Rotation Matrix Method for Analyzing Noisy Nonlinear Data

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References

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Ocean Modeling/Data Analysis

$$\mathbf{A}\hat{\mathbf{a}} = \mathbf{QY},$$

$$\mathbf{Y} = \bar{\mathbf{Y}} + \mathbf{Y}'$$

signal $\bar{\mathbf{Y}}$

noise **Y’**

Algebraic Equation

- *L*-dimensional state vector: **a**
- *P*-dimensional observation vector: **Y**
- Coefficient matrix: **A** *P times L*,
- Square matrix (*P* times *P*): **Q**
- $P > L$

η_1

ILL-POSED ALGEBRAIC EQUATION

$$\eta_1 = \frac{\left\| \mathbf{QY}' \right\|}{\left\| \mathbf{Q\bar{Y}} \right\|}, \qquad \eta_2 = \frac{L}{P}$$

$$\eta_3 = \frac{\max(\text{sigular values of } \mathbf{A})}{\min(\text{sigular values of } \mathbf{A})}$$

Imperfect Condition

- Noise to signal ratio and condition number

$$\eta_1 \geq 1, \qquad \eta_3 >> 1$$

Rotational Matrix Method

- **SAa = SQY**

- For the new matrix: **SA**

$$\tilde{\eta}_3^2 \,(1 + \tilde{\eta}_1^2) \rightarrow \min,$$

Optimization

$$\tilde{\eta}_1 \equiv \frac{\left\|\mathbf{SQY'}\right\|}{\left\|\mathbf{SQ\bar{Y}}\right\|} \qquad \tilde{\eta}_3 \equiv \frac{\left\|\mathbf{SQ\bar{Y}}\right\|}{\left\|\mathbf{a}\right\|}$$

$$J_1 = \left\|\mathbf{A}\right\|^{\mathbf{2}} - \tilde{\eta}_3^2\left[1 + \frac{2\,(\mathbf{SQ\bar{Y}} * \mathbf{SQY'})}{\left\|\mathbf{SQ\bar{Y}}\right\|^{\mathbf{2}}} + \tilde{\eta}_1^2\right] \rightarrow \max,$$