Nonlinear Model Predictability

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References

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Physical Reality

- $\boldsymbol{Y}$
- Physical Law: $d\boldsymbol{Y}/dt = \boldsymbol{h}(\boldsymbol{y}, t)$
- Initial Condition: $\boldsymbol{Y}(t_0) = \boldsymbol{Y}_0$

Atmospheric Models

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  - $\boldsymbol{X}$ is the prediction of **Y**
  - d $\boldsymbol{X}/$ dt = $\boldsymbol{f}(\boldsymbol{X}, t) + q(t)\, \boldsymbol{X}$
  - Initial Condition: $\boldsymbol{X}(t_0) = \boldsymbol{X}_0$
  - Stochastic Forcing:
  - $\langle q(t) \rangle = 0$
  - $\langle q(t)q(t') \rangle = q^2\delta(t-t')$

Model Error

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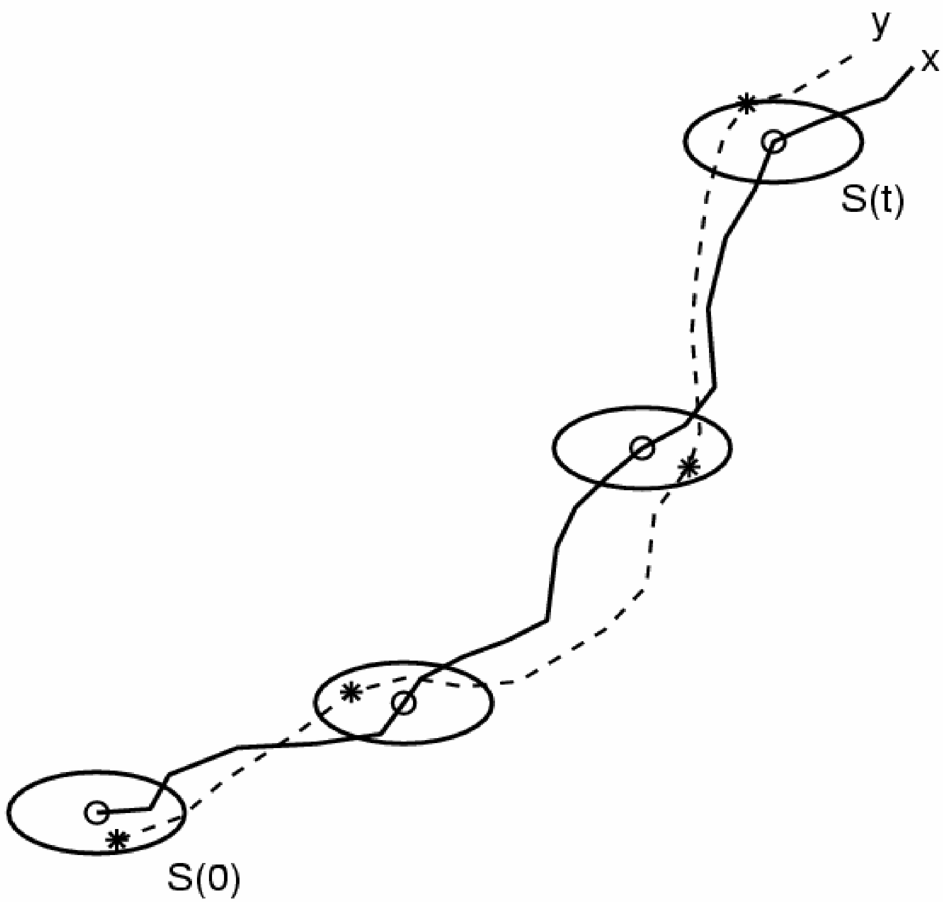
- $$Z = X - Y$$
- Initial: $Z_0 = X_0 - Y_0$

One Overlooked Parameter

- Tolerance Level ε
- Maximum accepted error

Valid Predict Period (VPP)



- VPP is defined as the time period when the prediction error first exceeds a pre-determined criterion (i.e., the tolerance level ε).

First-Passage Time

Conditional Probability Density Function

- Initial Error: $\boldsymbol{z}_0$
- $(t - t_0)$ ⇗ Random Variable
- Conditional PDF of $(t - t_0)$ with given $\boldsymbol{z}_0$ ⇗
  - $P[(t - t_0) \mid \boldsymbol{z}_0]$

Two Approaches to Obtain PDF of VPP

- Analytical (Backward Fokker-Planck Equation)
- Practical

Backward Fokker-Planck Equation

$$\frac{\partial P}{\partial t} - \left[\mathbf{f}(\mathbf{z}_0, t)\right]\frac{\partial P}{\partial \mathbf{z}_0} - \frac{1}{2} q^2 \mathbf{z}_0{}^2 \frac{\partial^2 P}{\partial \mathbf{z}_0 \partial \mathbf{z}_0} = 0$$

Moments

$$\tau_1(\mathbf{z}_0) = \int_{t_0}^{\infty} P(t_0, \mathbf{z}_0, t - t_0)(t - t_0)dt$$

$$\tau_2(\mathbf{z}_0) = \int_{t_0}^{\infty} P(t_0, \mathbf{z}_0, t - t_0)(t - t_0)^2 dt$$

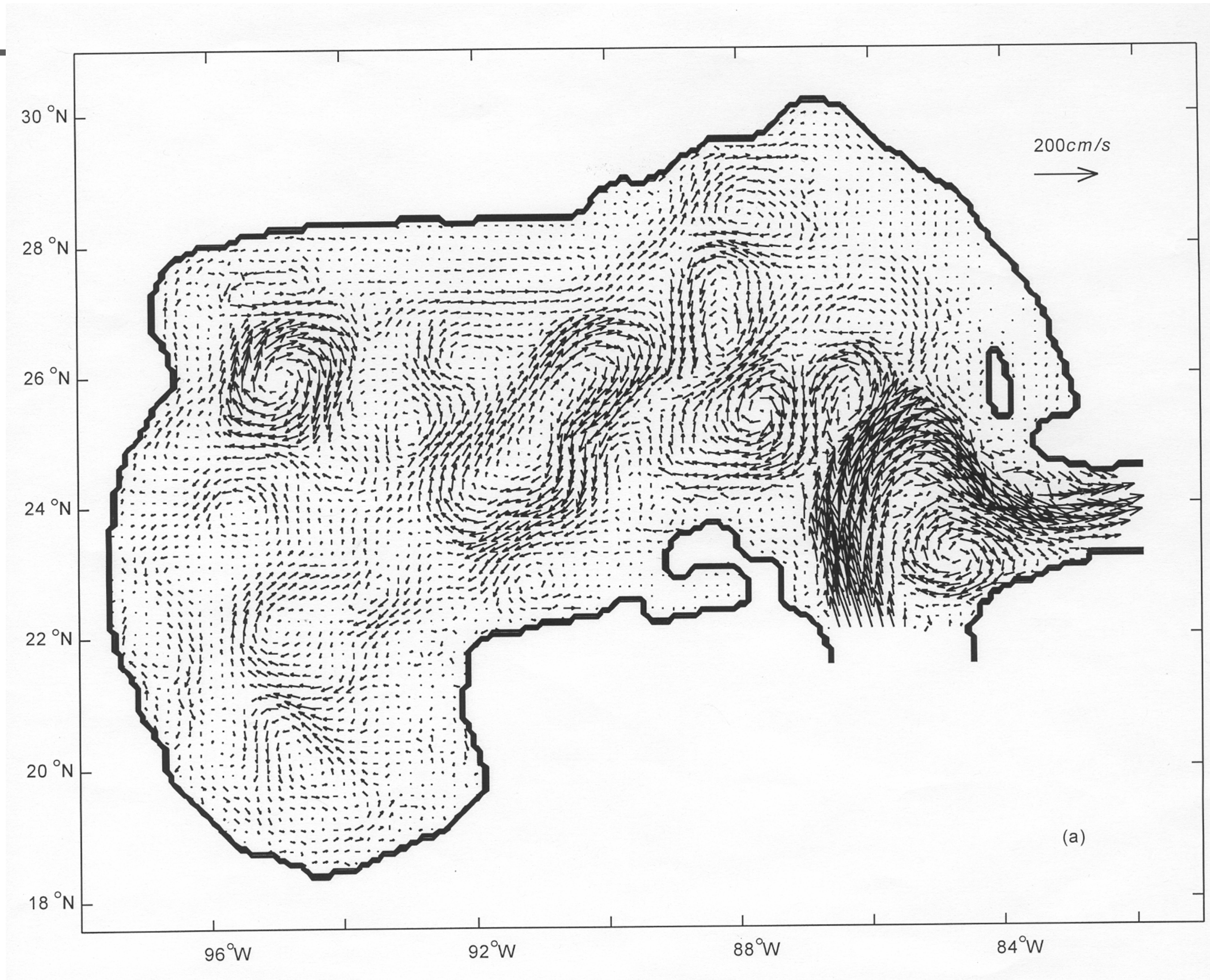
Extremely Long Predictability

- Non-Gaussian Distribution with Long Tail Towards Large FPT Domain.

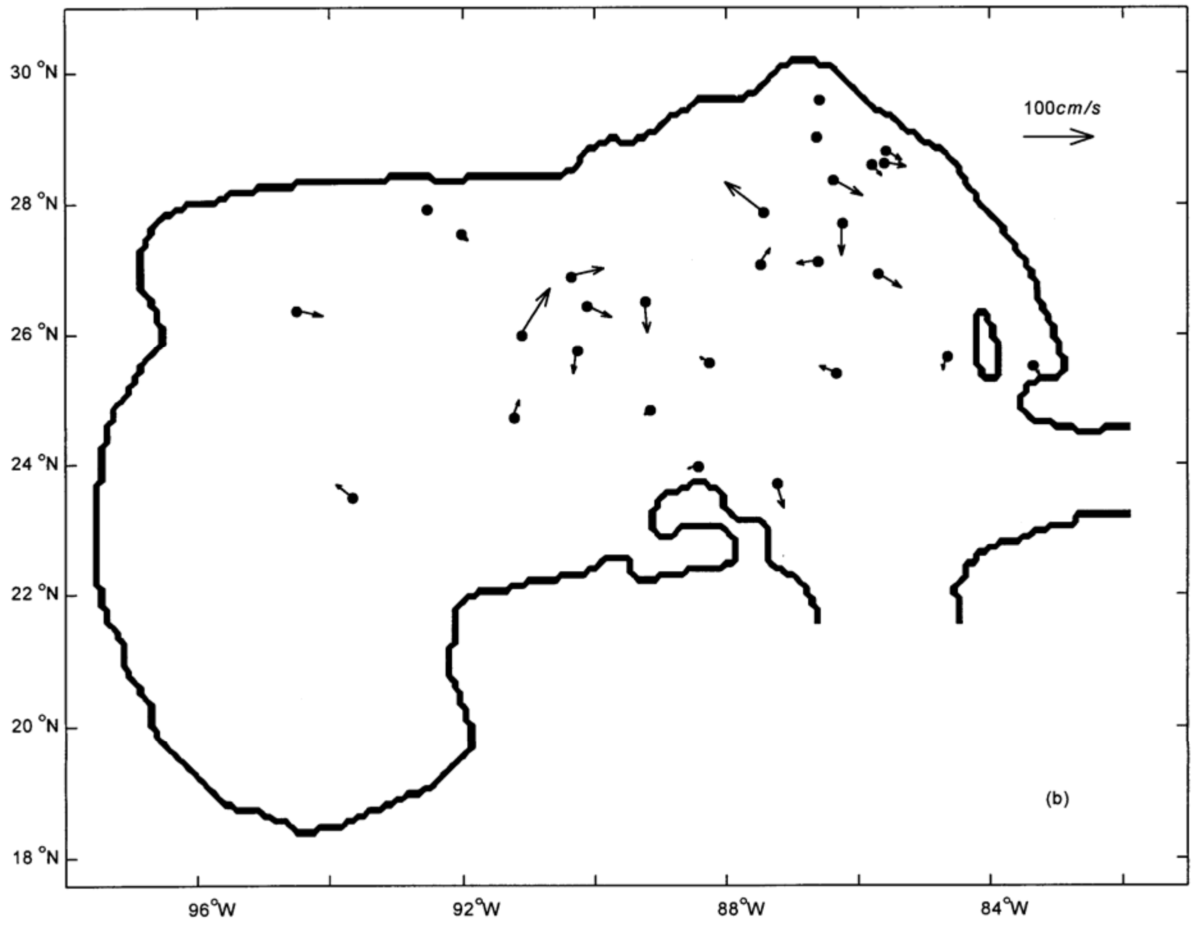
Gulf of Mexico Forecast System

- University of Colorado Version of POM
- 1/12° Resolution
- Real-Time SSH Data (TOPEX, ESA ERS-1/2) Assimilated
- Real Time SST Data (MCSST, NOAA AVHRR) Assimilated
- Six Months Four-Times Daily Data From July 9, 1998 for Verification

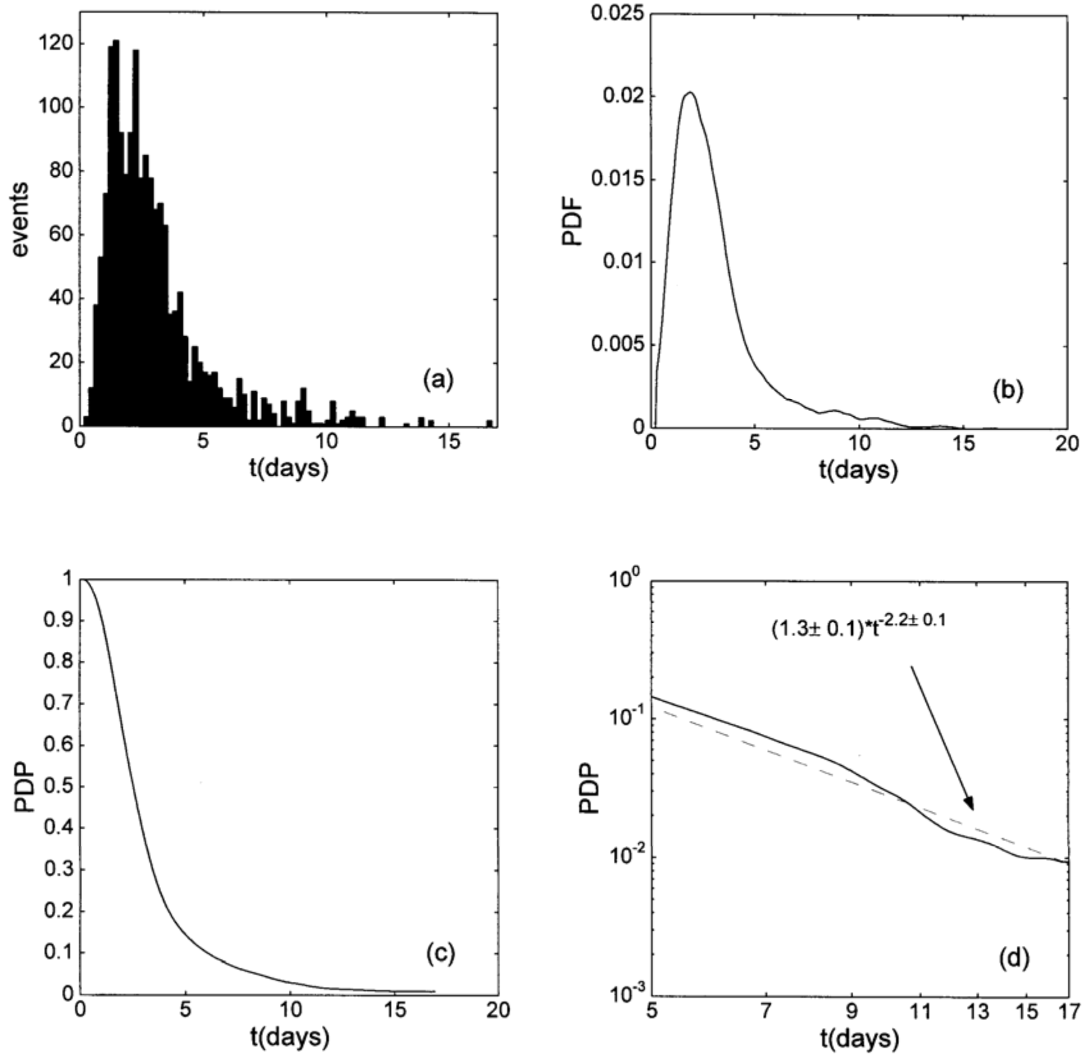
Model Generated Velocity Vectors at 50 m on 00:00 July 9, 1998



(Observational) Drifter Data at 50 m on 00:00 July 9, 1998



Statistical Characteristics of VPP for zero initial error and 22 km tolerance level (Non-Gaussion)



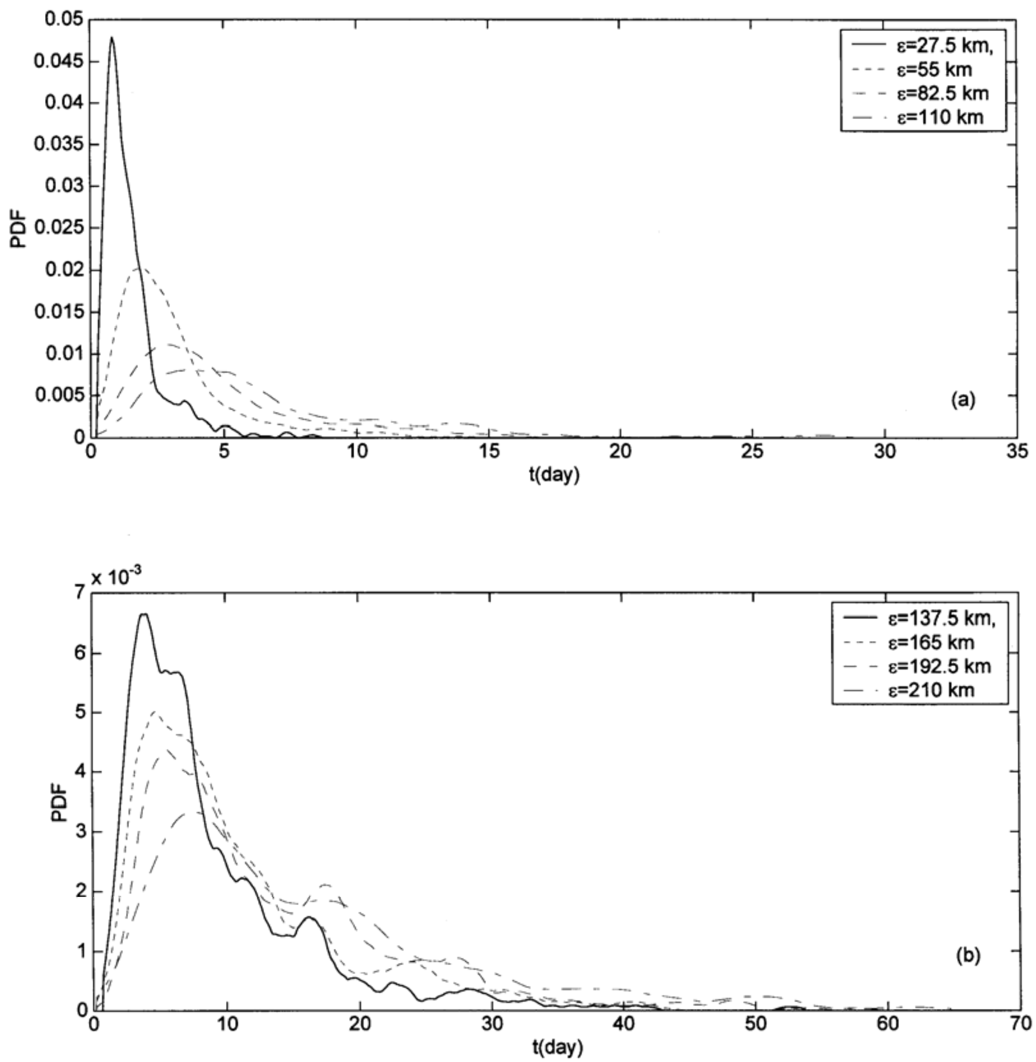
Probability Density Function of VPP calculated with different tolerance levels

Non-Gaussian distribution

with long tail toward large

values of VPP (Long-term

Predictability)



Error Mean and Variance

Error Mean $$\mathbf{L}_1 = \langle \mathbf{z} \rangle$$

Error Variance $$\mathbf{L}_2 = \left\langle \left(\mathbf{z} - \langle \mathbf{z} \rangle\right)^t \left(\mathbf{z} - \langle \mathbf{z} \rangle\right) \right\rangle$$

Exponential Error Growth

$$L_1 \propto e^{\sigma t}, \qquad L_2 \propto e^{\omega t},$$

Classical Linear Theory

No Long-Term Predictability

Power Law

$$L_1 \propto t^{\alpha}, \qquad L_2 \propto t^{\beta},$$

$$P(t_0, \mathbf{z}_0, \varepsilon, t - t_0) \sim t^{-\gamma} \quad \text{for large } t.$$

Long-Term Predictability May Occur

Scaling behavior of the

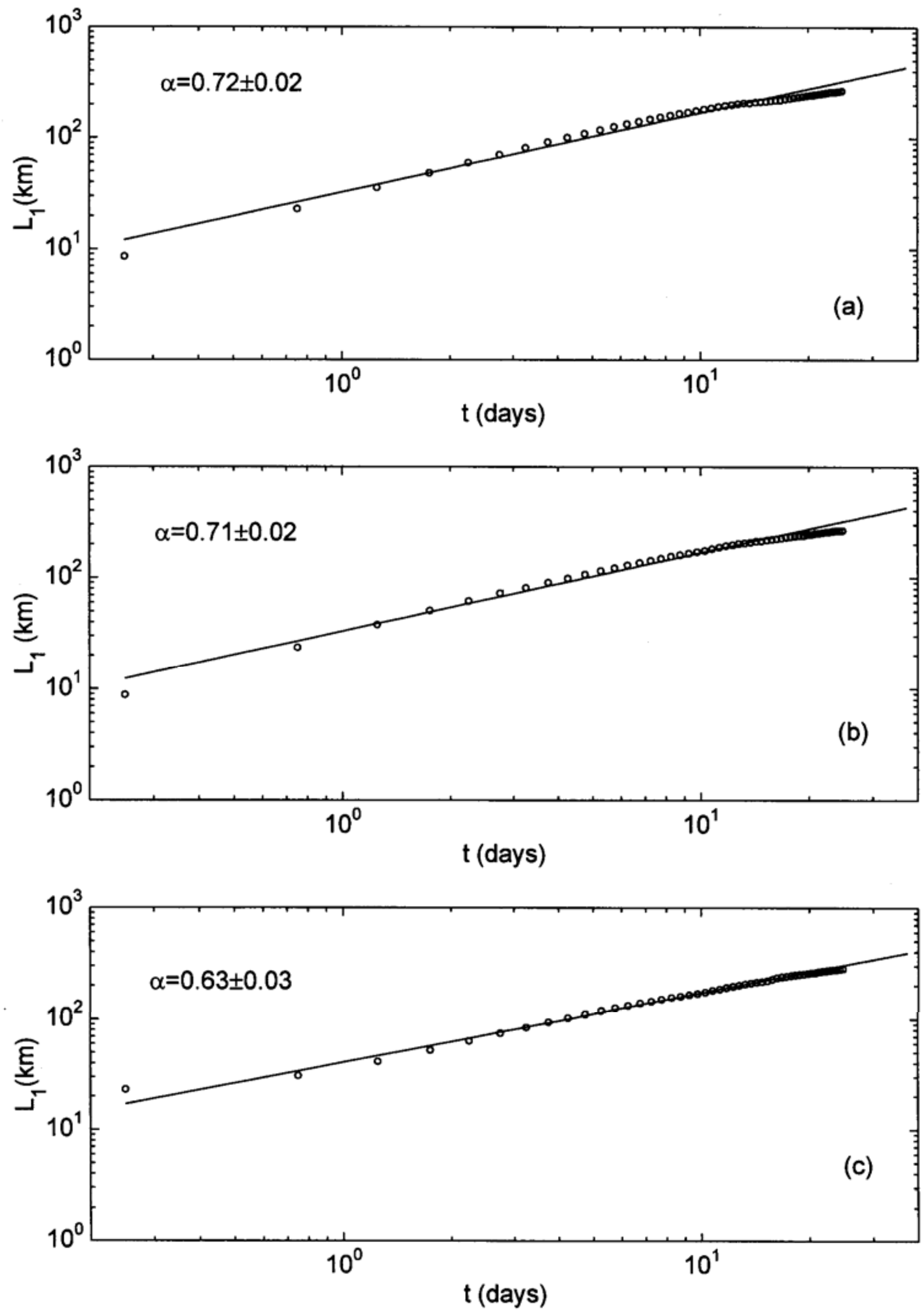
mean error (L_1) growth

for initial error levels:

(a) 0

(b) 2.2 km

(c) 22 km



Scaling behavior of the

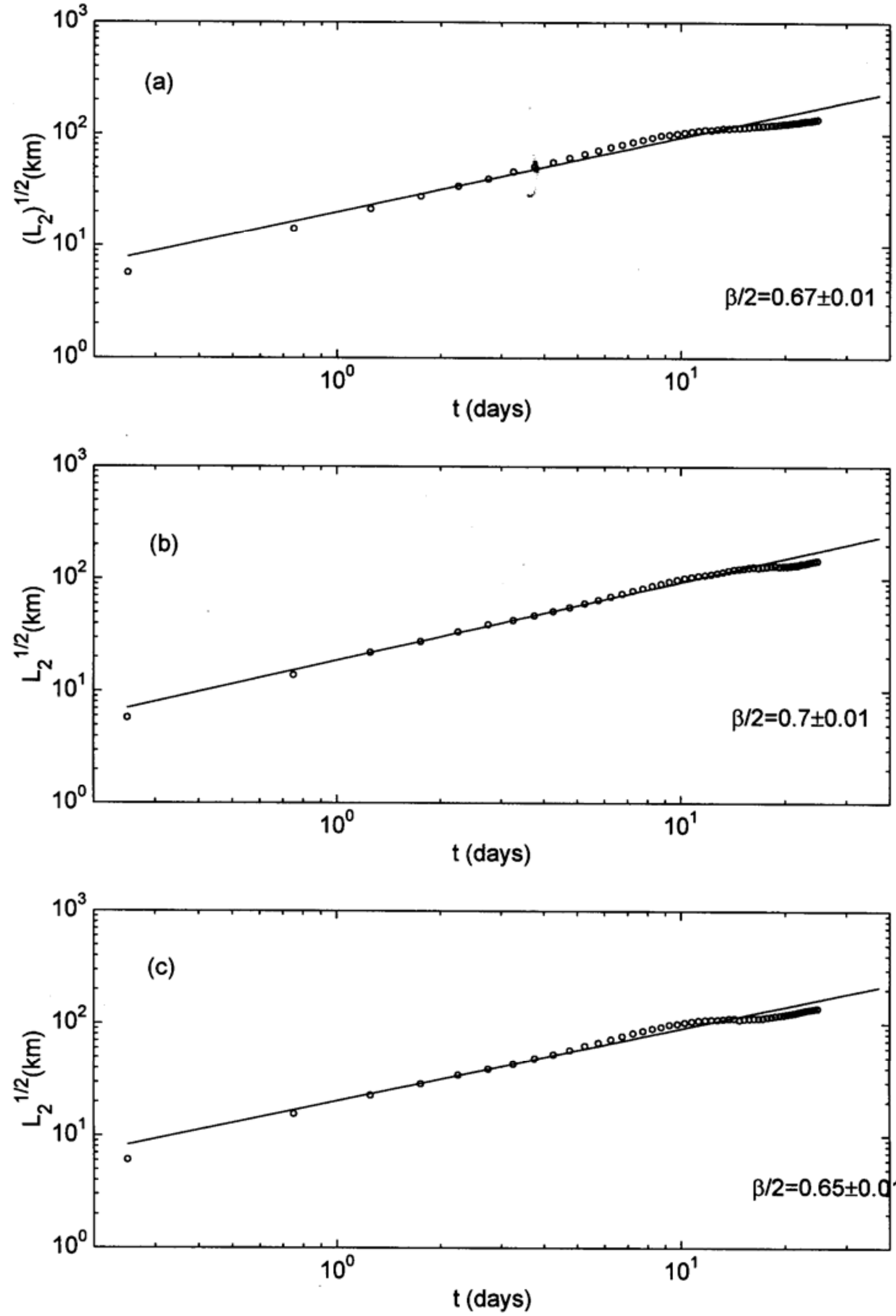
Error variance (L_2) growth

for initial error levels:

(a) 0

(b) 2.2 km

(c) 22 km



Predictability Tube

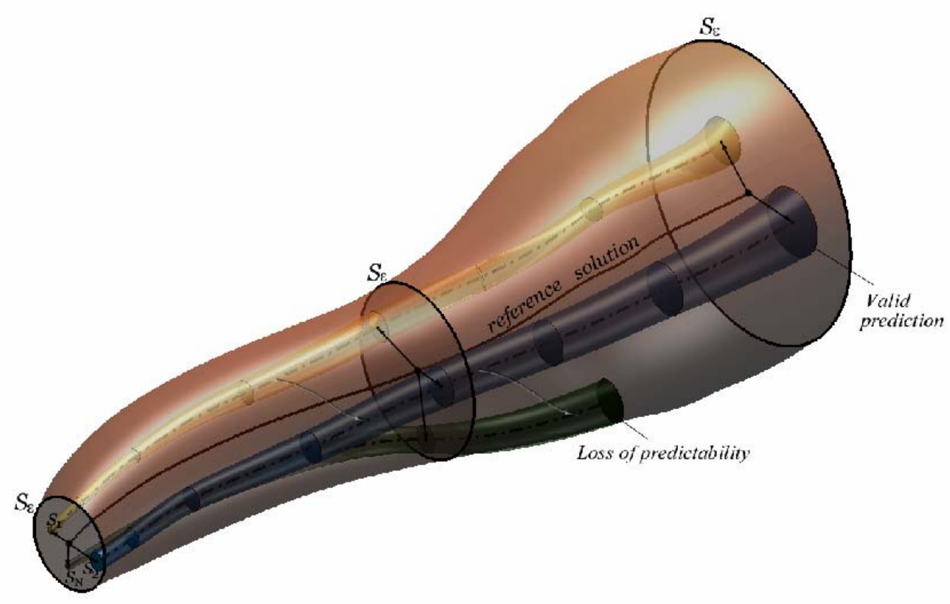


Fig. 31

Conclusions

- (1) Nonlinear model predictability can be effectively represented by FPT.
- (2) Backward Fokker-Planck equation is the theoretical framework for FPT.
- (3) Nonlinear stochastic-dynamic modeling