3D Rigid Body Impact Burial Prediction Model

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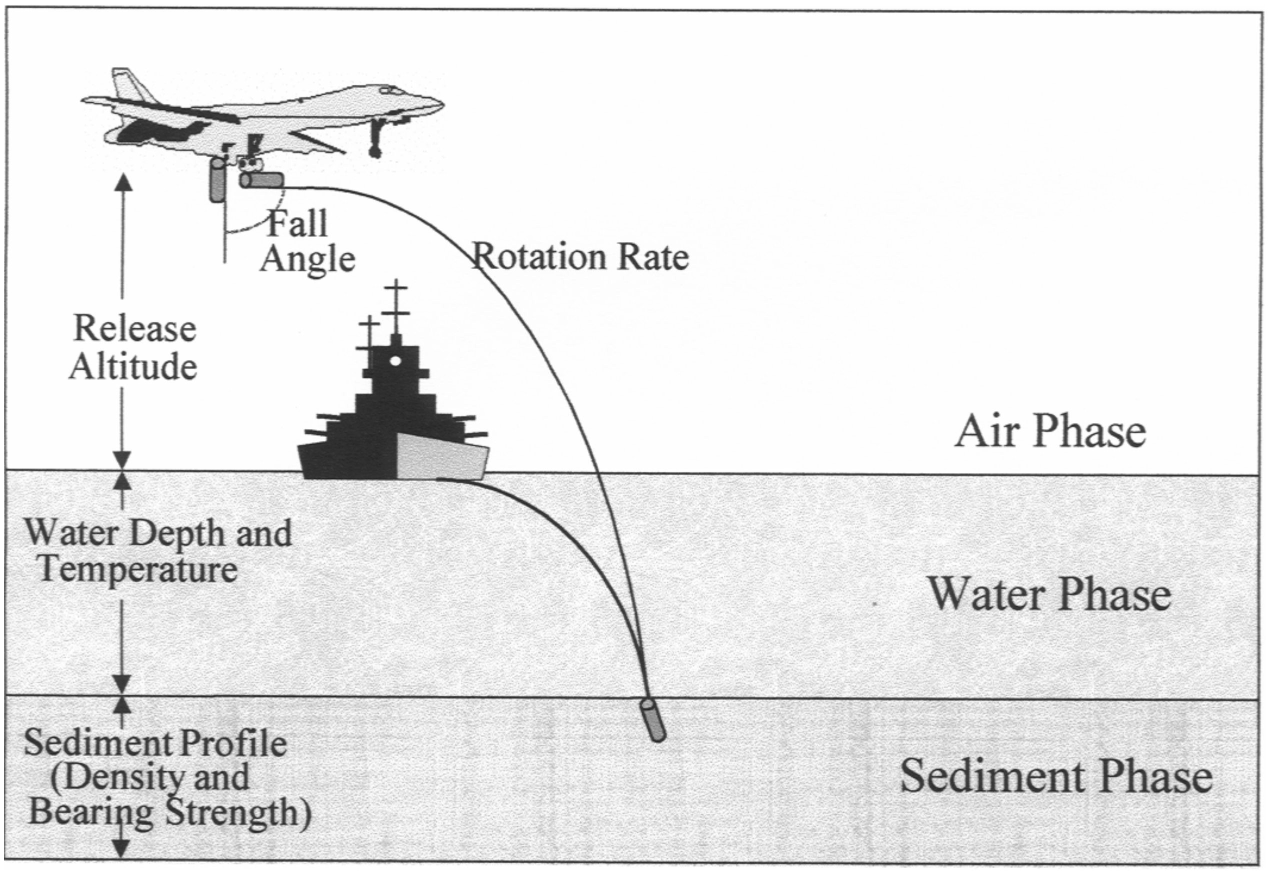
References

- Chu, P.C., C.W. Fan, A.D. Evans, and A. Gilles, 2003: Triple coordinate transforms for prediction of falling cylinder through the water column. Journal of Applied Mechanics, in press.

Hydrodynamic Characteristics



Complicated Scientific Problem

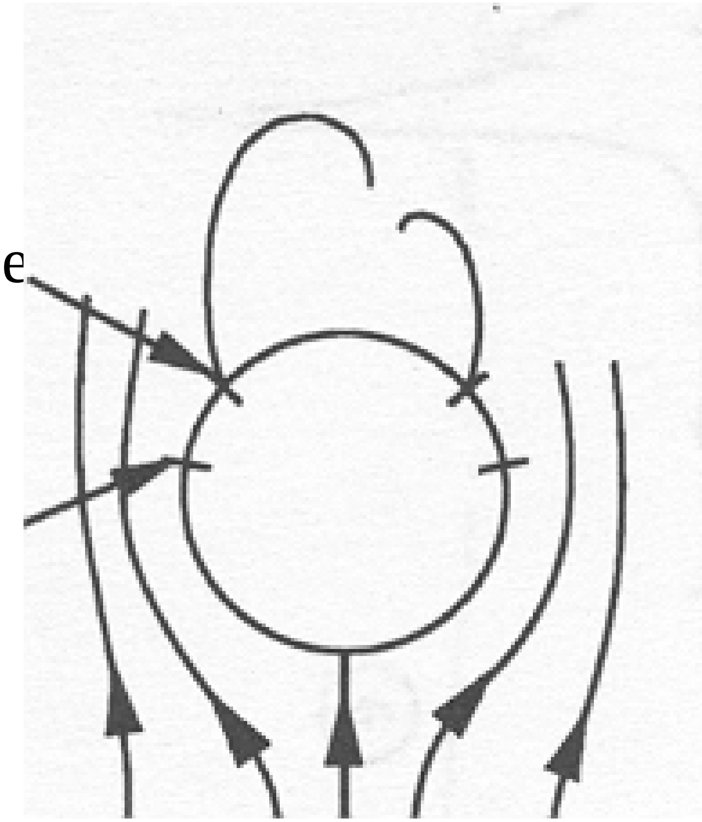
- Body-Fluid Interaction
- Highly Nonlinear
- Chaotic Behavior

Key Non-Dimensional Numbers

- Reynolds Number
- Keulegan-Carpenter (KC) Number (Body-Waves Interaction)
- Wave Period ~ 1 sec

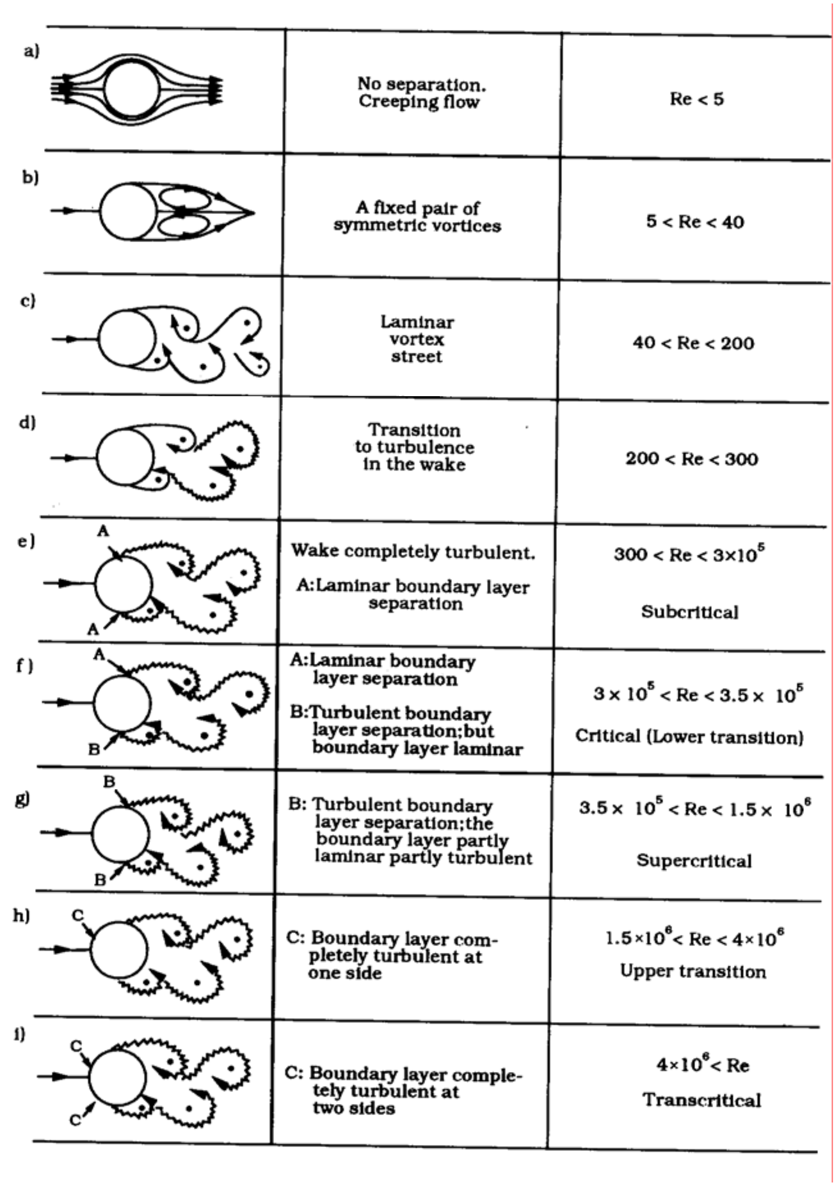
Flow Around the Falling Cylinder

- Turbulence
- Laminar



Falling Cylinder

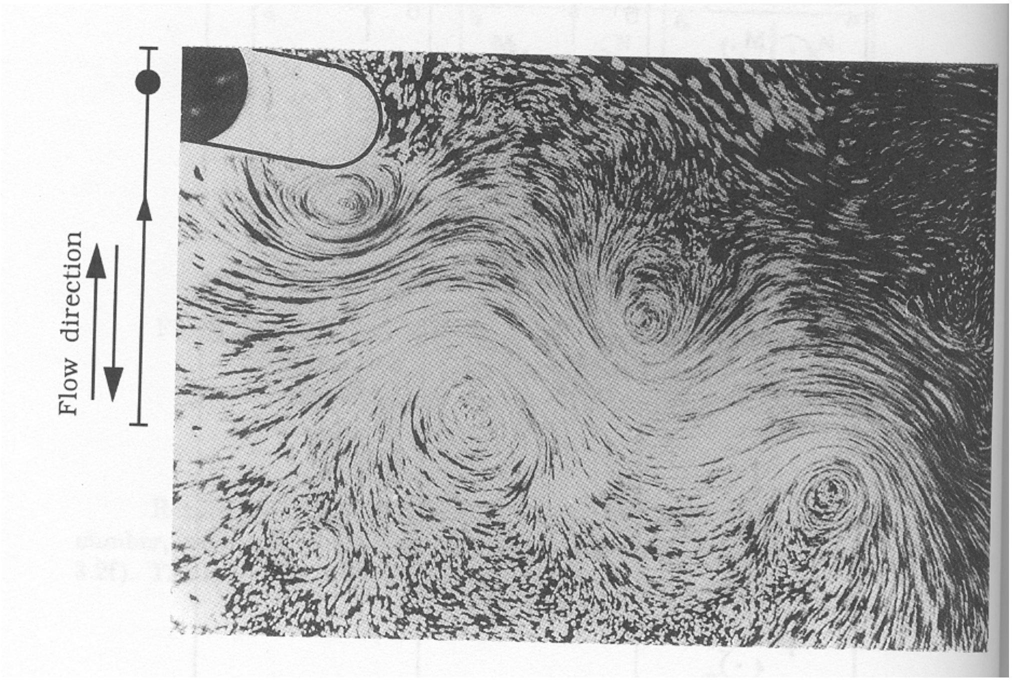
- Reynolds Number Much Larger Than 300



a)	No separation. Creeping flow	Re < 5
b)	A fixed pair of symmetric vortices	5 < Re < 40
c)	Laminar vortex street	40 < Re < 200
d)	Transition to turbulence in the wake	200 < Re < 300
e) A, A	Wake completely turbulent. A: Laminar boundary layer separation	$300 < Re < 3\times10^5$ Subcritical
f) A, B	A: Laminar boundary layer separation B: Turbulent boundary layer separation; but boundary layer laminar	$3\times10^5 < Re < 3.5\times10^5$ Critical (Lower transition)
g) B, B	B: Turbulent boundary layer separation; the boundary layer partly laminar partly turbulent	$3.5\times10^5 < Re < 1.5\times10^6$ Supercritical
h) C	C: Boundary layer completely turbulent at one side	$1.5\times10^6 < Re < 4\times10^6$ Upper transition
i) C, C	C: Boundary layer completely turbulent at two sides	$4\times10^6 < Re$ Transcritical

Falling Cylinder

- KC = 12 (Vortex Shedding)



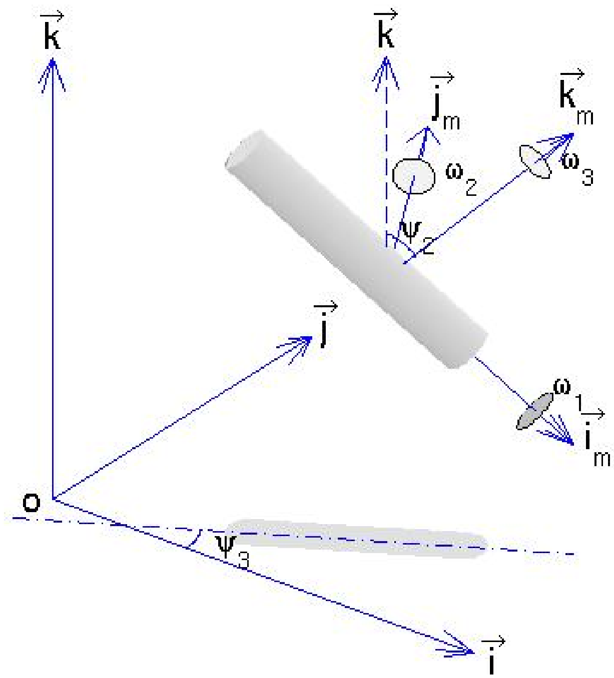
Chaotic Features

- Basic equations (6 unknowns) are nonlinear, similar to Lorenz system

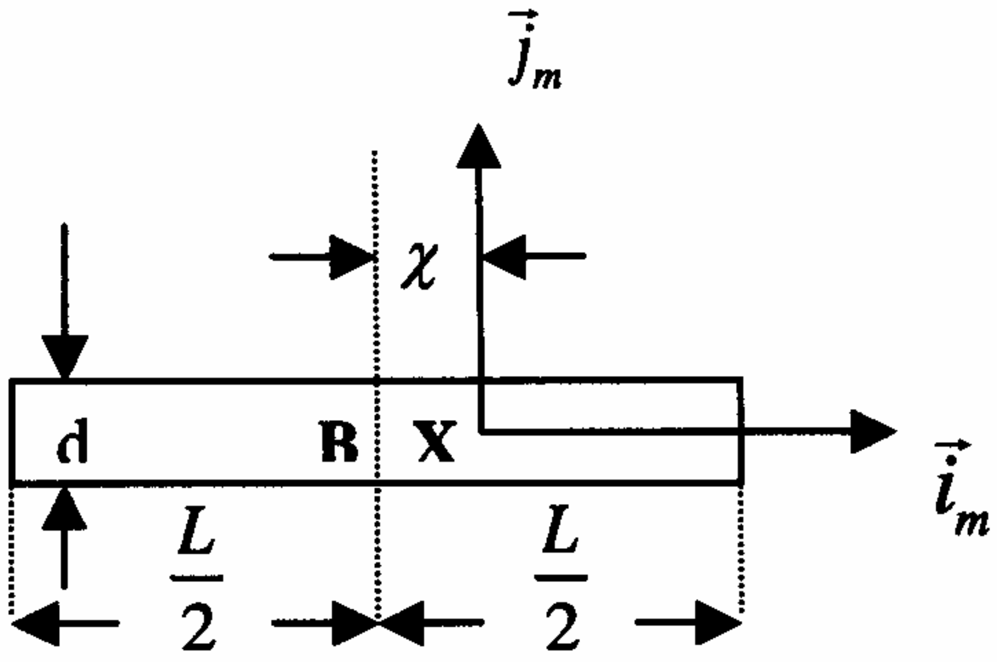
Triple Coordinate Transform

- earth-fixed coordinate (E-coordinate)
- cylinder's main-axis following coordinate (M-coordinate)
- hydrodynamic force following coordinate (F-coordinate).

E and M Coordinate Systems



Cylinder: X (COM), B (COV)



E-Coordinate, $F_E(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$

- COM Position: $\mathbf{X} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,

- Translation velocity:

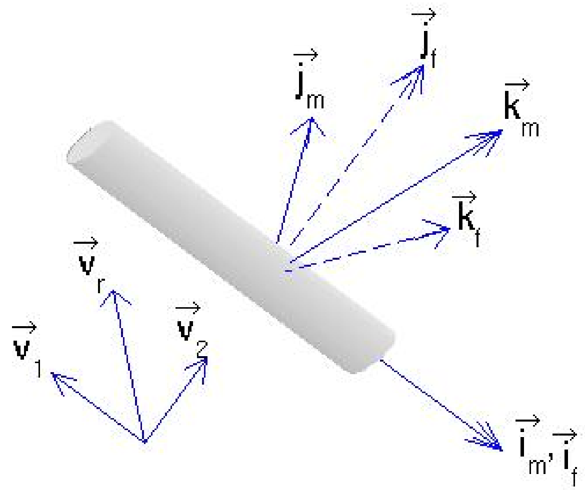
$$d\mathbf{X}/dt = \mathbf{V}, \quad \mathbf{V} = (u, v, w)$$

Transform Between E- and M-Coordinate Systems

$$
{}^{E}_{M}\mathbf{R}(\psi_2,\psi_3) \equiv \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}:
$$

$$
= \begin{bmatrix} \cos\psi_3 & -\sin\psi_3 & 0 \\ \sin\psi_3 & \cos\psi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi_2 & 0 & \sin\psi_2 \\ 0 & 1 & 0 \\ -\sin\psi_2 & 0 & \cos\psi_2 \end{bmatrix},
$$

F-Coordinate System



E- and F-Coordinate Transform

$$\mathbf{i}_F = \mathbf{i}_M = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}, \qquad \mathbf{j}_F = \mathbf{V}_2 / |\mathbf{V}_2|, \qquad \mathbf{k}_F = \mathbf{i}_F \times \mathbf{j}_F.$$

$$^E_F\mathbf{R}(\psi_2, \psi_3, \phi_{MF}) \equiv \begin{bmatrix} r_{11} & r'_{12} & r'_{13} \\ r_{21} & r'_{22} & r'_{23} \\ r_{31} & r'_{32} & r'_{33} \end{bmatrix},$$

Momentum Equation in E-Coordinate System

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ \left(1-\rho_w/\bar{\rho}\right)g \end{bmatrix} + \frac{1}{\bar{\rho}\Pi}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix},$$

Moment of Momentum Equation in M-Coordinate System

$$\mathbf{J} \cdot \frac{d\boldsymbol{\omega}}{dt} = \mathbf{M}_b + \mathbf{M}_h ,$$

M-Coordinate

The moment of gyration tensor for the axially Symmetric cylinder is a diagonal matrix

$$\mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix},$$

F-Coordiante

Hydrodynamic forces (drag and lift)
are easily calculated.

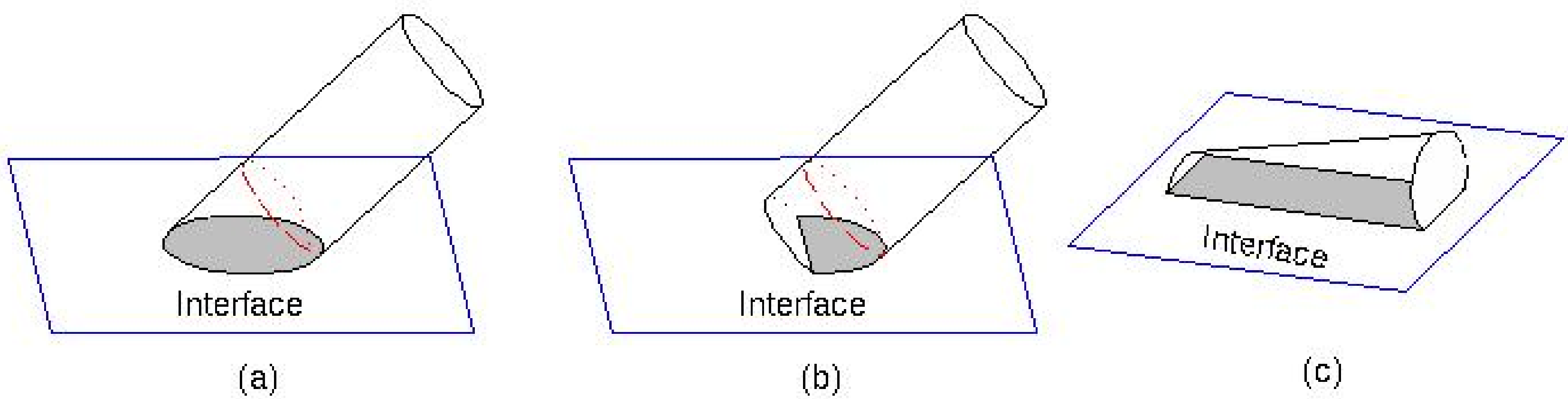
$$\mathbf{M}_{v1} = -C_{m1}\omega_1\mathbf{i}_F\,, \quad C_{m1} \equiv \pi\mu L d^2\,.$$

Moment of Momentum Equations

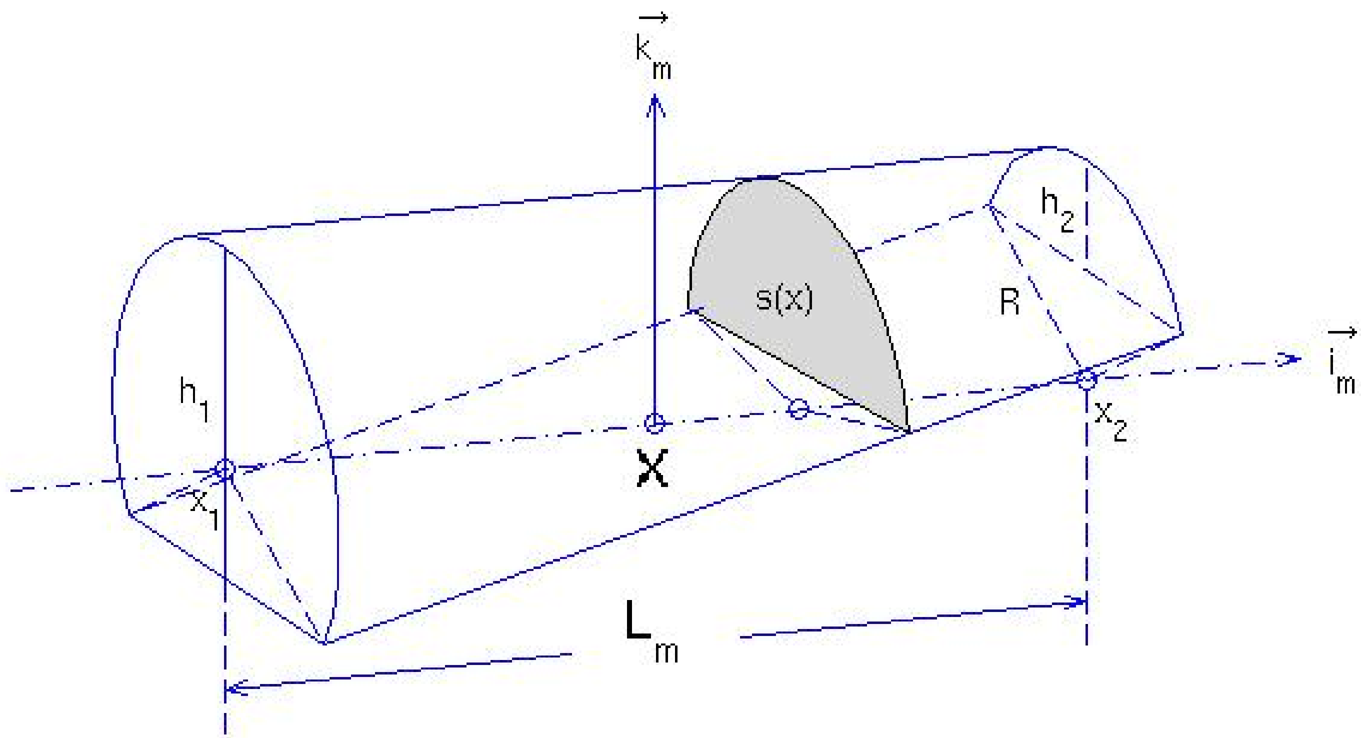
$$\frac{d\omega_1}{dt} = -a_1\omega_1 ,$$

$$\frac{d}{dt}\begin{bmatrix}\omega_2 \\ \omega_3\end{bmatrix} = -\mathbf{B}\cdot\begin{bmatrix}\omega_2 \\ \omega_3\end{bmatrix} + \boldsymbol{\alpha}_2 ,$$

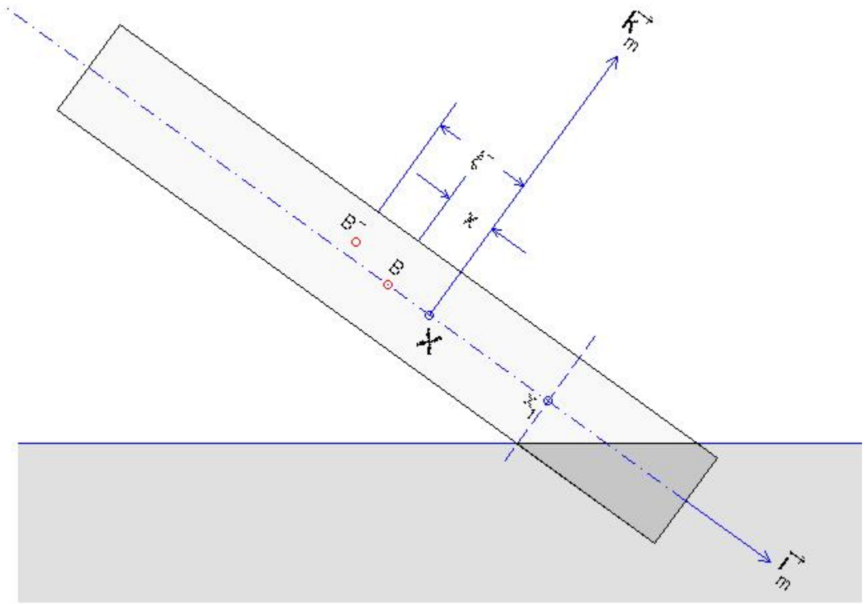
Interfacial Penetration Modeling



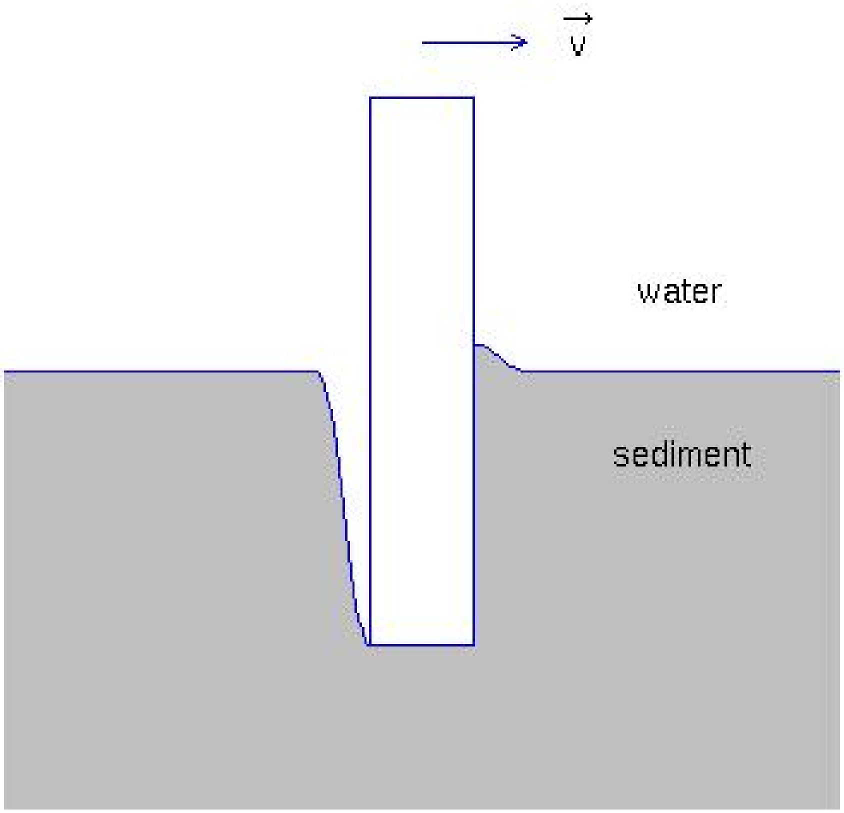
M-Coordinate System



Equivalent Cylinder



Penetration into Sediment



Experiment

- Hydrodynamic Model Development

- Behavior of Falling Cylinder in Water Column (Chaotic, Turbulent Wake, Eddy Shedding)

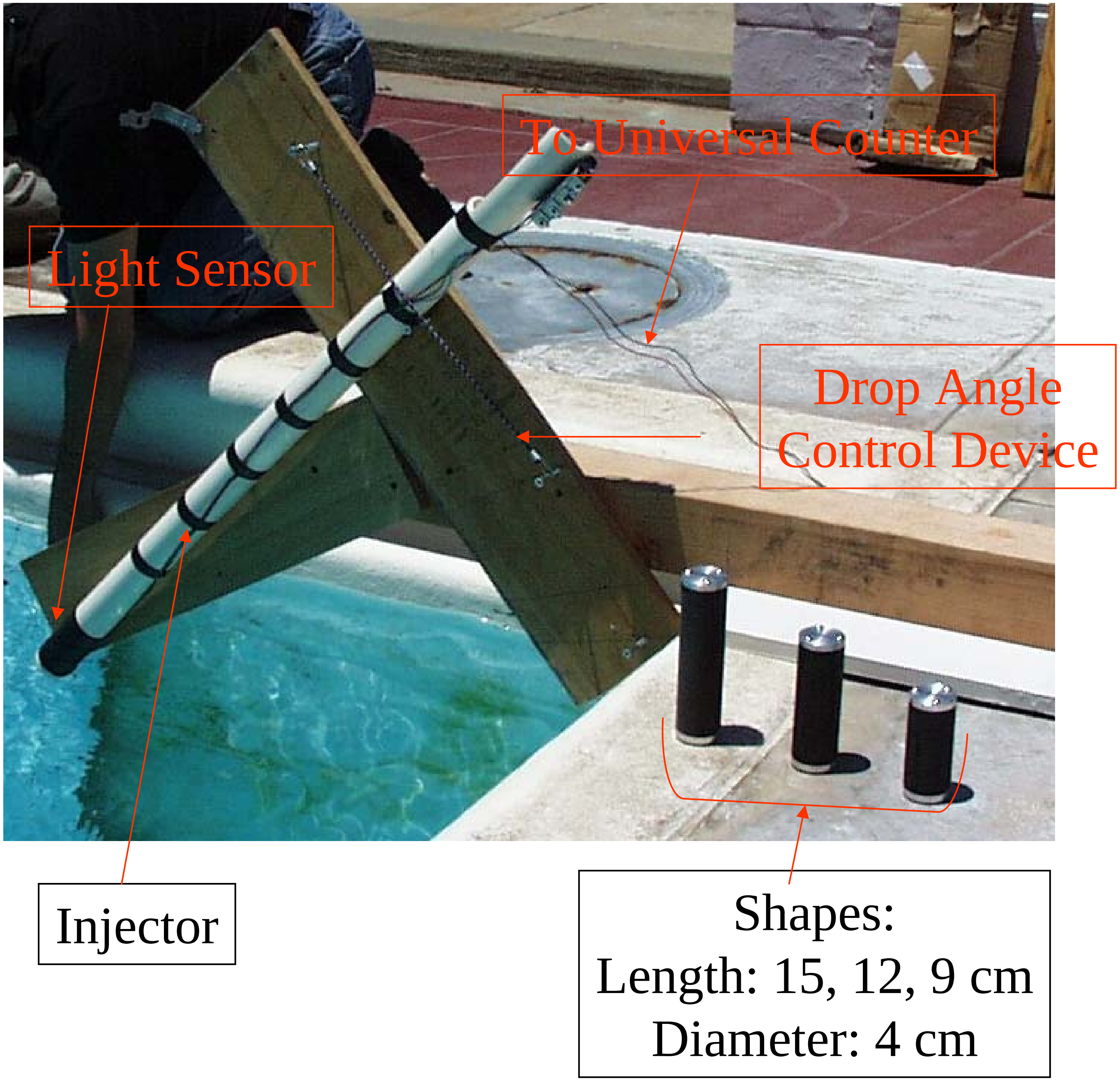
Experiment

- Cylinder Parameters:

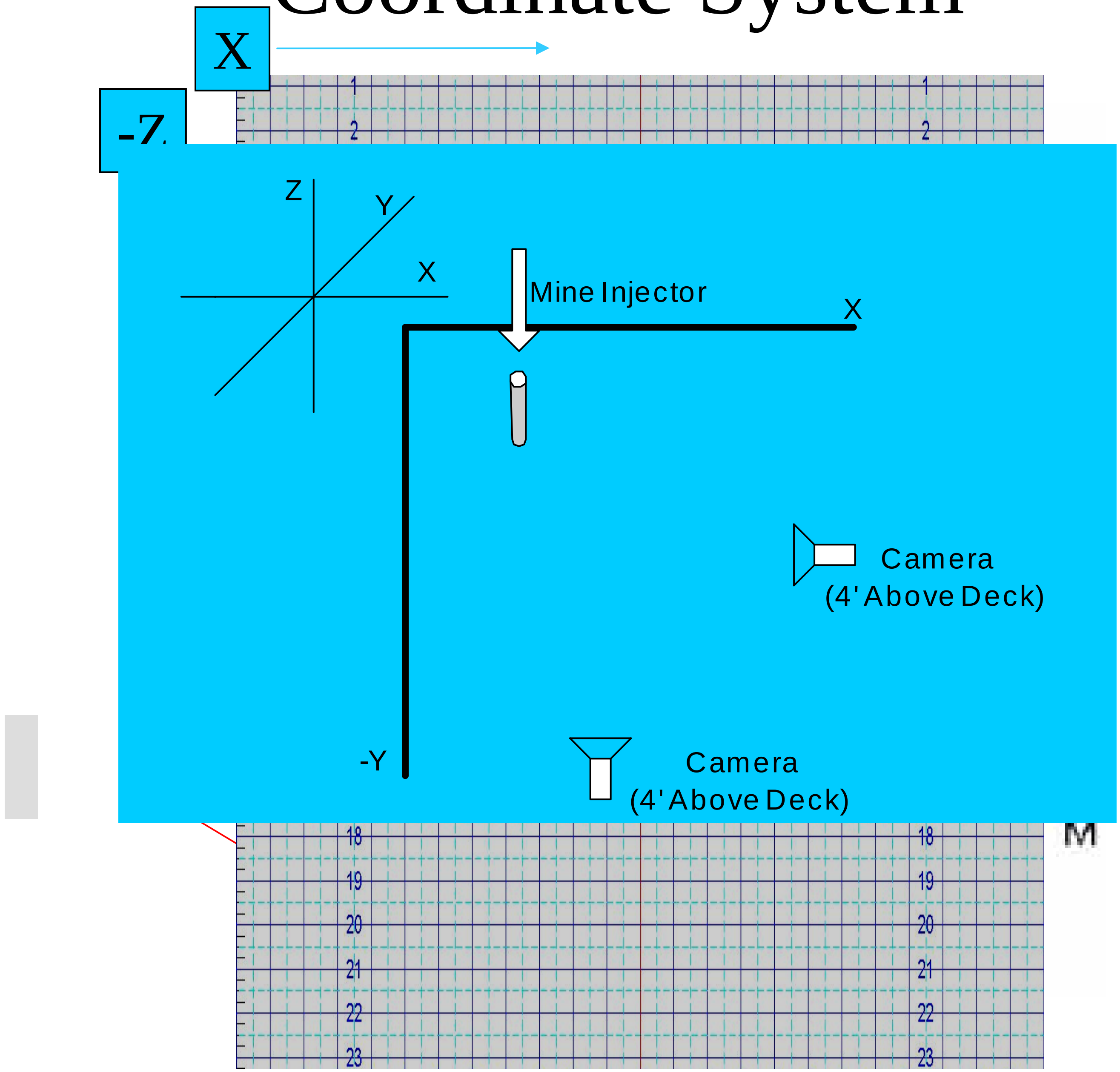
1. Density Ratio (1.68, 1.70, 1.88)
2. Center of Mass Position.
3. L/D ratio.

- Drop Parameters:

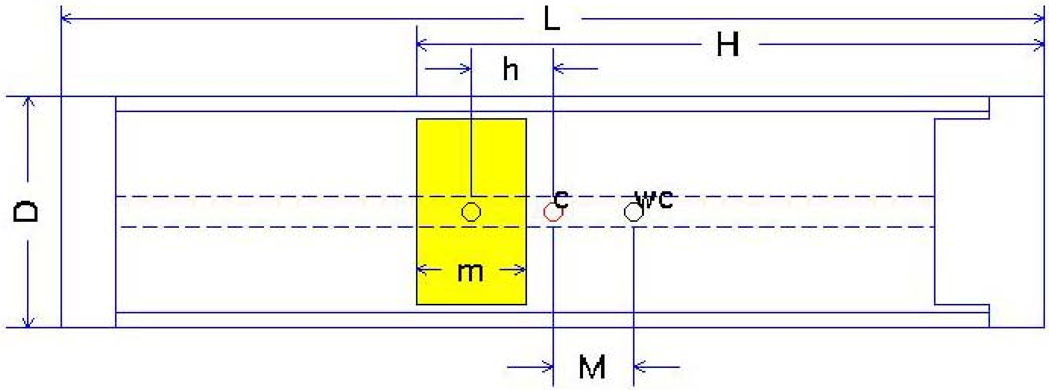
1. Drop Angles: 15º, 30º, 45º, 60º, 75º.
2. Release Velocity V_{init}


To Universal Counter
Light Sensor
Drop Angle
Control Device
Injector
Shapes:
Length: 15, 12, 9 cm
Diameter: 4 cm

Coordinate System



Center of Mass



MODEL # 1

L=15.1359cm D=4cm m=2.7cm

Weight=322.5 g Volume=190.2028 cm^3 Density=1.6956 g/cm^3

H:	10.380	8.052	5.725	cm
h:	−1.462	0.866	3.193	cm
M:	0.000	18.468	36.935	mm

MODEL # 2

L=12.0726cm D=4cm m=1.7cm

Weight=254.2 g Volume=151.709 cm^3 Density=1.6756 g/cm^3

H:	8.450	6.609	4.768	cm
h:	−1.564	0.277	2.119	cm
M:	0.000	12.145	24.290	mm

MODEL # 3

L=9.1199cm D=4cm m=1.47cm

Weight=215.3 g Volume=114.6037 cm^3 Density=1.8786 g/cm^3

H:	6.662	5.592	4.521	cm
h:	−1.368	−0.297	0.774	cm
M:	0.000	6.847	13.694	mm

Defined COM position as:

2 or -2: Farthest from volumetric center

1or -1

0: Coincides with volumetric center

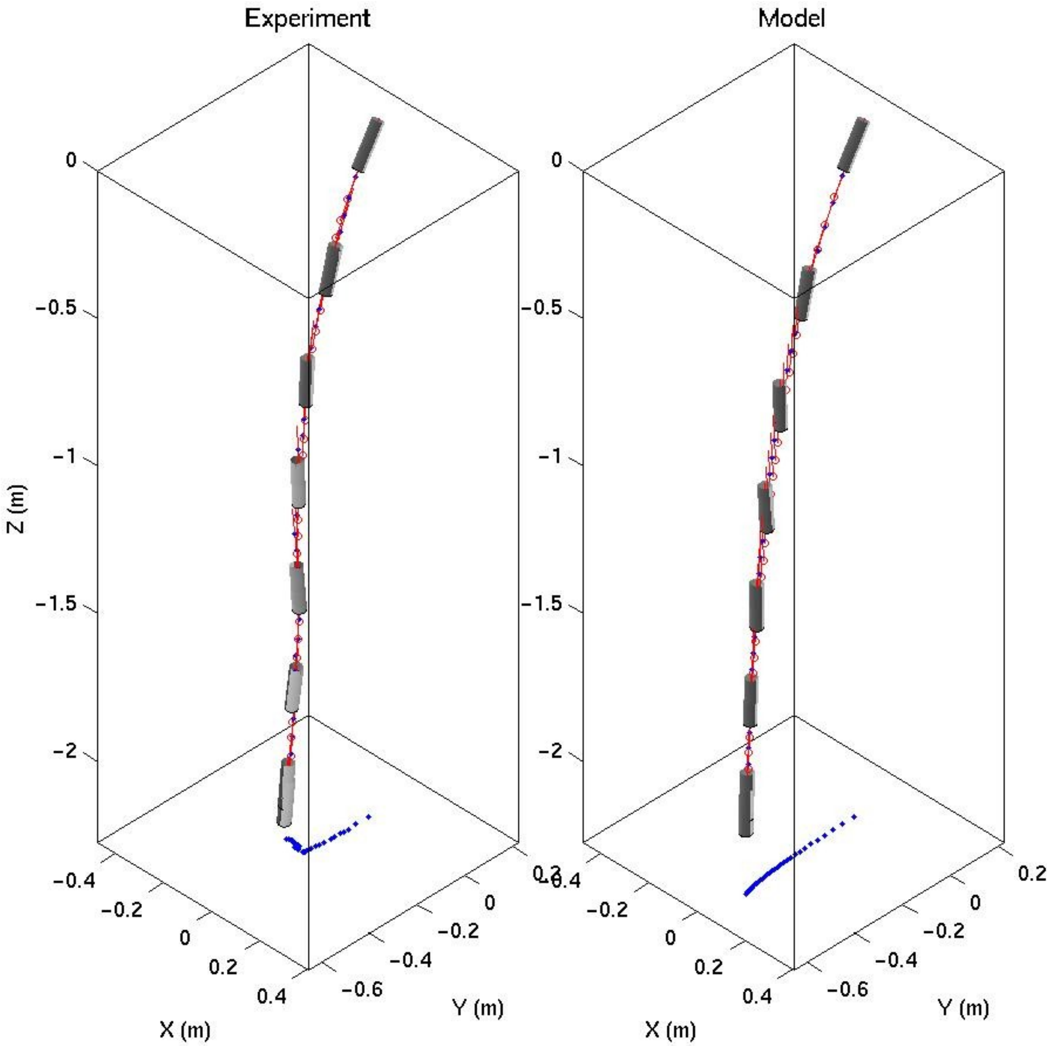
Data Analysis

1. Video converted to digital format.
2. Digital video from each camera analyzed frame by frame (30Hz) using video editing program.
3. Mine’s top and bottom position determined using background x-z and y-z grids. Positions manually entered into MATLAB for storage and later processing.
4. Analyzed 2-D data to obtain mine’s x,y and z center positions, attitude (angle with respect to z axis) and u,v, and w components.

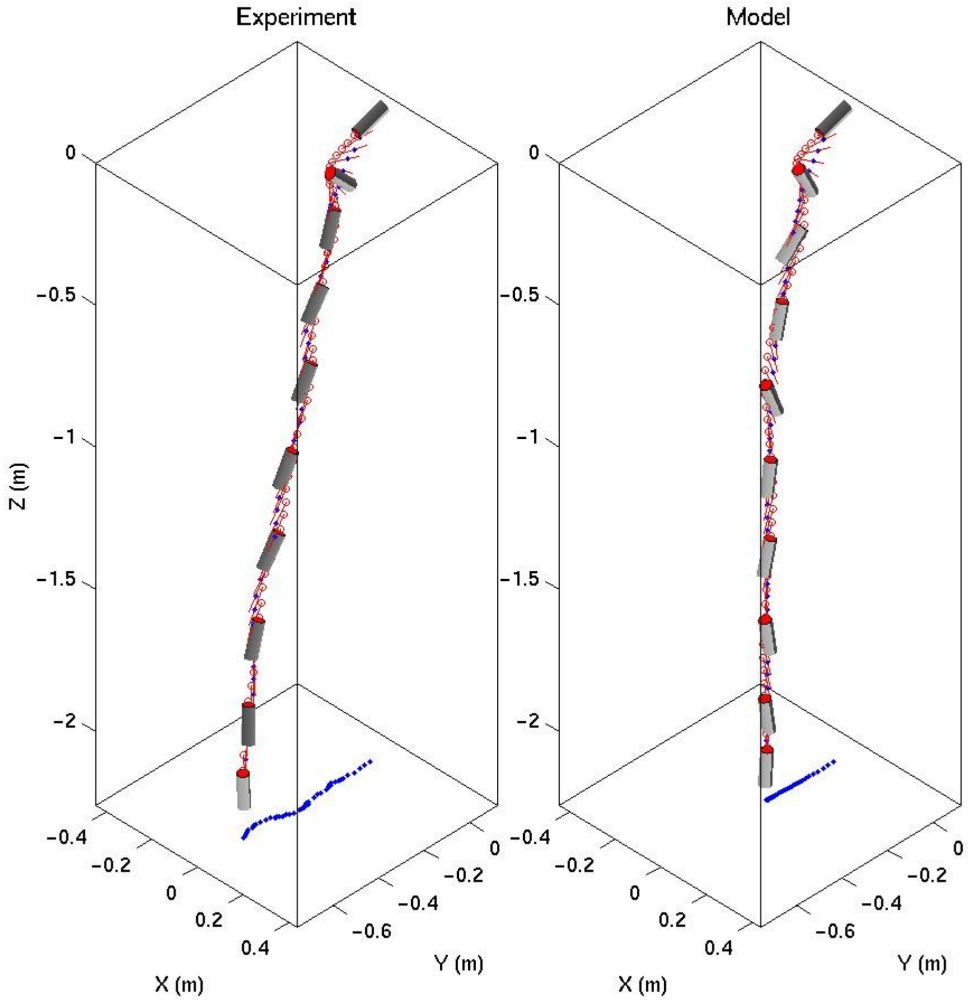
Sources of Error

1. Grid plane behind mine trajectory plane. Results in mine appearing larger than normal.
2. Position data affected by parallax distortion and binocular disparity.
3. Air cavity affects on mine motion not considered in calculations.
4. Camera plane not parallel to x-y plane due to pool slope.

Model-Data Comparison



Model-Data Comparison



Conclusions

- Triple coordinate transform is an effective method to predict movement of cylinder in water column
- Momentum equation (E-coordinate)
- Moment of momentum equation (M-coordinate)
- Hydrodynamic forces and torques (F-coordinate)