Adaptation of Global Volume Transport for Coastal Ocean Models

Peter C Chu and Chenwu Fan
Naval Postgraduate School,
Monterey, California, USA
Email: pcchu@nps.edu
http://www.oc.nps.navy.mil/~chu
One Difficulty in Coastal Ocean Modeling:
*Unknown Volume Transport at Open Boundaries*

- Subjective

- Can we determine the volume transport objectively?
A possible approach

Diagnostic Method
Existing Diagnostic Methods for Determination of Volume Transport

• From (T, S) Data – Total Geostrophic Flow (Reid 1985, 1994, …)

• Top 500 m Transport From (T, S) and Wind Data – Sverdrup Model (Godfrey 1989)
Reid (1994)

- No wind driven component
Three Components of the Ekman-Munk Model

- Ekman Model for Extra-Equatorial Regions
- Munk Model for the Equatorial Region
- Stokes Theorem for Determining $\Psi$ for Islands
Steady-State Large-Scale Dynamics (Mid-Latitudes)

\[- f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2} + A_h \nabla^2 u \]

\[f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2} + A_h \nabla^2 v \]

\[\frac{\partial p}{\partial z} = -\rho g \]

\[u_g = -\frac{1}{f \rho_0} \frac{\partial p}{\partial y}, \quad v_g = -\frac{1}{f \rho_0} \frac{\partial p}{\partial x} \]

\[\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]
(1) Ekman Number (Mid-Latitude)

\[ E = \frac{O(|A_h \nabla^2 \mathbf{V}|)}{O(|f \mathbf{V}|)} = \frac{A_h}{|f| L^2} \]

- \( L \sim 2 \times 10^5 \text{m} \quad A_h = 5 \times 10^5 \text{ m}^2\text{s}^{-1} \)

\[ E \simeq \frac{5 \times 10^5 \text{ m}^2\text{s}^{-1}}{(10^{-4} \text{s}^{-1}) \times (2 \times 10^5 \text{m})^2} = 0.125 \]

- Horizontal diffusion can be neglected
Ekman Model for Mid-Latitudes

\[-f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2}\]

\[f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2}\]
Vertically Integrated Ekman Model

\[-f(V - V_g) = A_z \frac{\partial u}{\partial z}|_{z=\eta} - A_z \frac{\partial u}{\partial z}|_{z=-H}\]

\[f(\bar{U} - U_g) = A_z \frac{\partial v}{\partial z}|_{z=\eta} - A_z \frac{\partial v}{\partial z}|_{z=-H}\]
Vertically Integrated Velocity

\[ U = \hat{U}_g + U_r + \frac{T_y}{f \rho_0}, \quad V = \hat{V}_g + V_r - \frac{T_x}{f \rho_0} \]
Vertically Integrated Velocity

\[(\hat{U}_g, \hat{V}_g) = \frac{g}{f \rho_0} \left( \int_{-H}^{\eta} \int_{-H}^{z} \frac{\partial \rho}{\partial y} dz' dz, - \int_{-H}^{\eta} \int_{-H}^{z} \frac{\partial \rho}{\partial x} dz' dz \right). \]

\[U_r = H u_{-H} - \frac{C_D}{f} \sqrt{u_{-H}^2 + v_{-H}^2} u_{-H}, \]

\[V_r = H v_{-H} + \frac{C_D}{f} \sqrt{u_{-H}^2 + v_{-H}^2} v_{-H}. \]

- CD ~ Bottom Drag Coefficient (0.0025)
Volume Transport Stream Function

\[ U = -\frac{\partial \psi}{\partial y}, \quad V = \frac{\partial \psi}{\partial x} \]
Poisson-\(\Psi\) Equation

\[ \nabla^2 \Psi = \Pi, \quad \Pi = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \]

\[ \Pi = \Pi_1 + \Pi_2 + \Pi_3 \]
Forcing Terms in The Poisson-$\Psi$ Equation

- $T, S$

- $T, S$ (P-vector Inverse Method)

- Wind Stress

\[ \Pi_1 = \left( \frac{\partial \hat{V}_g}{\partial x} - \frac{\partial \hat{U}_g}{\partial y} \right) \]

\[ \Pi_2 = \left( \frac{\partial V_r}{\partial x} - \frac{\partial U_r}{\partial y} \right) \]

\[ \Pi_3 = -\left[ \frac{\partial}{\partial x} \left( \frac{\tau_x}{f \rho_0} \right) + \frac{\partial}{\partial y} \left( \frac{\tau_y}{f \rho_0} \right) \right]. \]
Datasets

- Topography: DBDB5, 5’ resolution
- Annual and Monthly Mean T, S: NODC (Levitus et al. 1998)
- Annual and Monthly Mean Wind Stress: NCEP
Annual Mean \( \Pi \) Values
Ekman Number (Low-Latitudes) 
8° S – 8° N

- Horizontal diffusion cannot be neglected

\[ E \geq \frac{5 \times 10^5 \text{ m}^2\text{s}^{-1}}{(0.2 \times 10^{-4}\text{s}^{-1}) \times (2 \times 10^5 \text{m})^2} = 0.5 \]
(2) Munk Model (Equatorial Region)

- Vertically Integrated Vorticity Equation

\[
\nabla^2 \Pi = \frac{\beta}{A_h} V - \frac{1}{A_h} \left[ \frac{\partial}{\partial x} \left( \frac{\tau_y}{\rho_0} \right) - \frac{\partial}{\partial y} \left( \frac{\tau_x}{\rho_0} \right) \right],
\]

- Boundary Conditions:

\[\Pi \text{ Values at } 8^\circ \text{ S} - 8^\circ \text{ N}\]
Integration of Munk Model

• Obtaining $\Pi$ Values in the tropics
(3) Solving Poisson $\psi$-Equation

$$\nabla^2 \psi = \Pi,$$

- With known forcing term for the globe
- We need to know $\Psi$-Values at islands.
Boundary Conditions for $\psi$

$\psi = C_1$

$\psi = C_2$
Multi-Connected Domain
Stokes Theorem

\[ -\oint_{\delta \Omega_j} \mathbf{V} \cdot d\mathbf{s} + \oint_{\delta \omega_j} \mathbf{V} \cdot d\mathbf{s} = \iint_{C_j} \mathbf{k} \cdot (\nabla \times \mathbf{V}) \, dx \, dy \]

\[ \oint_{\delta \Omega_j} \nabla \Psi \cdot n \, ds = \oint_{\delta \omega_j} \mathbf{V} \cdot d\mathbf{s} - \iint_{C_j} \mathbf{k} \cdot (\nabla \times \mathbf{V}) \, dx \, dy \]
Minimum Circuit Method

\[ \int_{\delta \Omega_j} \nabla \Psi \cdot n ds \rightarrow \Gamma_j \quad \text{as} \quad C_j \rightarrow 0 \]

\[ \Gamma_j \equiv \int_{\delta \omega_j} V \cdot ds \]
Stokes Theorem
Ψ-Values at Islands (Annual Mean)
Global Volume Transport
Streamfunction
Indonesia Throughflow
(Pacific – Indian Link)
Indonesia Throughflow (Fine et al. 1994)
Australia-Bali Section Transports
(Fieux et al. 1994)

Table II-14: Australia–Bali Section Transports
(in Sverdrups; adapted from Fieux et al., 1994)

<table>
<thead>
<tr>
<th>Layers</th>
<th>Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–200 db</td>
<td>−23.1</td>
</tr>
<tr>
<td>200–500 db</td>
<td>−2.7</td>
</tr>
<tr>
<td>Total 0–500 db</td>
<td>−25.8</td>
</tr>
<tr>
<td>500–2000 db</td>
<td>+9.6</td>
</tr>
<tr>
<td>Total 0–2000 db</td>
<td>−16.2</td>
</tr>
</tbody>
</table>

*Minus signs indicate westward flow.
Annual Mean Transports Calculated Using Ekman-Munk Model
South Atlantic Link to the Indian Ocean: Agulhas Current System (Schmitz 1996)
Agulhas Current System
(from the Ekman-Munk Model)
Atlantic Linkage to Pacific Ocean
Malvinas Confluence
Conclusions

• Ekman-Munk model has capability to diagnose the volume transport from wind and hydrographic data
• Stokes theorem is effective for determining streamfunction at islands
• Annual and monthly mean global volume transport data are useful for coastal modeling
• The adaptation package is easily incorporated into any coastal model