How Long Can an Atmospheric Model Predict?

Peter C Chu and Leonid Ivanov
Naval Ocean-Atmospheric Prediction Laboratory
Naval Postgraduate School
Monterey, California 93943, USA
pcchu@nps.edu
http://www.oc.nps.navy.mil/~chu

References

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Physical Reality

- $\boldsymbol{Y}$
- Physical Law: $d\boldsymbol{Y}/dt = \boldsymbol{h}(\boldsymbol{y}, t)$
- Initial Condition: $\boldsymbol{Y}(t_0) = \boldsymbol{Y}_0$

Atmospheric Models

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  - ***X*** is the prediction of **Y**
  - $d\,\boldsymbol{X}/\,dt = \boldsymbol{f}(\boldsymbol{X}, t) + q(t)\,\boldsymbol{X}$
  - Initial Condition: $\boldsymbol{X}(t_0) = \boldsymbol{X}_0$
  - Stochastic Forcing:
  - $<q(t)> = 0$
  - $<q(t)q(t')> = q^2\delta(t-t')$

Model Error

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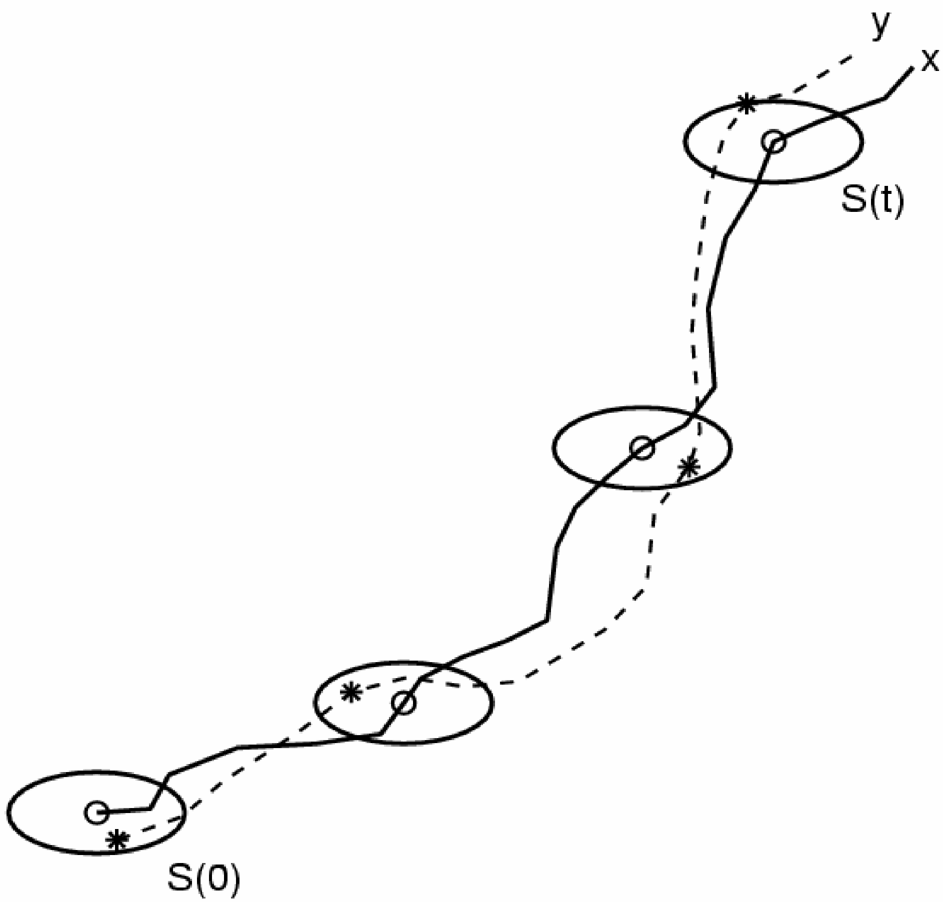
- $Z = X - Y$

- Initial: $Z_0 = X_0 - Y_0$

One Overlooked Parameter

- Tolerance Level ε
- Maximum accepted error

Valid Predict Period (VPP)



- VPP is defined as the time period when the prediction error first exceeds a pre-determined criterion (i.e., the tolerance level ε).

First-Passage Time

Conditional Probability Density Function

- Initial Error: $\boldsymbol{z}_0$
- $(t - t_0)$ ⇗ Random Variable
- Conditional PDF of $(t - t_0)$ with given $\boldsymbol{z}_0$ ⇗
  - $P[(t - t_0) \mid \boldsymbol{z}_0]$

Two Approaches to Obtain PDF of VPP

- Analytical (Backward Fokker-Planck Equation)
- Practical

Backward Fokker-Planck Equation

$$\frac{\partial P}{\partial t} - \left[\mathbf{f}(\mathbf{z}_0, t)\right]\frac{\partial P}{\partial \mathbf{z}_0} - \frac{1}{2} q^2 \mathbf{z}_0{}^2 \frac{\partial^2 P}{\partial \mathbf{z}_0 \partial \mathbf{z}_0} = 0$$

Moments

$$\tau_1(\mathbf{z}_0) = \int_{t_0}^{\infty} P(t_0, \mathbf{z}_0, t - t_0)(t - t_0)dt$$

$$\tau_2(\mathbf{z}_0) = \int_{t_0}^{\infty} P(t_0, \mathbf{z}_0, t - t_0)(t - t_0)^2 dt$$

Example: Maximum Growing Maniford of Lorenz System (Nicolis, 1992)

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$$\frac{d\xi}{dt} = (\sigma - g\xi^2) + v(t)\xi, \qquad 0 \le \xi < \infty$$

$$\langle v(t) \rangle = 0, \qquad \langle v(\mathrm{t}) v(\mathrm{t}') \rangle = q^2 \delta(t - t').$$

$$\sigma = 0.64, \qquad g = 0.3, \qquad q^2 = 0.2.$$

Mean and Variance of VPP

$$(\sigma\xi_0 - g\xi_0^2)\frac{d\tau_1}{d\xi_0} + \frac{q^2\xi_0^2}{2}\frac{d^2\tau_1}{d\xi_0^2} = -1$$

$$(\sigma\xi_0 - g\xi_0^2)\frac{d\tau_2}{d\xi_0} + \frac{q^2\xi_0^2}{2}\frac{d^2\tau_2}{d\xi_0^2} = -2\tau_1$$

$$\tau_1 = 0, \qquad \tau_2 = 0 \qquad \text{for} \quad \xi_0 = \varepsilon.$$

$$\frac{d\tau_1}{d\xi_0} = 0, \qquad \frac{d\tau_2}{d\xi_0} = 0 \qquad \text{for} \quad \xi_0 = \xi_{noise}.$$

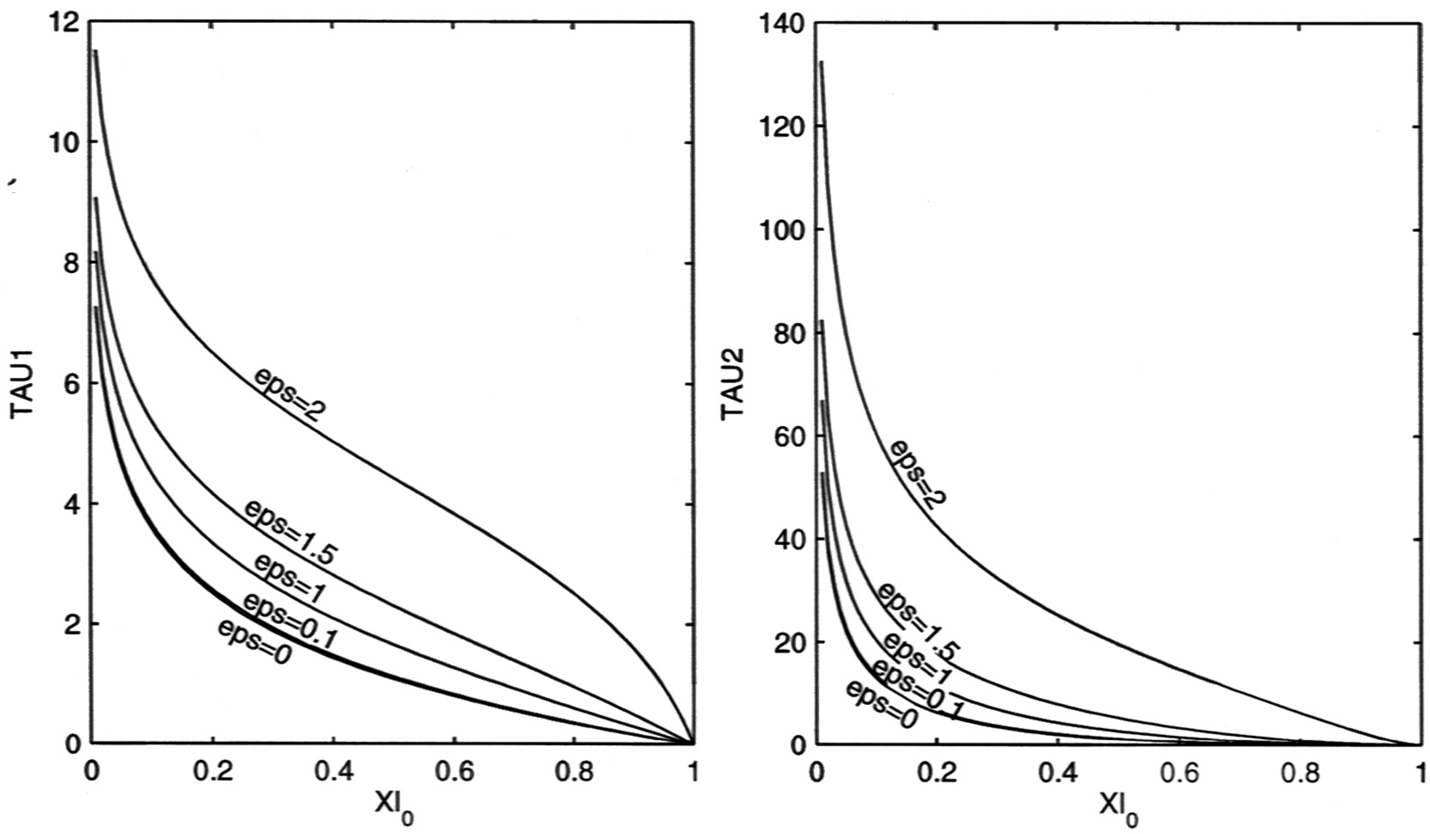
Analytical Solutions

$$\tau_1(\bar{\xi}_0,\bar{\xi}_{noise},\varepsilon)=\frac{2}{q^2}\int_{\bar{\xi}_0}^{1} y^{-\frac{2\sigma}{q^2}}\exp\left(\frac{2\varepsilon g}{q^2}y\right)\left[\int_{\bar{\xi}_{noise}}^{y} x^{\frac{2\sigma}{q^2}-2}\exp\left(-\frac{2\varepsilon g}{q^2}x\right)dx\right]dy$$

$$\tau_2(\bar{\xi}_0,\bar{\xi}_{noise},\varepsilon)=\frac{4}{q^2}\int_{\bar{\xi}_0}^{1} y^{-\frac{2\sigma}{q^2}}\exp\left(\frac{2\varepsilon g}{q^2}y\right)\left[\int_{\bar{\xi}_{noise}}^{y} \tau_1(x)x^{\frac{2\sigma}{q^2}-2}\exp\left(-\frac{2\varepsilon g}{q^2}x\right)dx\right]dy$$

$$\bar{\xi}_0=\xi_0/\varepsilon\,,\qquad \bar{\xi}_{noise}=\xi_{noise}/\varepsilon$$

Dependence of tau1 & tau2 on Initial Condition Error (ξ_0/ε)



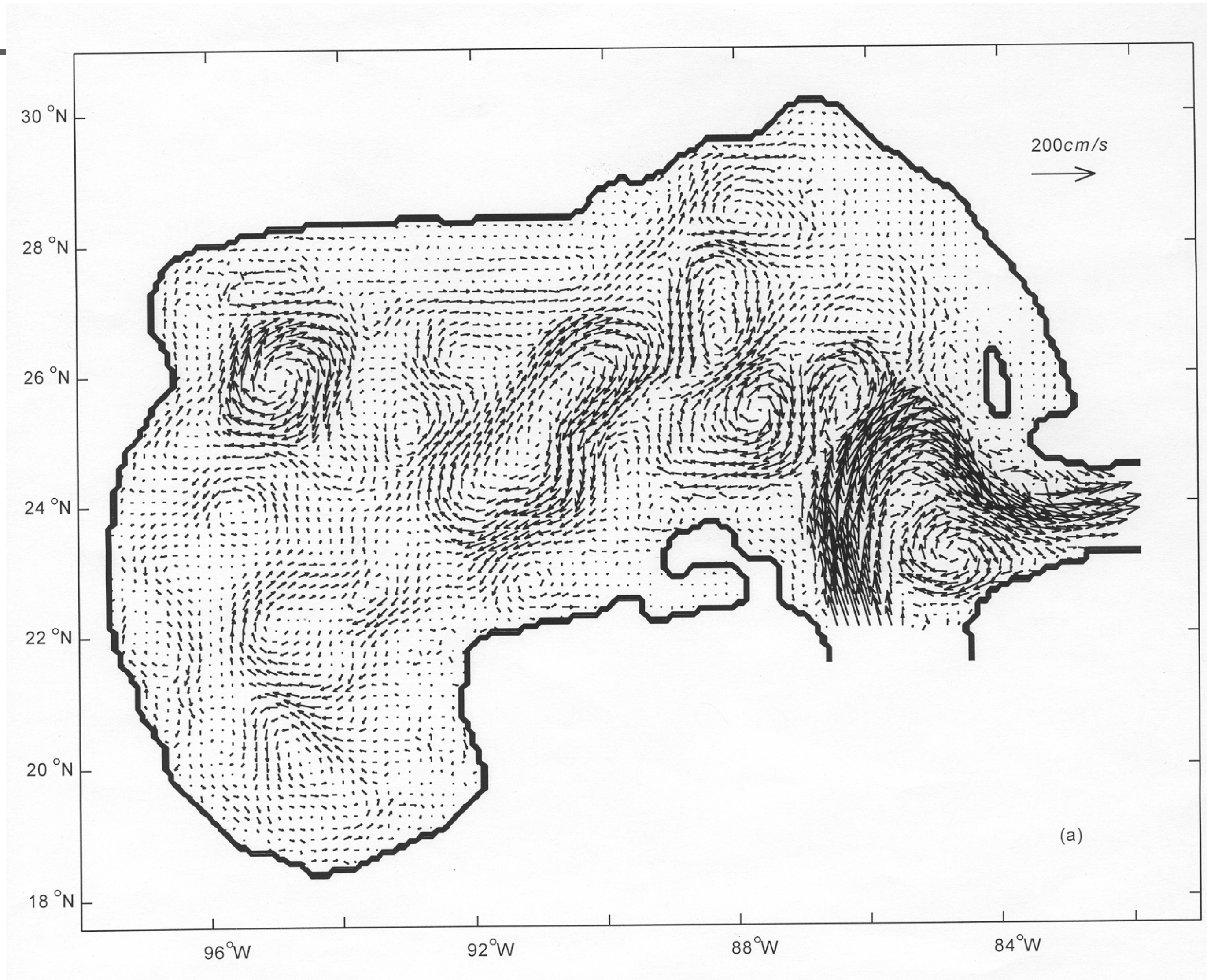
Practical Approach

Gulf of Mexico Ocean Prediction System

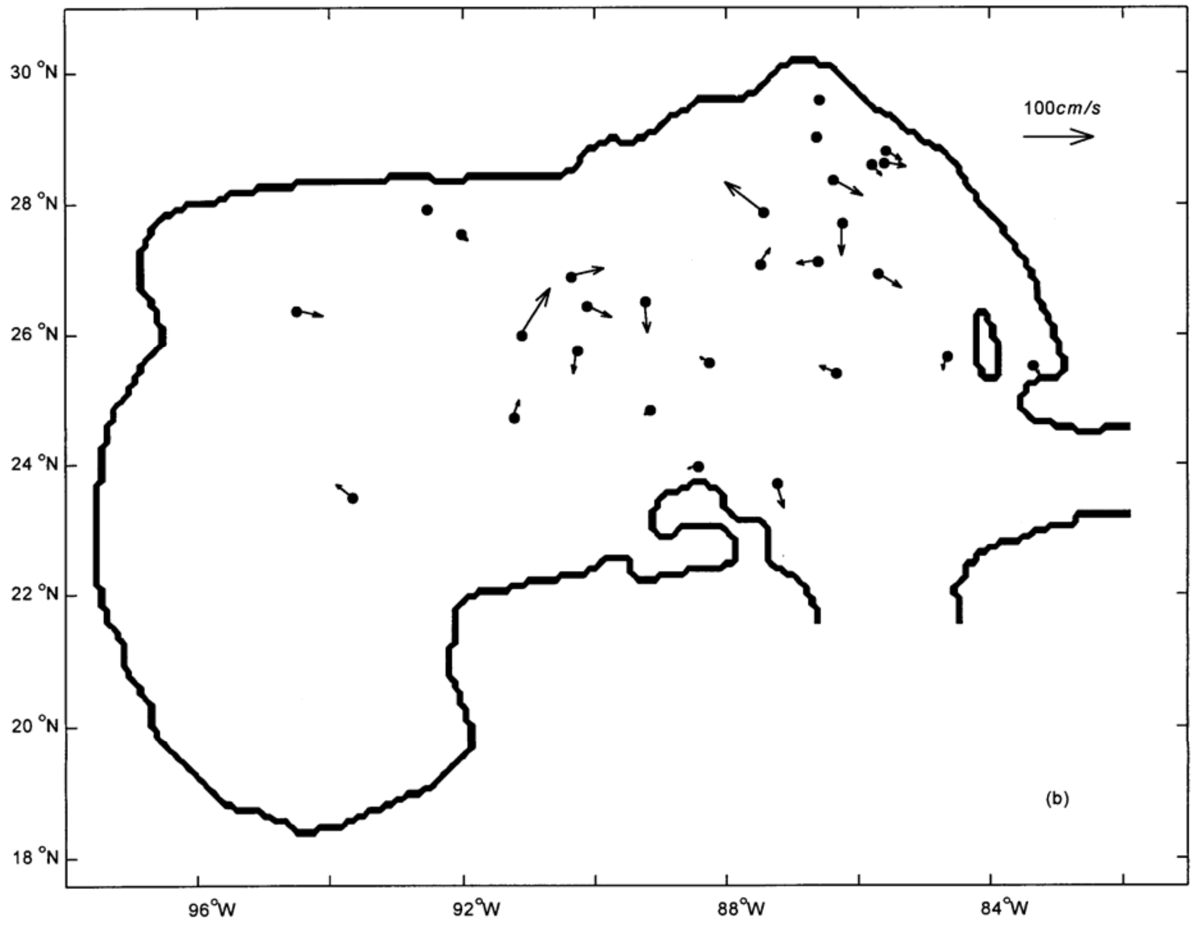
Gulf of Mexico Forecast System

- University of Colorado Version of POM
- 1/12^o Resolution
- Real-Time SSH Data (TOPEX, ESA ERS-1/2) Assimilated
- Real Time SST Data (MCSST, NOAA AVHRR) Assimilated
- Six Months Four-Times Daily Data From July 9, 1998 for Verification

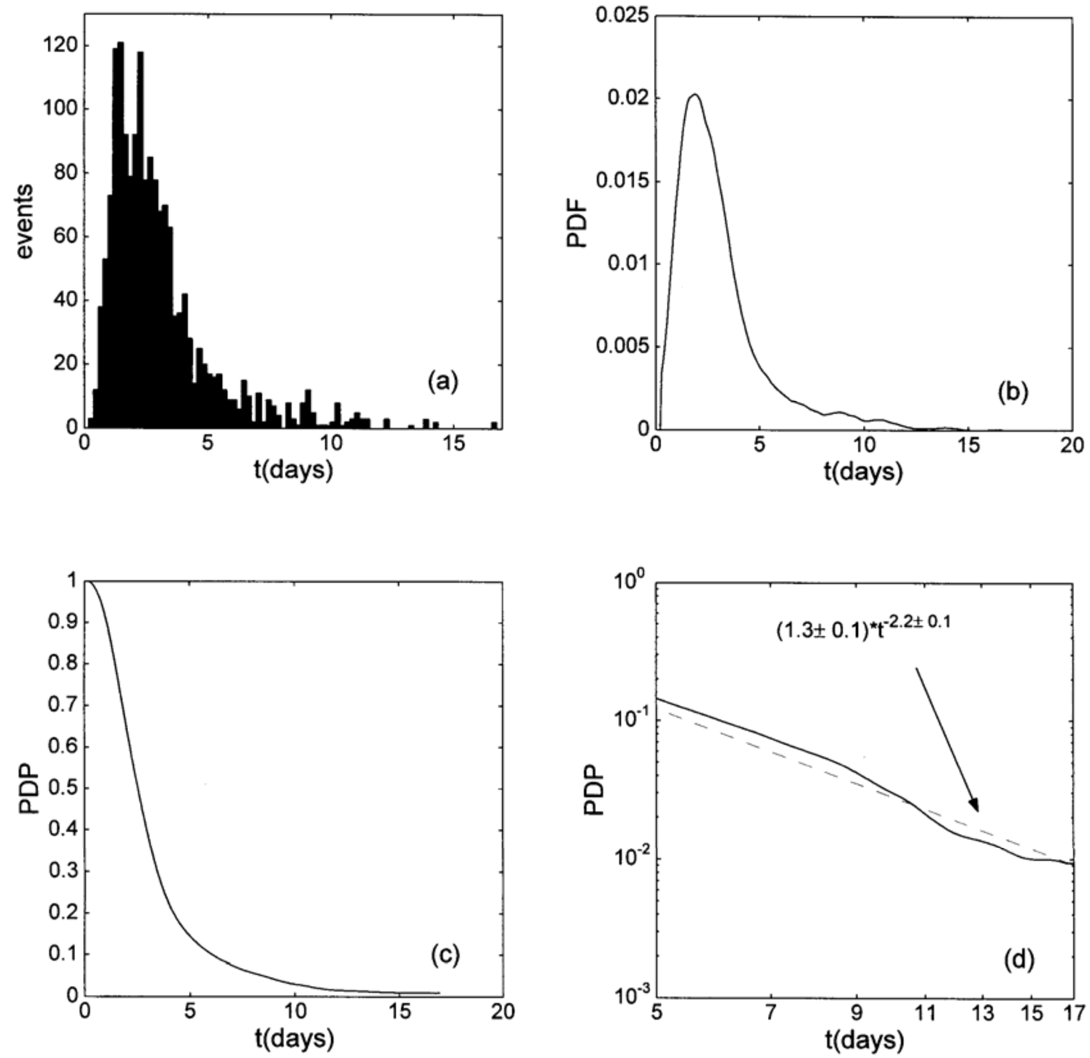
Model Generated Velocity Vectors at 50 m on 00:00 July 9, 1998



(Observational) Drifter Data at 50 m on 00:00 July 9, 1998



Statistical Characteristics of VPP for zero initial error and 22 km tolerance level (Non-Gaussion)



Conclusions

- (1) VPP (i.e., FPT) is an effective prediction skill measure (scalar).

- (2) Theoretical framework for FPT (such as Backward Fokker-Planck equation) can be directly used for model predictability study.