Velocity Observation

• 31 near-surface (10-14 m) current meter moorings during LATEX from April 1992 to November 1994

• Drifting buoys deployed at the first segment of the Surface Current and Lagrangian-drift Program (SCULP-I) from October 1993 to July 1994.
Moorings and Buoys
LTCS current reversal detected from SCULP-I drift trajectories.
Conclusions

• (1) Data analysis is important for coastal modeling and prediction.

• (2) KZ filter reduces model-data incompatibility.

• (3) OSD is an effective method for establishing gidden data from sparse and noisy ocean observations.