Hydrostatic Correction for Terrain-Following Ocean Models

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How to effectively reduce the sigma error is a challenge task for TOMS.

Pressure Gradient Error

Uses a coordinate system which is scaled with the depth

$$\sigma = \frac{z - \zeta}{H + \zeta}.$$

Makes use of coordinate surfaces which are located below the bottom depth in Level models.

Aspect ratio	Ocean	Atmosphere
$\frac{\Delta h_{\text{topography}}}{H_{\text{depth}}}$	$\sim \mathcal{O}(1)$	$\ll \mathcal{O}(1)$

- Atmosphere: Hybrid (σ, p) models are common.
- Ocean: ρ and σ surfaces may intersect at large angle.

Pressure Gradient Error

Pressure gradient in Sigma coordinates

$$\left.\frac{\partial P}{\partial x}\right|_{z=\mathrm{const}} = \left.\frac{\partial P}{\partial x}\right|_{\sigma=\mathrm{const}} - \frac{\sigma}{h}\frac{\partial h}{\partial x}\frac{\partial P}{\partial \sigma}$$

(1) **Pressure grad along σ-surfaces**

(2) **Correction for vertical component in (1)**

In case of large slopes:

$$\mathcal{O}(2) = \mathcal{O}(1)$$

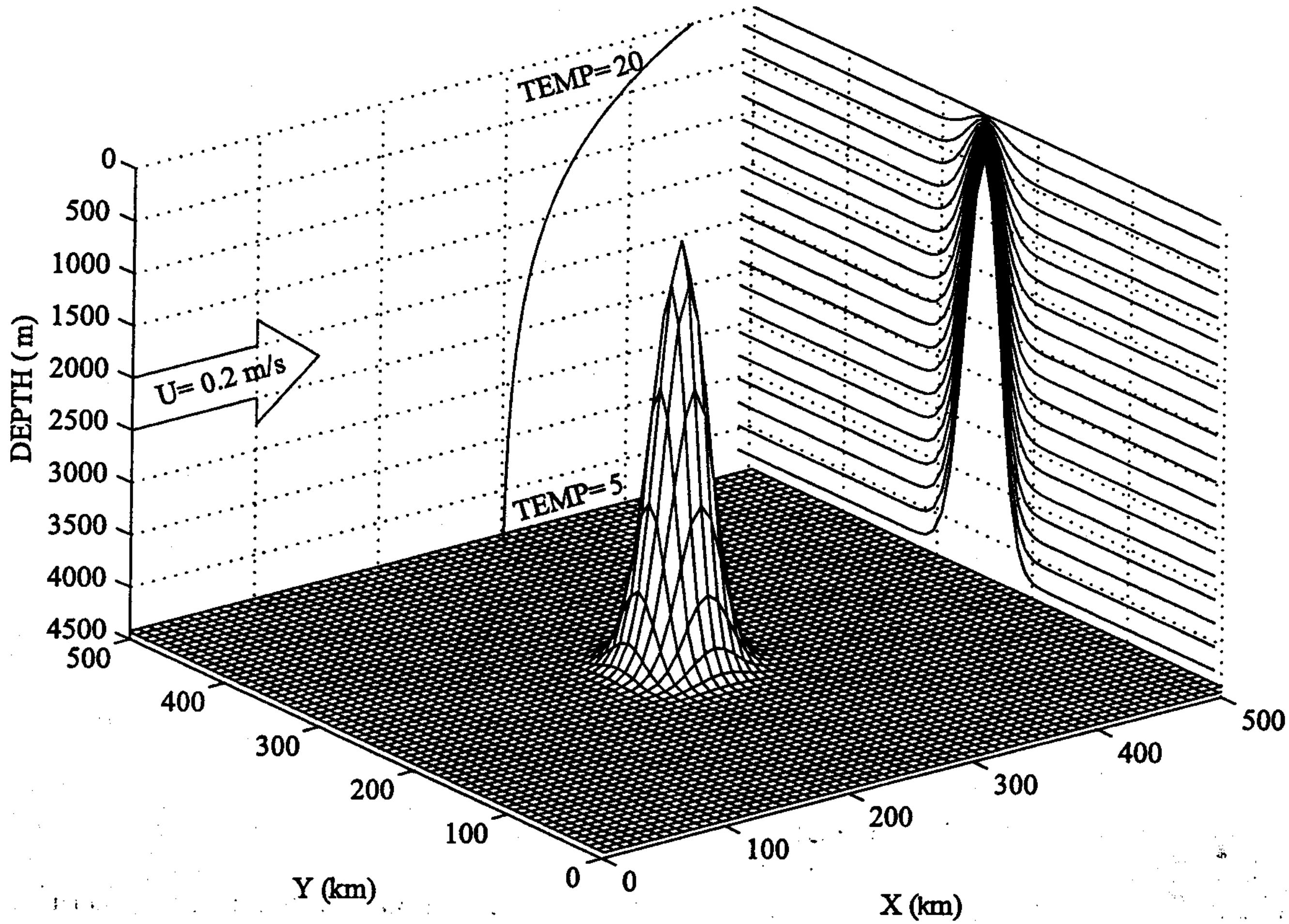
$$(1) + (2) \ll (1) \vee (2)$$

Thus, truncation errors may be significant.

Original Thought: High-Order Schemes

- Ordinary Five-Point Sixth-Order Scheme (Chu and Fan, 1997 JPO)
- Three-Point Sixth-Order Combined Compact Difference (CCD) Scheme (Chu and Fan, 1998 JCP)
- Three-Point Sixth-Order Nonuniform CCD Scheme (Chu and Fan, 1999, JCP)
- Three-Point Sixth-Order Staggered CCD Scheme (Chu and Fan, 2000, Math. & Comp. Modeling)
- Accuracy Progressive Sixth-Order Scheme (Chu and Fan, 2001, JTECH)

Verification of Current Schemes by Ezer, Arango, Shchepetkin (OCEMOD, 2002)



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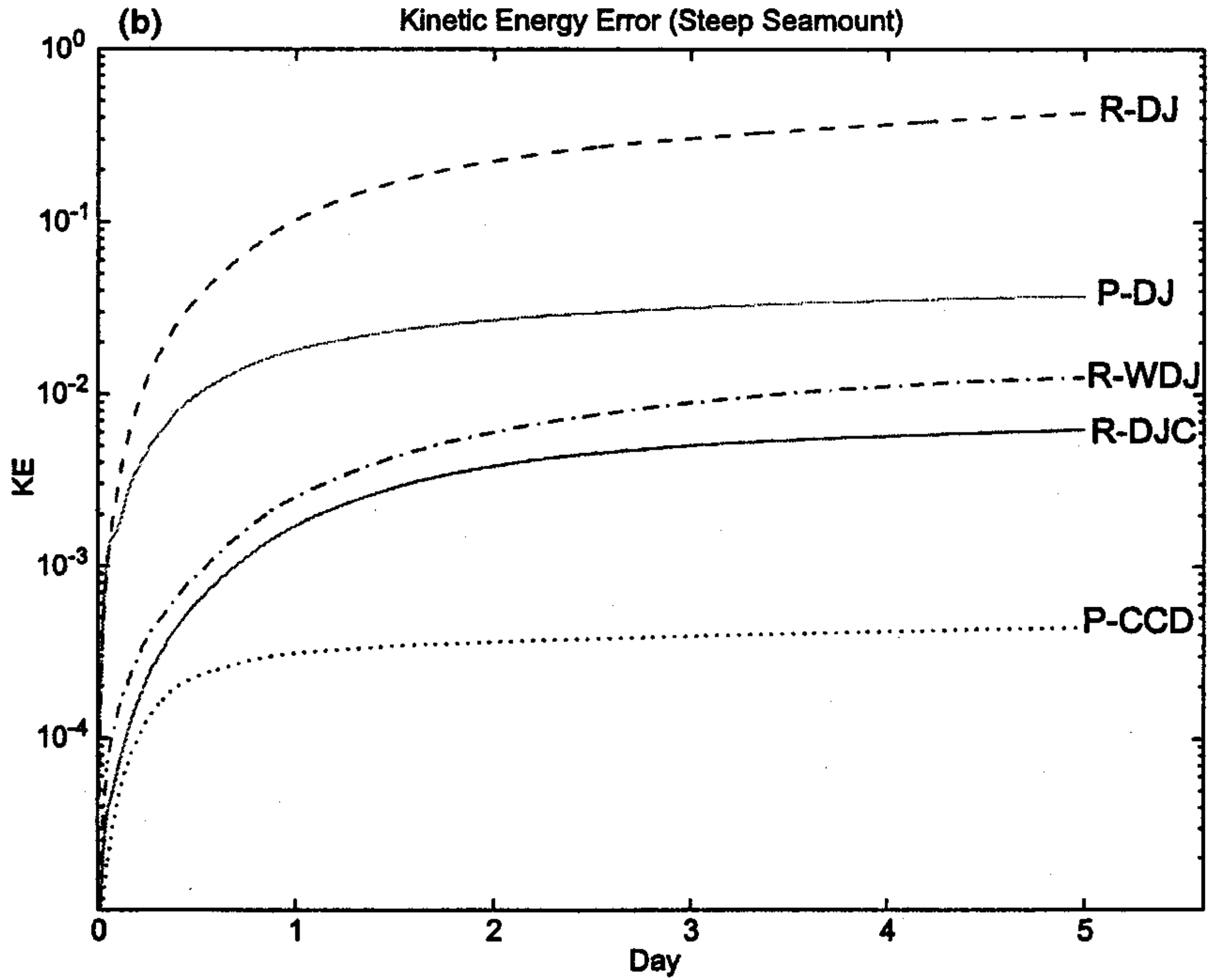

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Table 1
Comparisons of computational costs for POM and ROMS when using different features

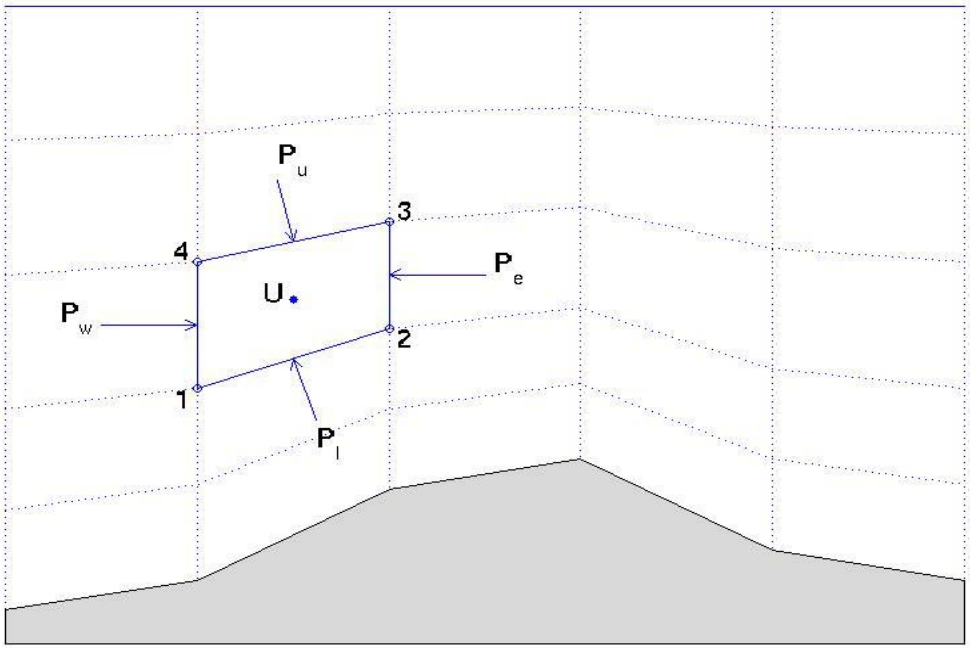
Basic model	Model features[a]	Computation time per grid point per time step (in 10^{-3} s)	Computation time for one day of integration (in s)
POM	CADV2	12.5	21.3
POM	UADV2	13.2	
POM	MPDATA2	14.0	
POM	MPDATA3	17.1	
POM	CCD6	207.4	
POM	ZINT	40.0	
ROMS	CADV2	17.3	21.6
ROMS	CADV4	19.3	
ROMS	UADV3	20.0	

How to reduce errors with low computer cost?

Hydrostatic Correction

- Chu, P.C., and C.W. Fan, 2003: Hydrostatic correction for reducing horizontal pressure gradient errors in sigma coordinate models. Journal of Geophysical Research, J. Geophys. Res. Vol. 108, No. C6, 3206, 10.1029/2002JC001668.

Finite Volume Consideration



$$\mathbf{F} = L_y \oint_C p\mathbf{n}ds$$

Horizontal Pressure Gradient

$$\frac{\partial p}{\partial x} \equiv \frac{F_x}{L_y \Delta S} = -\frac{1}{\Delta S}\left(\int_1^2 p_l dz + \int_2^3 p_e dz + \int_3^4 p_u dz + \int_4^1 p_w dz \right)$$

Second-Order Scheme

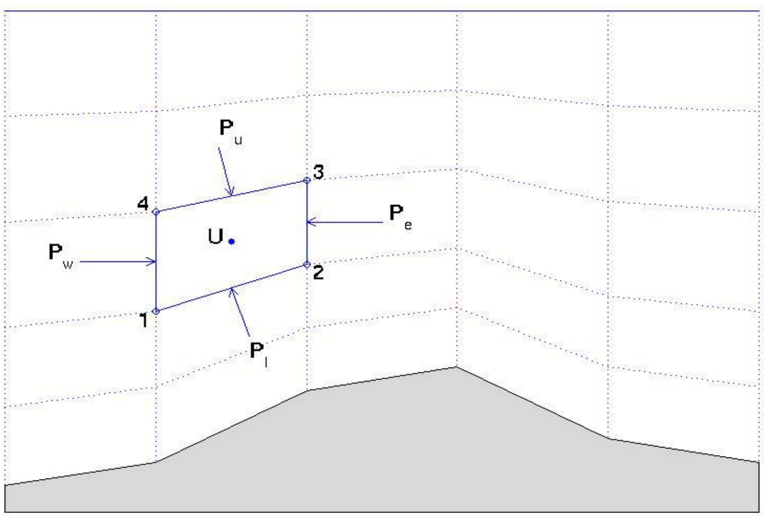
$$\frac{\Delta p}{\Delta x} = \frac{1}{\Delta S}\left[\begin{array}{l} \overline{p}_l(z_{i+1,k+1} - z_{i,k+1}) + \overline{p}_e(z_{i+1,k} - z_{i+1,k+1}) \\ +\overline{p}_u(z_{i,k} - z_{i+1,k}) + \overline{p}_w(z_{i,k+1} - z_{i,k}) \end{array}\right]$$

$$\int_1^2 p_l dz = \overline{p}_l(z_{i+1,k+1} - z_{i,k+1}), \quad \int_2^3 p_e dz = \overline{p}_e(z_{i+1,k} - z_{i+1,k+1}),$$

$$\int_3^4 p_l dz = \overline{p}_u(z_{i,k} - z_{i+1,k}), \quad \int_4^1 p_w dz = \overline{p}_w(z_{i,k+1} - z_{i,k}),$$

$$\left(\frac{\Delta p}{\Delta x}\right)_{i,k} = \frac{\left(p_{i+1,k+1} - p_{i,k}\right)\left(H_{i+1}\sigma_k - H_i\sigma_{k+1}\right) + \left(p_{i+1,k} - p_{i,k+1}\right)\left(H_i\sigma_k - H_{i+1}\sigma_{k+1}\right)}{\Delta x_i \Delta\sigma_k \left(H_i + H_{i+1}\right)}$$

Steep Topography – Hermit Polynomial Integration



$$\xi = \frac{l - l_1}{\Delta l}, \quad \Delta l \equiv l_2 - l_1$$

$$\int_{l_1}^{l_2} p(l)\,dl = \Delta l \int_0^1 \Psi(\xi)\,d\xi$$

Hermit Polynomial Integration

$$\int_{l_1}^{l_2} p\,dl = \int_0^1 \Psi(\xi)d\xi = \frac{\Delta l}{2}\left(p_1 + p_2\right) + \frac{\Delta l^2}{12}\left(\left(\frac{\partial p}{\partial l}\right)_1 - \left(\frac{\partial p}{\partial l}\right)_2\right)$$

$$\Psi\left(\xi\right) = p_{l_1}\Phi_1 + p_{l_2}\Phi_2 + \Delta l\left(\frac{\partial p}{\partial l}\right)_{l_1}\Phi_3 + \Delta l\left(\frac{\partial p}{\partial l}\right)_{l_2}\Phi_4$$

$$\Phi_1 = 1 - 3\xi^2 + 2\xi^3, \quad \Phi_2 = 3\xi^2 - 2\xi^3,$$

$$\Phi_3 = \xi - 2\xi^2 + \xi^3, \quad \Phi_4 = \xi^3 - \xi^2$$

Hydrostatic Correction

$$\left(\frac{\Delta p}{\Delta x}\right)_{i,k} \simeq \frac{\left[\left(p_{i+1,k+1} - p_{i,k}\right)\left(z_{i+1,k} - z_{i,k+1}\right) + \left(p_{i+1,k} - p_{i,k+1}\right)\left(z_{i,k} - z_{i+1,k+1}\right)\right]}{\left(x_{i+1} - x_i\right)\left(z_{i,k} + z_{i+1,k} - z_{i,k+1} - z_{i+1,k+1}\right)} + \Omega_{ik}$$

$$\Omega_{ik} = \frac{g\Gamma_{ik}}{6\left(x_{i+1} - x_i\right)\left(z_{i,k} + z_{i+1,k} - z_{i,k+1} - z_{i+1,k+1}\right)},$$

$$\Gamma_{ik} \equiv [\left(H_{i+1}\sigma_k - H_{i+1}\sigma_{k+1}\right)^2\left(\rho_{i+1,k} - \rho_{i+1,k+1}\right) - \left(H_i\sigma_k - H_i\sigma_{k+1}\right)^2\left(\rho_{i,k} - \rho_{i,k+1}\right)$$

$$+\left(H_{i+1}\sigma_{k+1} - H_i\sigma_{k+1}\right)^2\left(\rho_{i+1,k+1} - \rho_{i,k+1}\right) - \left(H_{i+1}\sigma_k - H_i\sigma_k\right)^2\left(\rho_{i+1,k} - \rho_{i,k}\right)]$$

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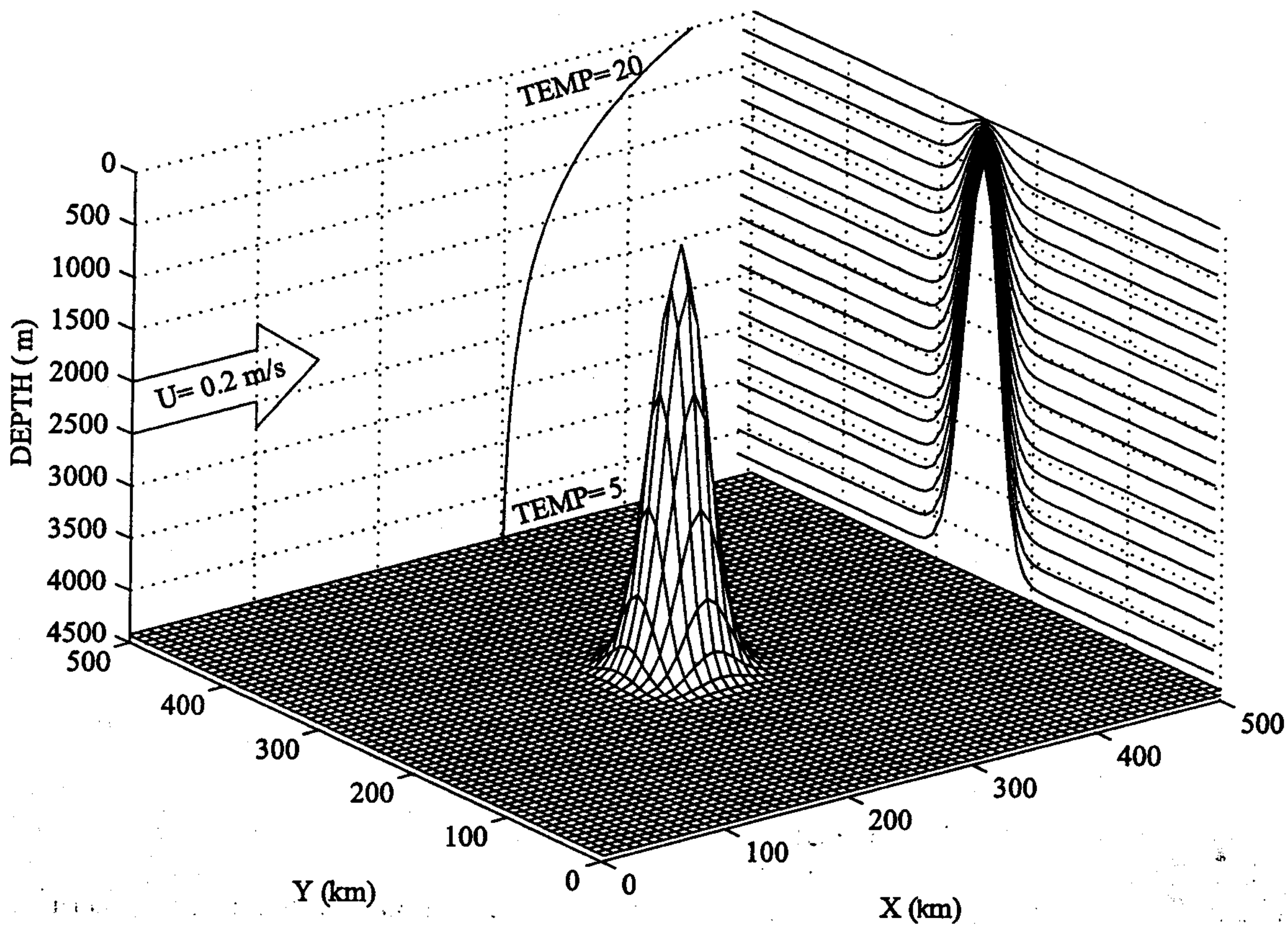
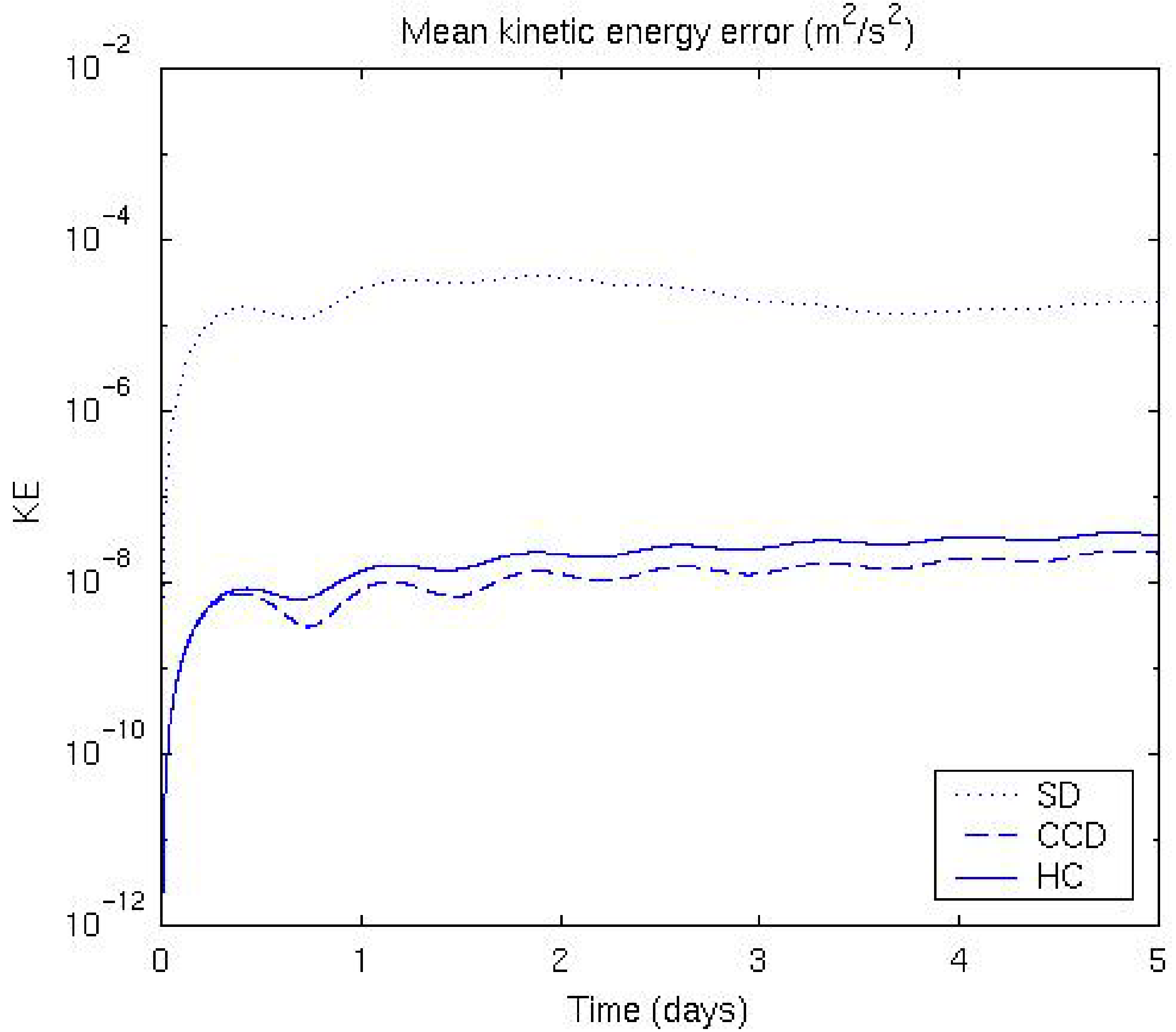
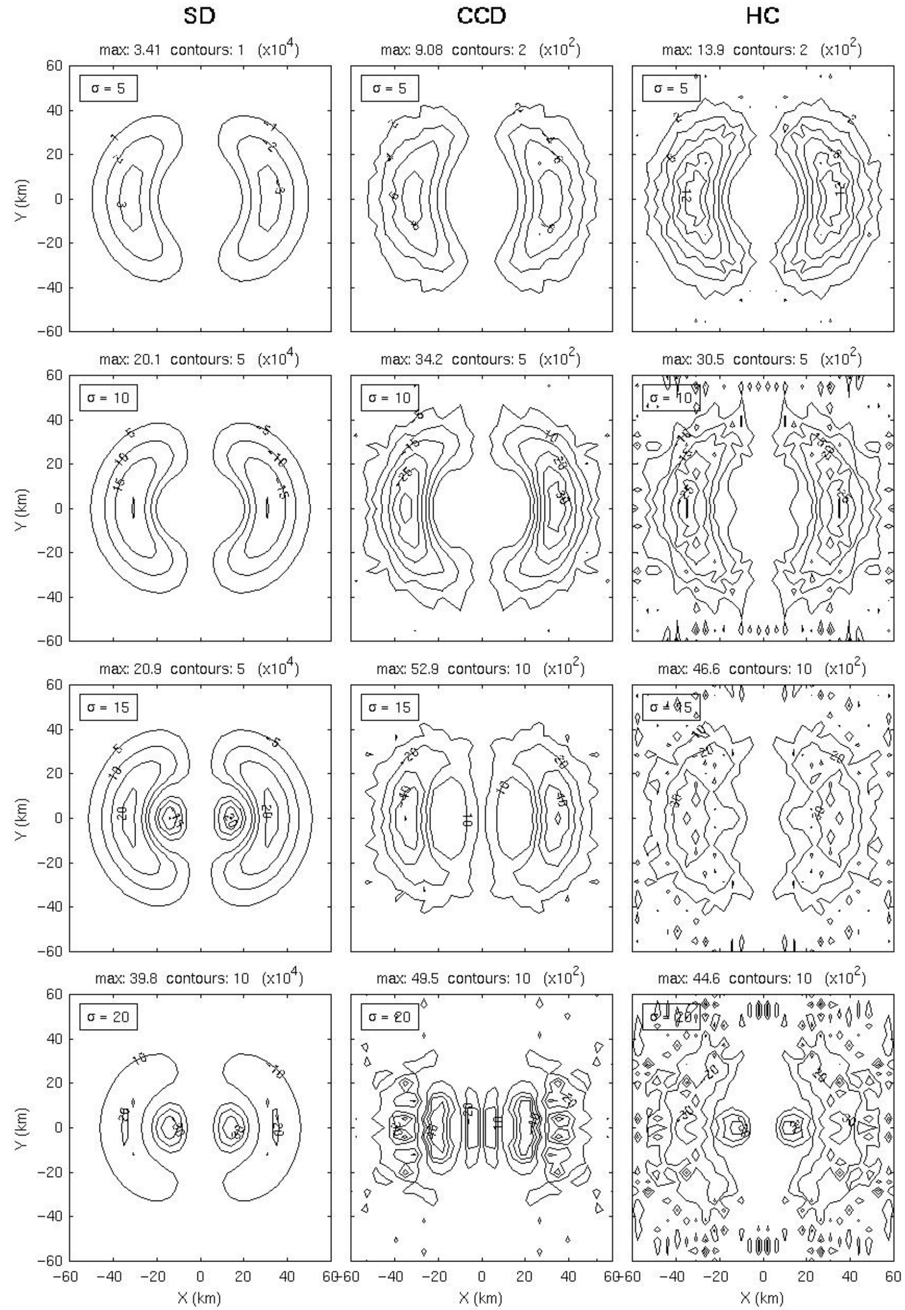
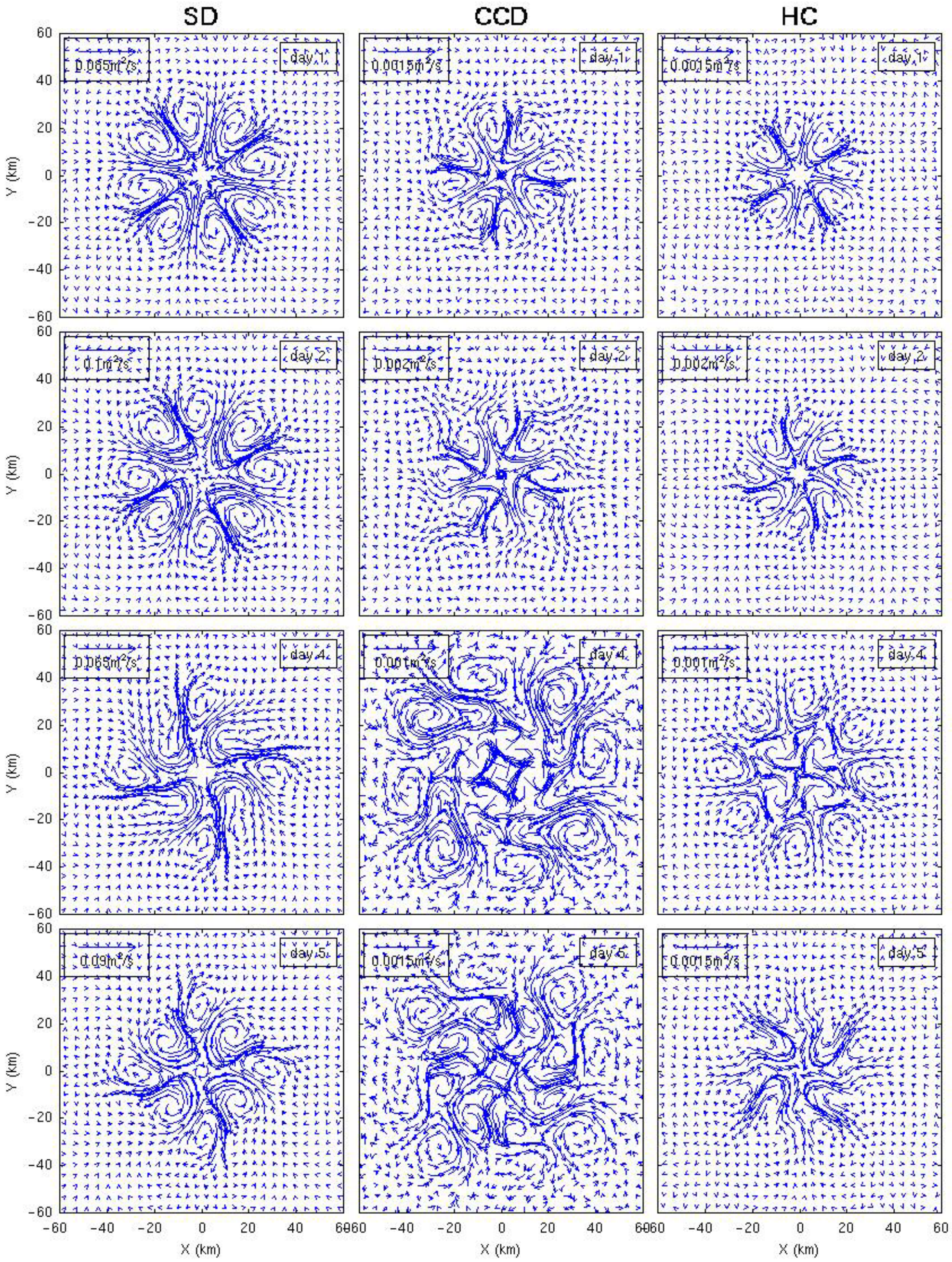


HC has the same accuracy as CCD Scheme




SD
CCD
HC
max: 3.41 contours: 1 (x10^4)
max: 9.08 contours: 2 (x10^2)
max: 13.9 contours: 2 (x10^2)
σ = 5
max: 20.1 contours: 5 (x10^4)
max: 34.2 contours: 5 (x10^2)
max: 30.5 contours: 5 (x10^2)
σ = 10
max: 20.9 contours: 5 (x10^4)
max: 52.9 contours: 10 (x10^2)
max: 46.6 contours: 10 (x10^2)
σ = 15
max: 39.8 contours: 10 (x10^4)
max: 49.5 contours: 10 (x10^2)
max: 44.6 contours: 10 (x10^2)
σ = 20
Y (km)
X (km)

- HC has the same accuracy as the sixth-order CCD scheme



CPU time (minute) at the end of 5 day runs of the POM seamount test case using the SD and HC schemes

Scheme	Second-order	HC
CPU Time	171.51	176.92
Ratio	1	1.03

Conclusions

- Hydrostatic correction scheme is a simple scheme with high accuracy and efficiency.
- The accuracy of the HC scheme is similar to the sixth-order CCD scheme.
- The CPU cost is similar to the ordinary second-order scheme.