IUGG 2007: MS008 Ensembles and Probabilistic Forecasting
July 2-3, 2007, Perugia, Italy

Prediction of Climate Indices Using the First Passage Time

Peter C Chu
Naval Ocean-Atmospheric Prediction Laboratory
Department of Oceanography
Naval Postgraduate School
Monterey, California 93943, USA
pcchu@nps.edu
http://www.oc.nps.navy.mil/~chu

Collaborators

- Leonid Ivanov, Tanya Margolina, Chenwu Fan
  Naval Postgraduate School
- Oleg Melcheniko, University of Hawaii
- Yuri Poberezhny, Marine Hydrophysical Institute, Sevastopol, Ukraine
- Lashmi Kantha, University of Colorado at Boulder
- Catherine Nicolis, Royal Meteorological Institute of Belgium

References

- Chu, P.C., 1999: Two kinds of predictability in Lorenz system. Journal of the Atmospheric Sciences, 56, 1427-1432.
- Chu, P.C., L.M. Ivanov, T. M. Margolina, and O.V. Melnichenko, On probabilistic stability of an atmospheric model to various amplitude perturbations. Journal of the Atmospheric Sciences, 59, 2860-2873.
- Chu, P.C., L.M. Ivanov, C.W. Fan, 2002: Backward Fokker-Planck equation for determining model valid prediction period. Journal of Geophysical Research, 107, C6, 10.1029/2001JC000879.
- Chu, P.C., L. Ivanov, L. Kantha, O. Melnichenko, and Y. Poberezhny, 2002: Power law decay in model predictability skill. Geophysical Research Letters, 29 (15), 10.1029/2002GLO14891.
- Chu, P.C., L.M. Ivanov, L.H. Kantha, T.M. Margolina, and O.M. Melnichenko, and Y.A, Poberenzhny, 2004: Lagrangian predictabilty of high-resolution regional ocean models. Nonlinear Processes in Geophysics, 11, 47-66.
- Ivanov, L. M., and P.C. Chu, 2007: On stochastic stability of regional ocean models to finite-amplitude perturbations of initial conditions. Dynamics of Atmosphere and Oceans, in press.
- Chu, P.C., 2007: First passage time analysis on climate indices. Journal of Atmospheric and Oceanic Technology, in press.

Spatial Correlation of Surface Pressure of Djakarta (Berlage 1966)

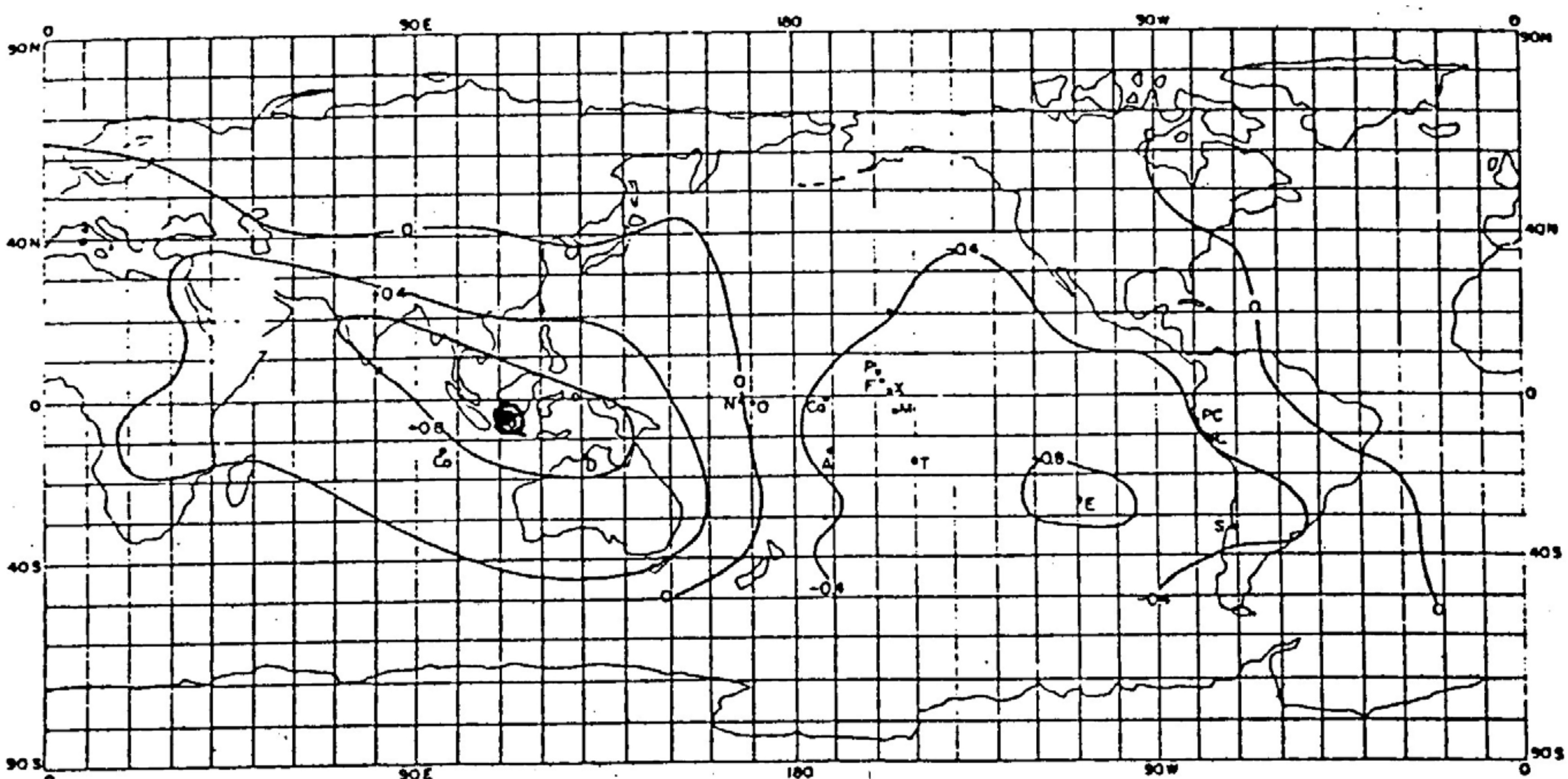


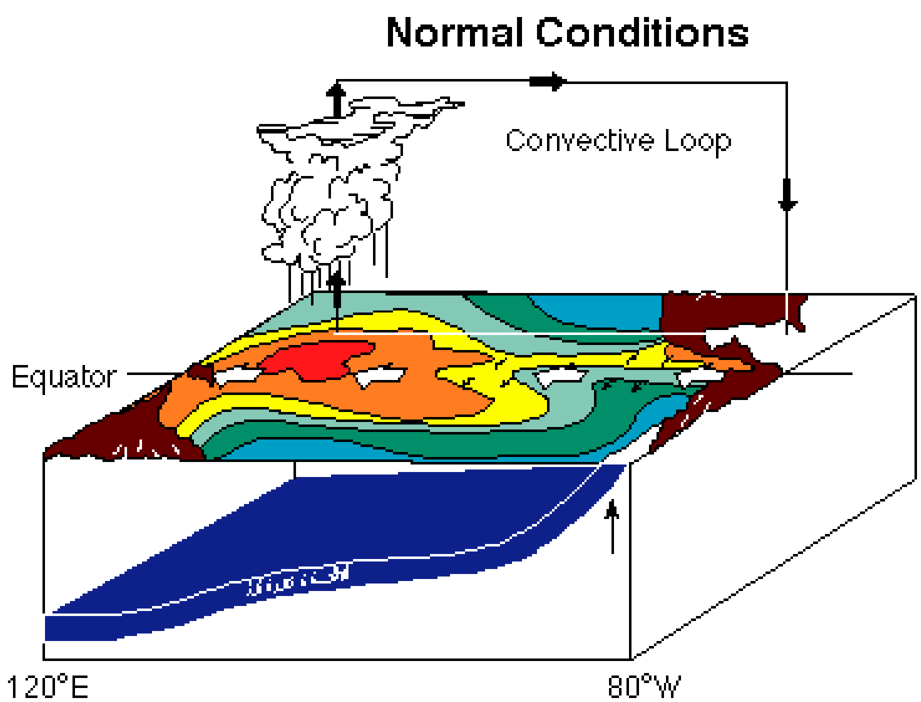
FIG. 1. Schematic map (after Berlage, 1966) showing isopleths of correlation of monthly mean station pressure with that of Djakarta, Indonesia (Dj). Other localities shown are Cocos Island (CO), Port Darwin (D), Nauru (N), Ocean Island (O), Palmyra (P), Christmas Island (X), Fanning (F), Malden Island (M), Apia, Samoa (A), Tahiti (T), Easter Island (E), Puerto Chicama (PC), Lima (L) and Santiago (S).

Southern Oscillation Index (SOI)

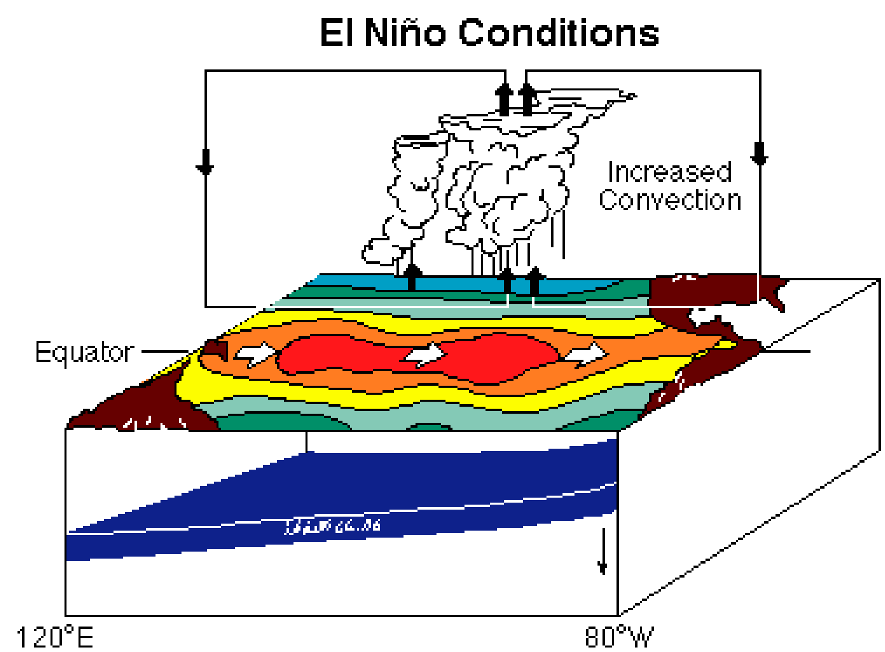
$$P_{diff} = p_{Tahiti} - p_{Dawin}$$

$$s(t) = 10 \times \frac{p_{\mathrm{diff}}(t) - \langle p_{\mathrm{diff}} \rangle}{SD(p_{\mathrm{diff}})}$$

Positive SOI

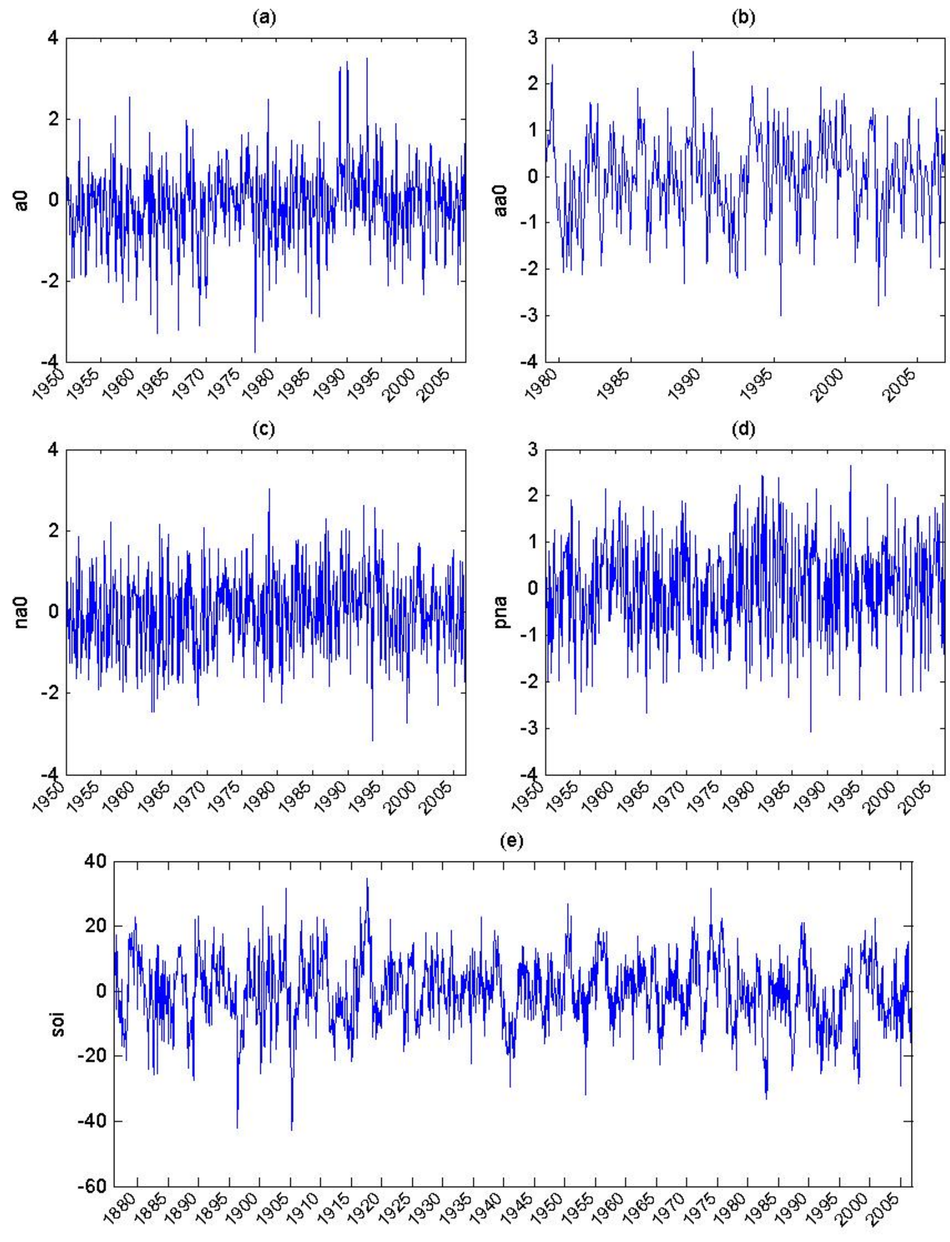


Sustained Negative SOI



Commonly Used Climate Indices

- Arctic Oscillation (AO)
- Antarctic Oscillation (AAO)
- North Atlantic Oscillation (NAO)
- Pacific/North American Pattern (PNA)
- Southern Oscillation Index (SOI)



Two Approaches → Index Prediction

- Forward Method
  - Collette and Ausloos (2004)
  - Lind et al. (2005)
- Backward Method
  - Chu (2007)

Forward Method

- Predicts the change of the index ρ at time *t* with a given temporal increment τ .

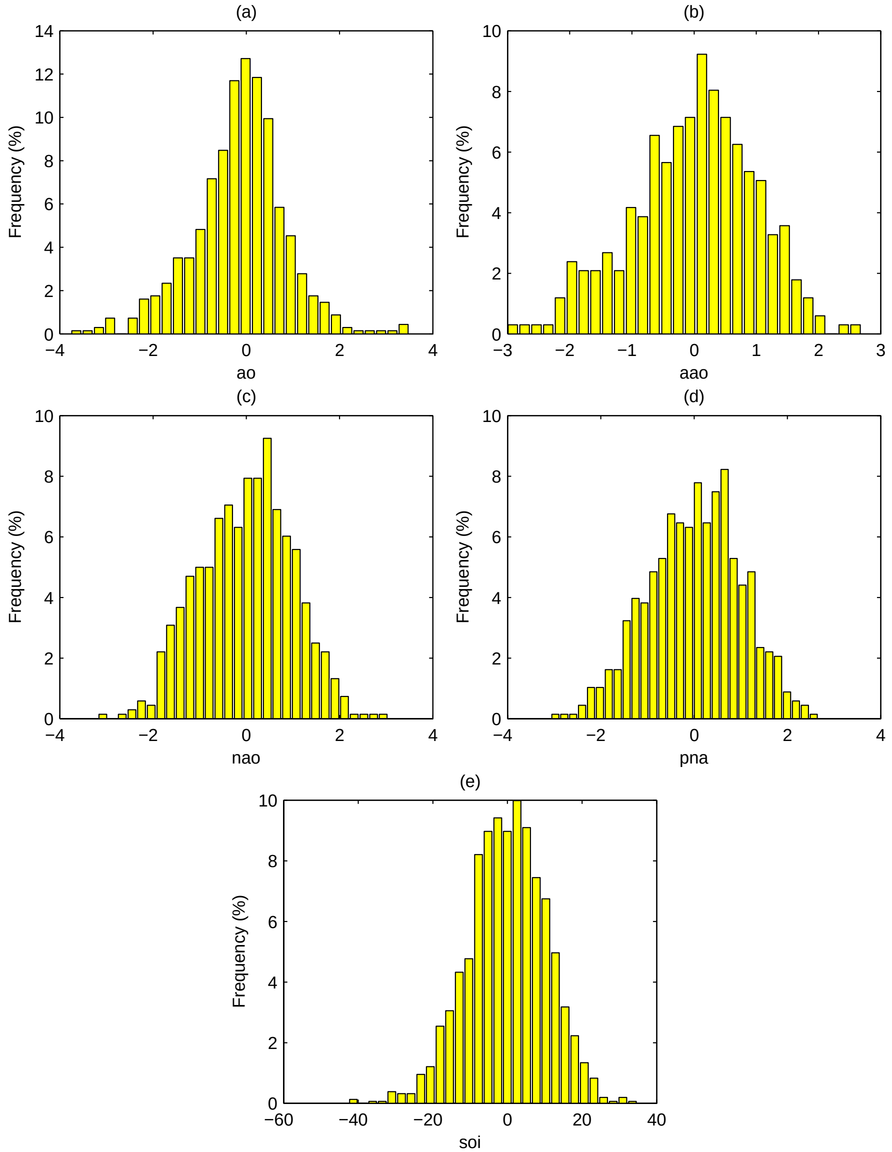
- Due to stochastic nature, the probability density function (PDF), should be first constructed.

PDFs of Monthly Mean Indices

- AO,   AAO

- NAO, PNA

- SOI



Example- NAO Index

- NAO describes a large-scale meridional vacillation in atmospheric mass between the anticyclone over Azores and the subpolar low pressure system over Iceland.

- Traditionally, the state of the NAO dipole system is characterized by an index, the so-called NAO index, which is basically the difference between the pressure at the high NAO pole (Azores) and the pressure at the low pole (Iceland).

Example – NAO Index $\rho(t)$

- Collette and Ausloos (2004) →
  - Brownian fluctuation

- Lind et al. (2005) → $\rho(t)$ → Langevin equation

$$\frac{d\rho(t)}{dt} = D^{(1)}\left[\rho(t),t\right] + \eta(t)\sqrt{D^{(2)}\left[\rho(t),t\right]}$$

($D^{(1)}$, $D^{(2)}$) → (Drift, Diffusion) Coefficients

$\eta(t)$ is a Langevin force
(δ - correlated Gaussian noise).

Backward Method

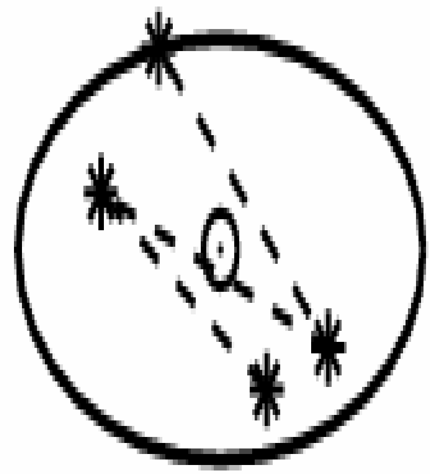
- This method predicts the typical time span (τ) needed to generate a fluctuation in the index of a given increment (ρ).

- This method uses FPT.

First-Passage Time (FPT)

ρ → Radius

FPT → Time when the particle first passes through the boundary.



FPT Problem

- Given a fixed value of an index reduction (ρ), the corresponding time span (positive) is estimated for which the index reduction

$$\gamma_{\Delta t}(t) = s(t + \Delta t) - s(t)$$

reaches the level for the first time,

$$\tau_{\rho}(t) = \inf\{\Delta t > 0 \mid \gamma_{\Delta t}(t) \le -\rho\}$$

which is called the FPT. FPT is a random variable.

- PDF of FPT

$$p(\tau_\rho)$$

- CDF of FPT

$$P(\tau_\rho) = \int_{\tau_\rho}^{\infty} p(\tau)\,d\tau$$

Moments of FPT

$$M_k(\rho) = k\int_0^\infty p(\tau)\tau^{k-1}d\tau, \qquad k = 1,\ldots,\infty$$

$$\langle \tau_\rho \rangle = M_1$$

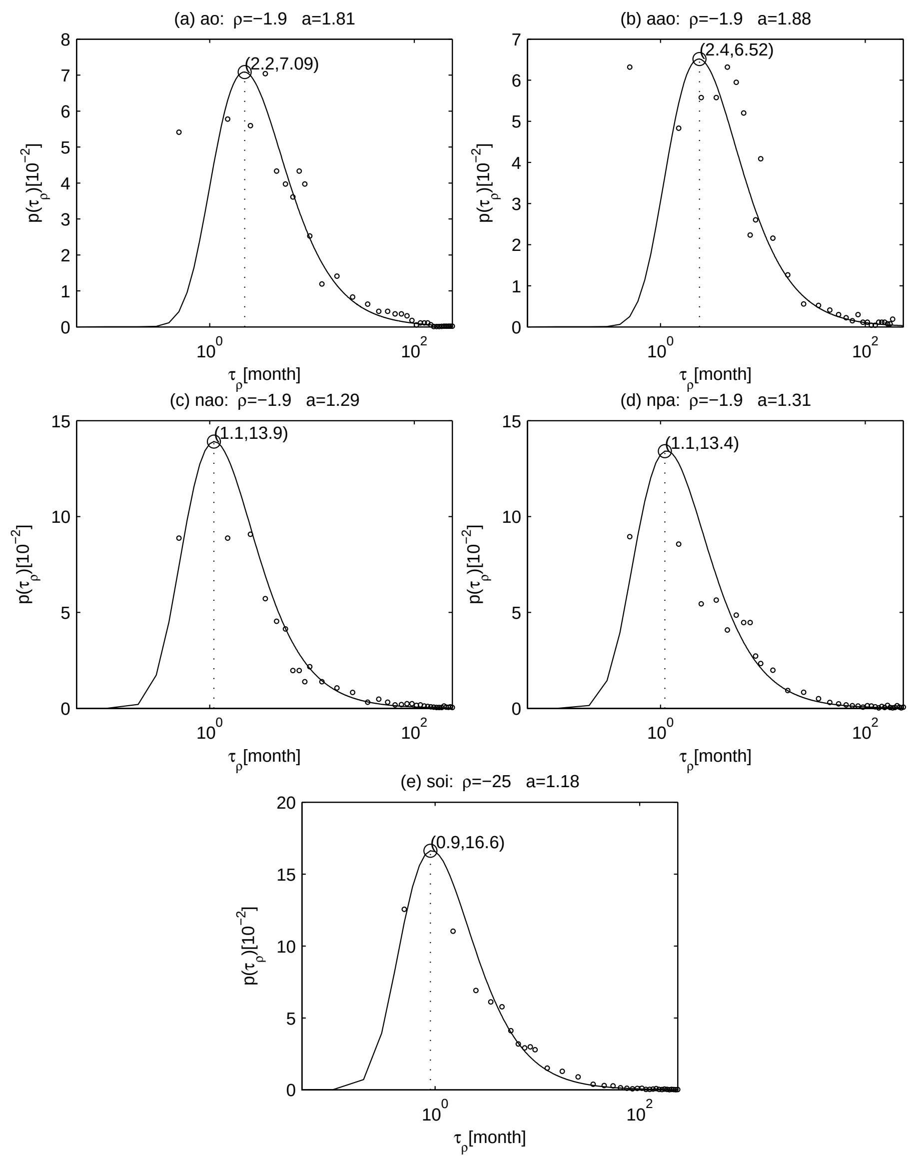
$$\langle \delta\tau_\rho^2 \rangle = M_2 - M_1^2$$

PDF of FPT (from index data) → Inverse Gaussian Distribution

- AO AAO

- NAO PNA

- SOI



PDF of FPT
(Analytical, Chu et al. 2002a)

- Backward Fokker-Planck equation

$$\frac{\partial p}{\partial t} - \left[D^{(1)}(\rho, t) \right] \frac{\partial p}{\partial \rho} - \frac{1}{2} \eta^{(2)} D^{(2)}(\rho, t) \frac{\partial^2 p}{\partial \rho^2} = 0.$$

Analytical Solution of the Backward Fokker-Planck Equation

- For Brownian fluctuation (e.g., NAO monthly index, Collette and Ausloos 2004), the backward Kokker-Planck Equation has analytical solution (Ding and Rangaranjian 1995)

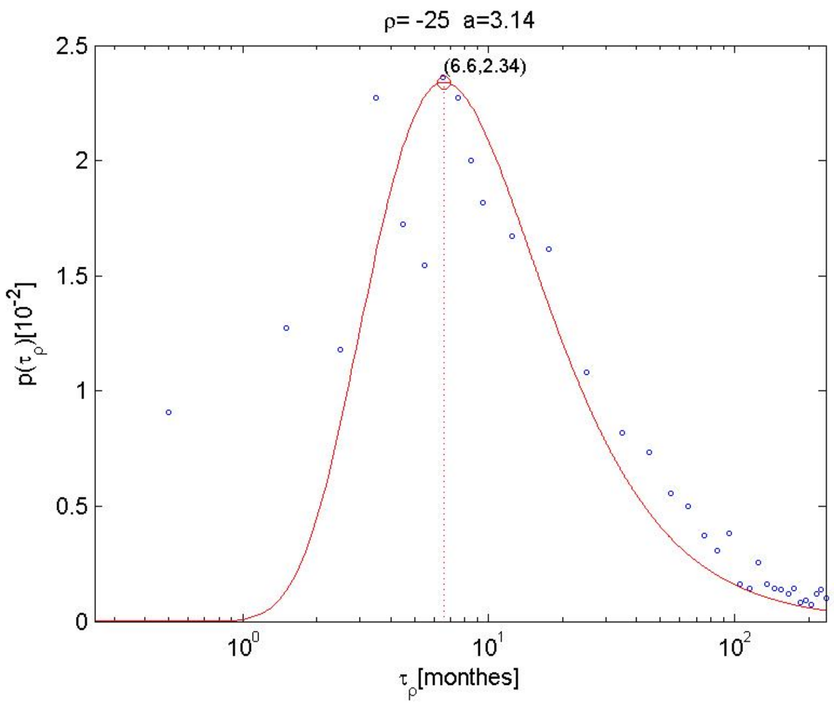
$$p(\tau) = \frac{1}{\sqrt{\pi}} \frac{a}{\tau^{3/2}} \exp\left(-\frac{a^2}{\tau} \right)$$

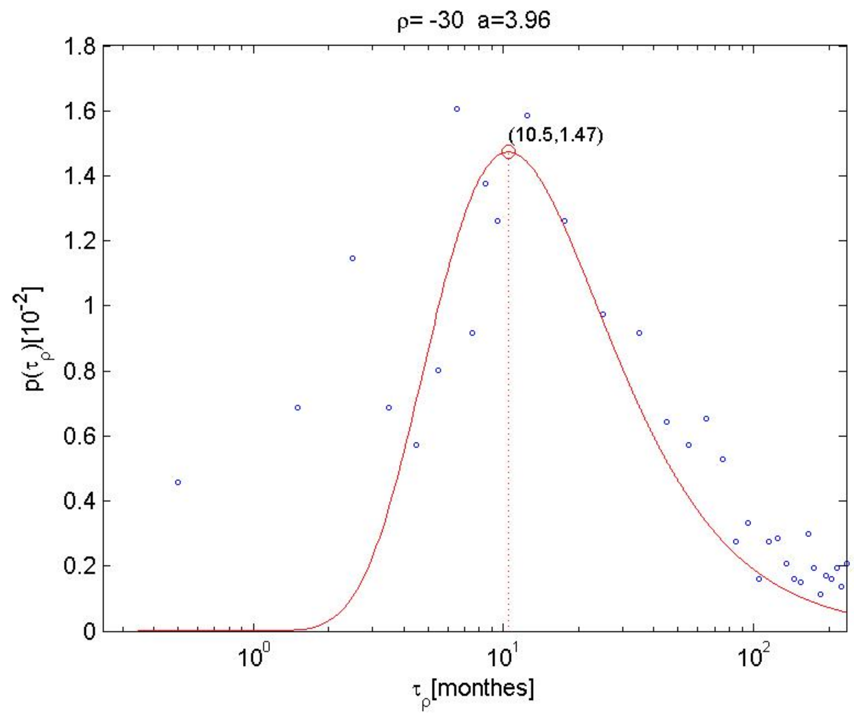
- The parameter ‘a’ depends on the index reduction ρ

Dependence of $p(\tau_\rho)$ on ρ (SOI)

ρ = -25 → a = 3.14

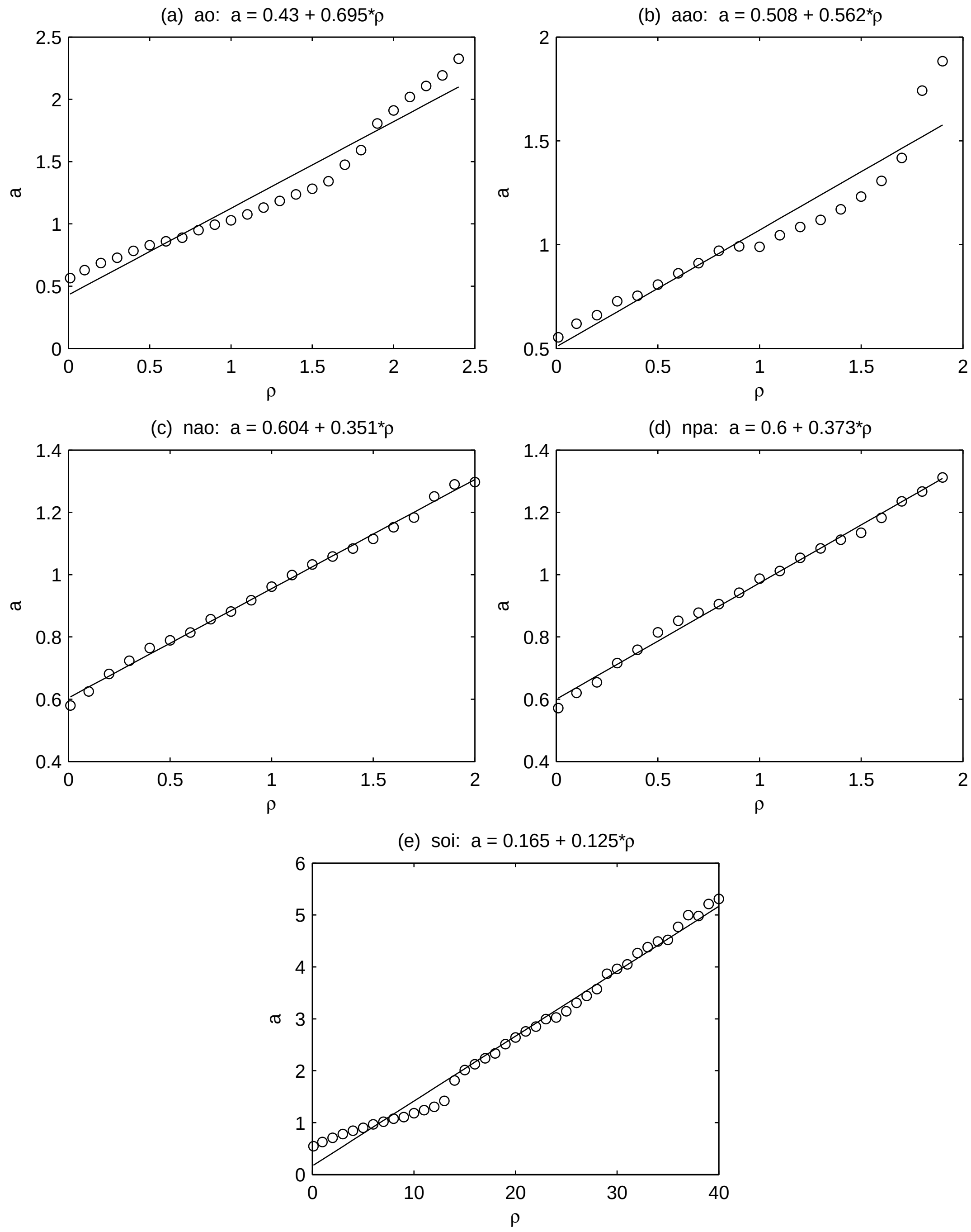
ρ = -30 → a = 3.96





- Linear relationship between the parameter 'a' in the analytical PDF & the index reduction ρ

$$a = \alpha_1 + \alpha_2 \rho$$



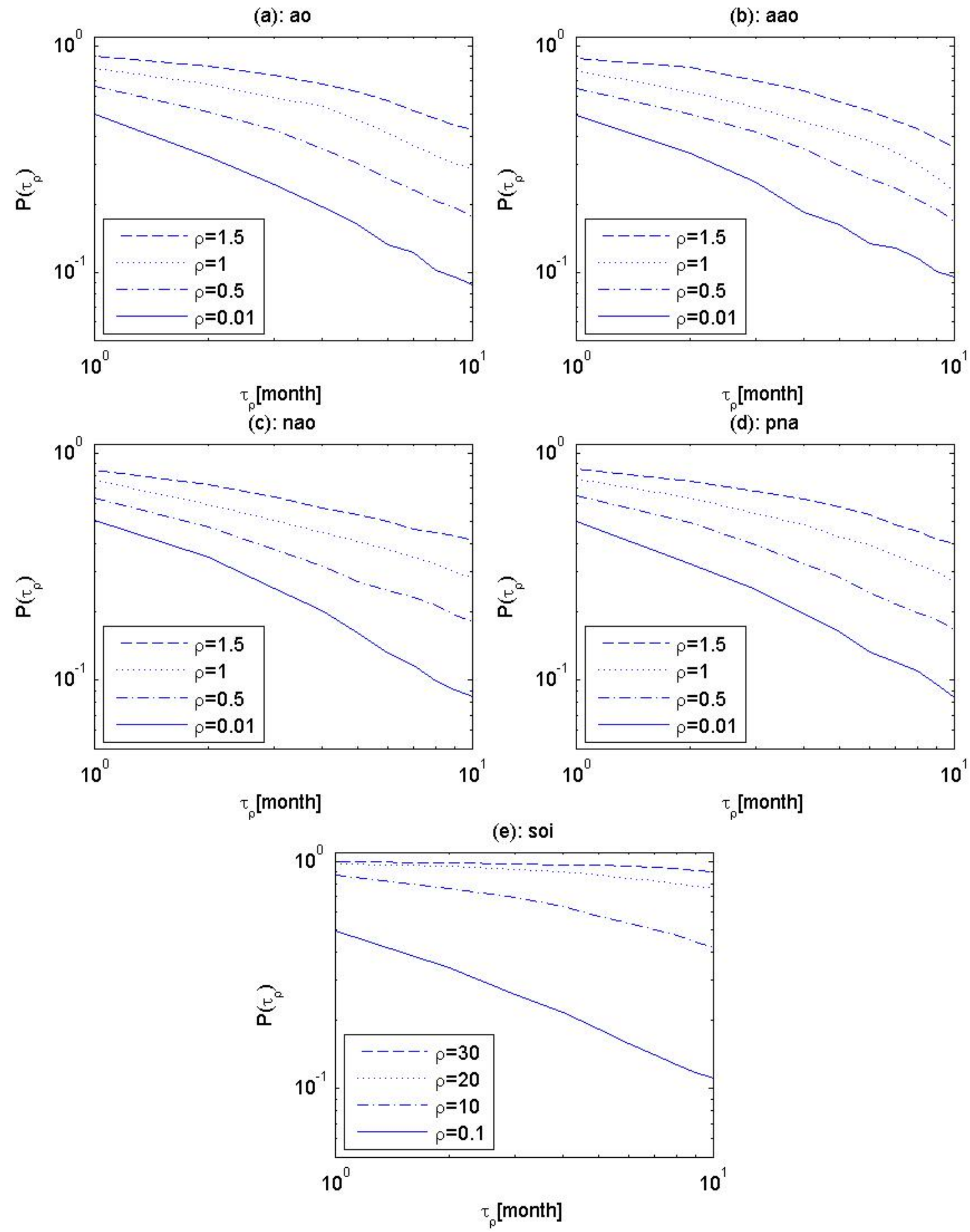
	AO	AAO	NAO	PNA	SO
α_1	0.430	0.508	0.604	0.600	0.165
α_2	0.696	0.562	0.351	0.373	0.125

- Empirical CDF of FPT $P(\tau_\rho)$

for various values of index reduction

→ Power Law

$$P(\tau) \sim \tau^{H-1}$$

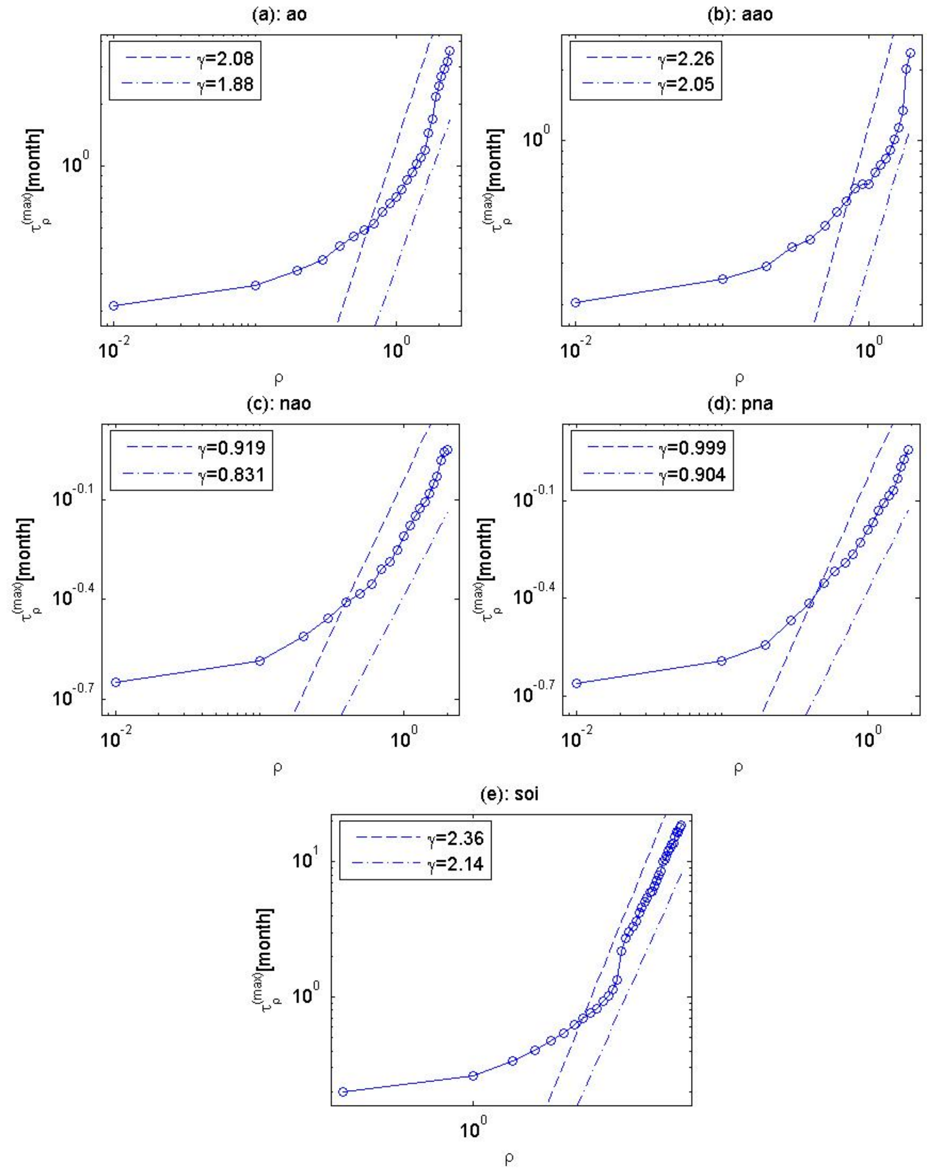


Mode of $p(\tau_\rho)$ → Optimal FPT $\tau_\rho^{(\max)}$

$\tau_\rho^{(\max)} = 2a^2/3$

$\tau_\rho^{(\max)} \sim \rho^\gamma$ for large ρ

$\gamma \sim 2.0$



Results

- FPT presents a new way to detect the temporal variability of the climate indices. It predicts a typical time span (τ) needed to generate an index reduction of a given increment (ρ).
- FPTs for the five climate indices satisfy the inverse Gaussian distribution → Brownian Fluctuation.
- $\tau_{\rho}^{(\max)}$ can be used as most probable time period needed for the low-frequency atmospheric circulation pattern to sustain.
- Power-law features