Hydrodynamics of Falling Mine in Water Column

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Mine Impact Burial Prediction

- Urgent Navy Problem
- Complicated Scientific Problem

Naval Mine Threat

Inexpensive Force Multiplier

Roberts (FFG-58), Tripoli (LPH-10), Princeton (CG-59)
Damages $125 Million;
Mines Cost $30K

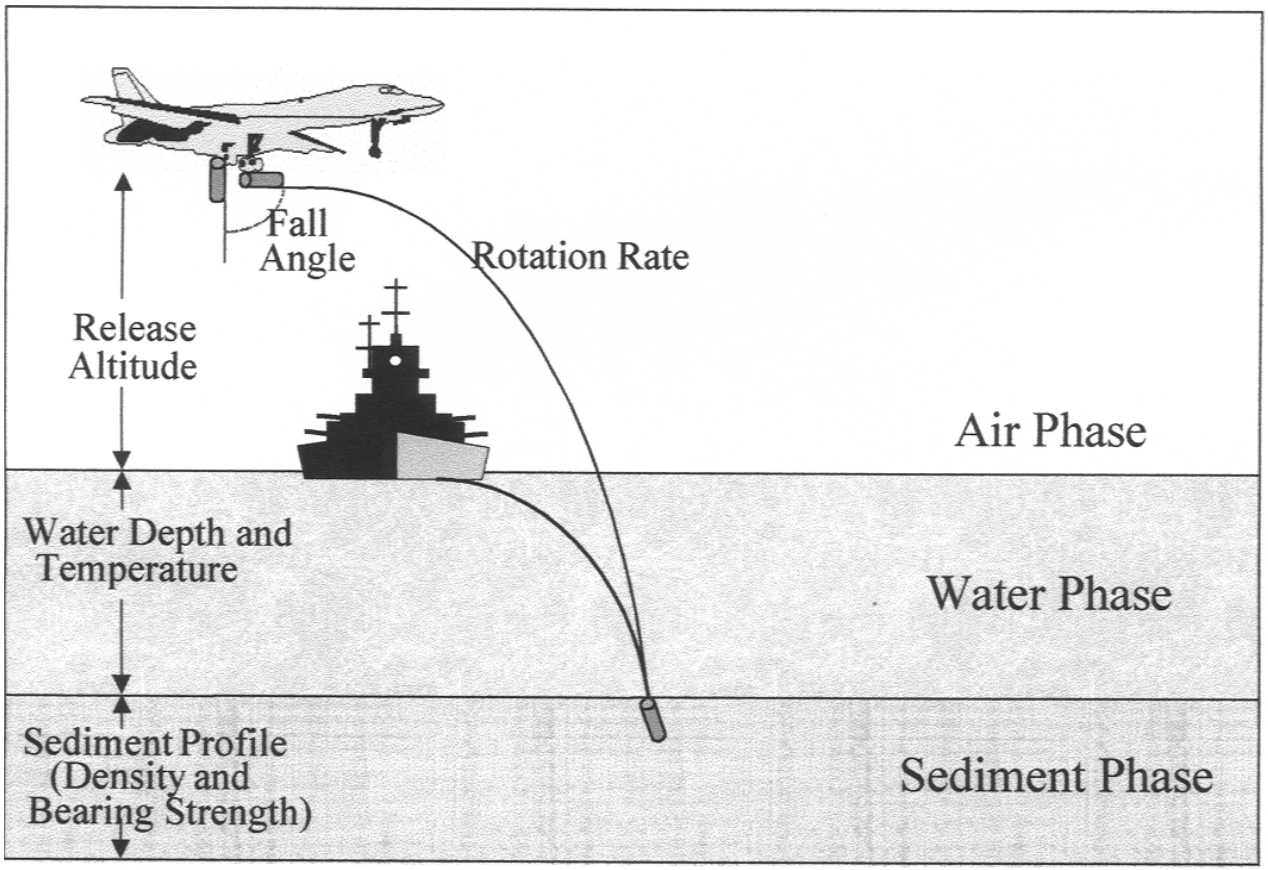
Widely Available

- Over 50 Countries
  (40% Increase in 10 Yrs)
  - Over 300 Types
    (75% Increase in 10 Yrs)
  - 32 Countries Produce
    (60% Increase in 10 Yrs)
  - 24 Countries Export
    (60% Increase in 10 Yrs)

Numerous Types

WWII Vintage to Advanced Technologies (Multiple Sensors, Ship Count Routines, Anechoic Coatings Non-Ferrous Materials)

Hydrodynamic Characteristics



Complicated Scientific Problem

- Body-Fluid Interaction
- Highly Nonlinear
- Chaotic Behavior

Development of Navy's Impact Burial Prediction Model (IBPM)

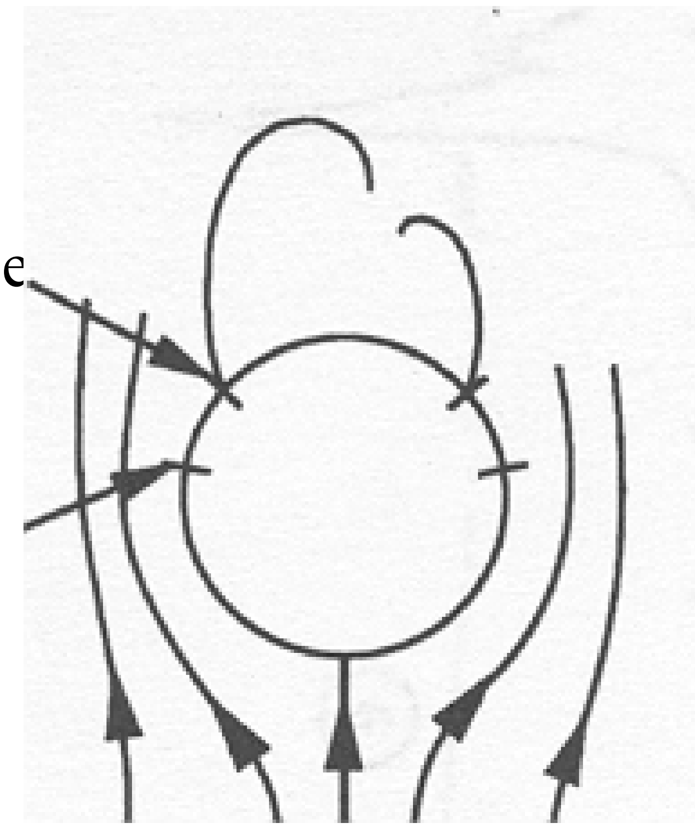
- IBPM was designed to calculate mine trajectories for air, water and sediment phases.
- Arnone & Bowen Model (1980) – Without Rotation.
- Improved IBPM (Satkowiak, 1987-88) – With Rotation.
- Improvements made by Hurst (1992): IMPACT25/28
- Sensitivity studies (Chu et al., 1999, 2000, Taber 1999, Smith 2000).

Key Non-Dimensional Numbers

- Reynolds Number
- Keulegan-Carpenter (KC) Number (Mine-Waves Interaction)
- Wave Period ~ 1 sec

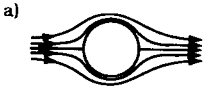
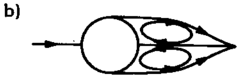
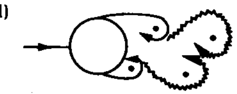
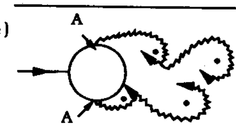
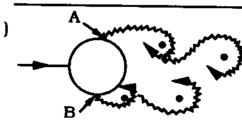
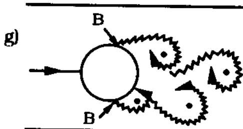
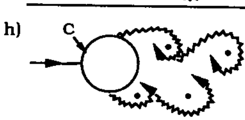
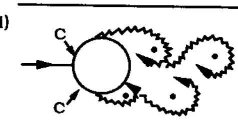
Flow Around the Falling Mine

- Turbulence
- Laminar



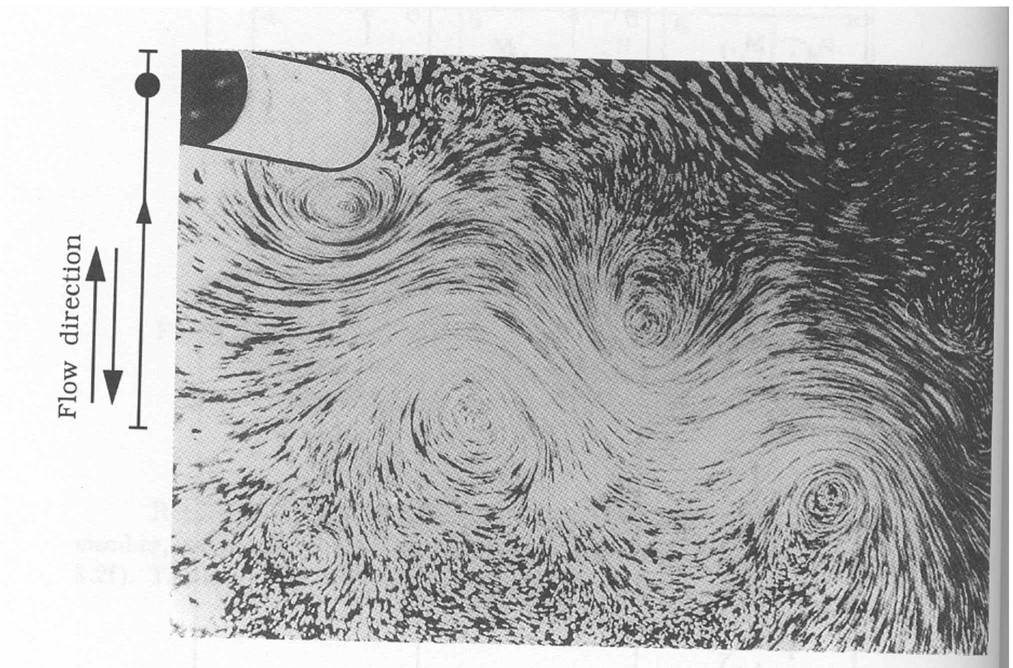
Falling Mine

- Reynolds Number Much Larger Than 300

a) 	No separation. Creeping flow	$Re < 5$
b) 	A fixed pair of symmetric vortices	$5 < Re < 40$
c) 	Laminar vortex street	$40 < Re < 200$
d) 	Transition to turbulence in the wake	$200 < Re < 300$
e) 	Wake completely turbulent. A: Laminar boundary layer separation	$300 < Re < 3\times10^5$ Subcritical
f) 	A: Laminar boundary layer separation B: Turbulent boundary layer separation; but boundary layer laminar	$3\times10^5 < Re < 3.5\times10^5$ Critical (Lower transition)
g) 	B: Turbulent boundary layer separation; the boundary layer partly laminar partly turbulent	$3.5\times10^5 < Re < 1.5\times10^6$ Supercritical
h) 	C: Boundary layer completely turbulent at one side	$1.5\times10^6 < Re < 4\times10^6$ Upper transition
i) 	C: Boundary layer completely turbulent at two sides	$4\times10^6 < Re$ Transcritical

Falling Mine

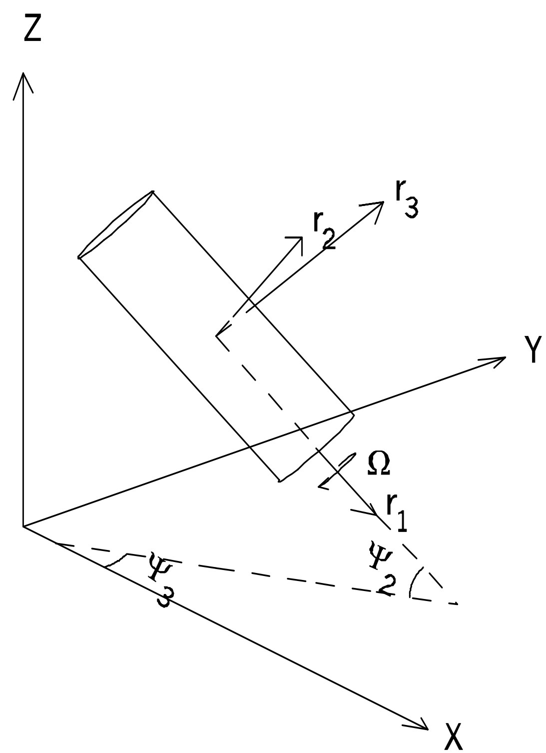
- KC = 12 (Vortex Shedding)



Chaotic Features

- Basic equations (6 unknowns) are nonlinear, similar to Lorenz system

Six Unknowns



Momentum Equations

$$\frac{dV_1}{dt} + \omega_2 V_3 - \omega_3 V_2 = \frac{\rho - \rho_w}{\rho} \sin \psi_2 + \frac{F_1^*}{\rho \Pi}$$

$$\frac{dV_2}{dt} + \omega_3 V_1 = \frac{F_2^*}{\rho \Pi}$$

$$\frac{dV_3}{dt} - \omega_2 V_1 = -\frac{\rho - \rho_w}{\rho} \cos \psi_2 + \frac{F_3^*}{\rho \Pi}$$

Moment of Momentum Equations

$$\frac{d\Omega}{dt} + \frac{J_3 - J_2}{J_1}\omega_2\omega_3 = \frac{LM_1^*}{gJ_1}$$

$$\frac{d\omega_2}{dt} = \frac{\chi\Pi(\rho_w - \rho)L}{J_2}\cos\psi_2 + \frac{LM_2^*}{gJ_2}$$

$$\frac{d\omega_3}{dt} = \frac{LM_3^*}{gJ_3}$$

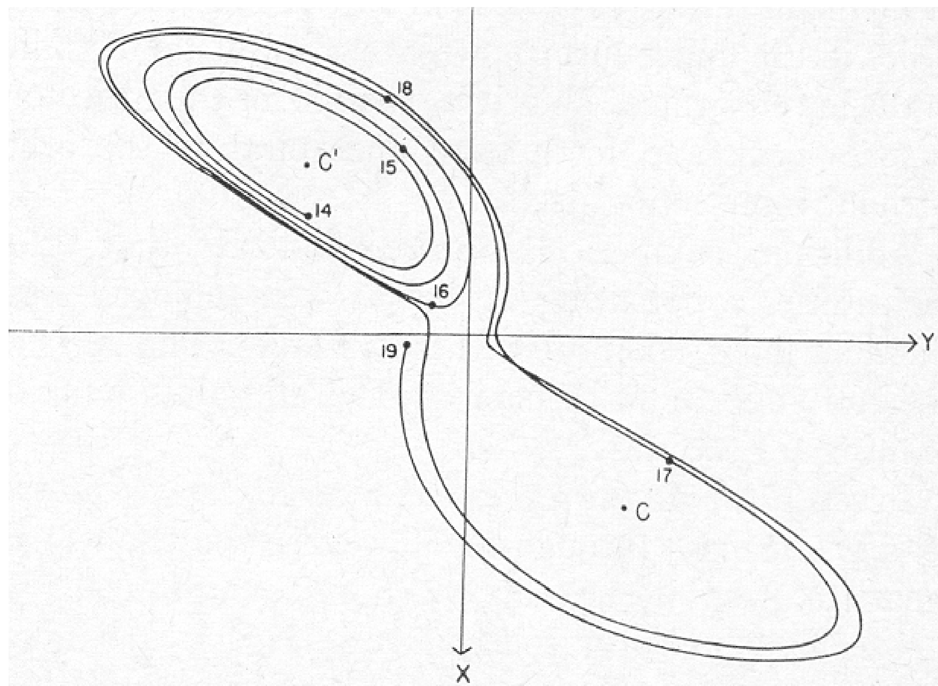
Lorenz System (1963)

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

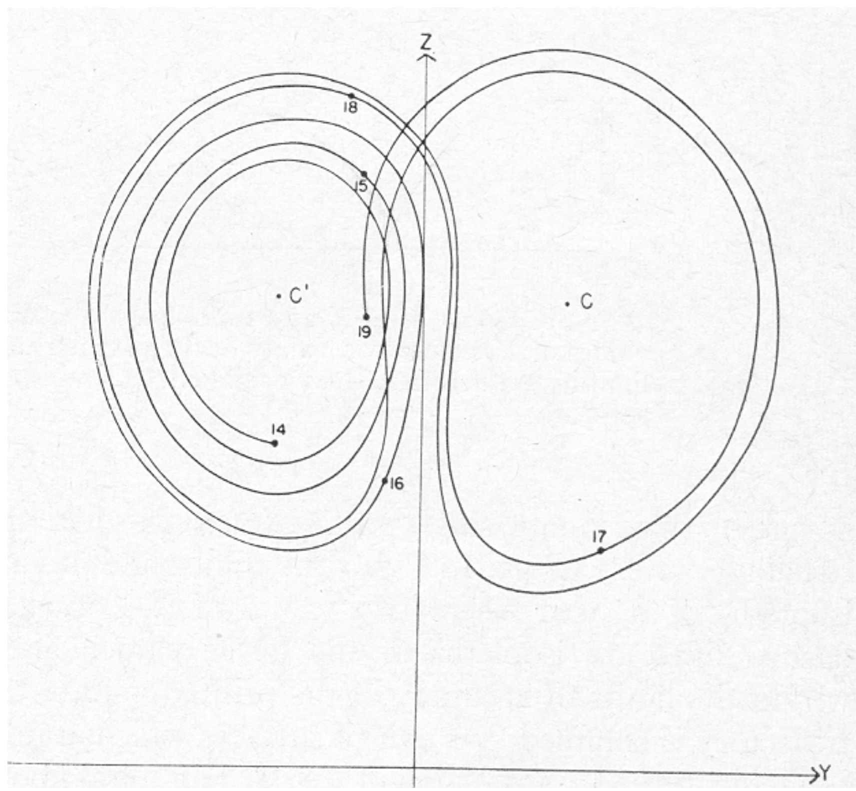
$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ.$$

Chaos (Butterfly Pattern) from the Lorenz System



Chaos (Butterfly Pattern) from the Lorenz System



Mine Drop Experiment (MIDEX)

- Hydrodynamic Model Development
- Behavior of Falling Mine in Water Column

  (Chaotic, Turbulent Wake, Eddy Shedding)

Model Evaluation

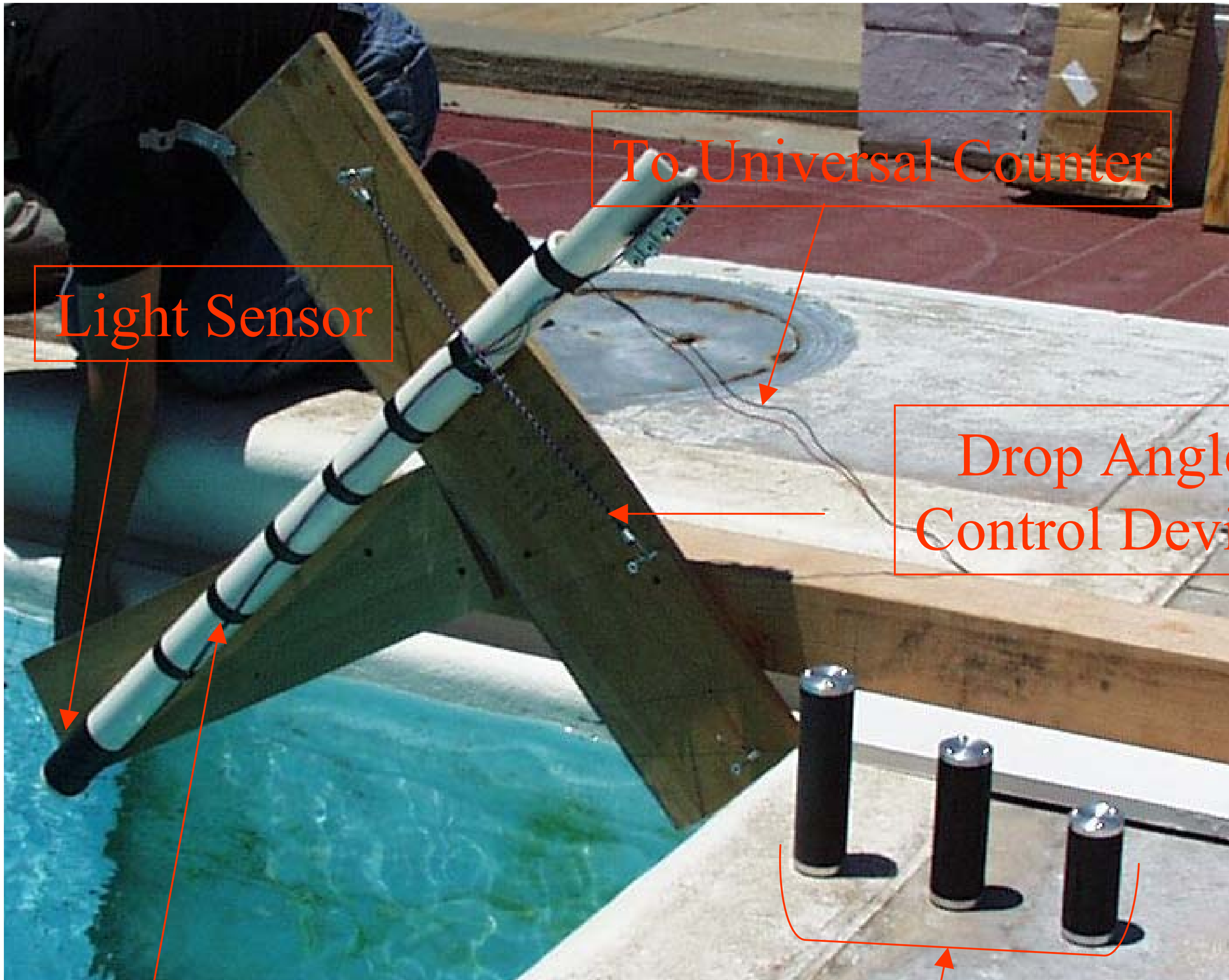
Mine Drop Experiment (MIDEX)

- Mine Parameters:

1. Density Ratio (1.68, 1.70, 1.88)
2. Center of Mass Position.
3. L/D ratio.

- Drop Parameters:

1. Drop Angles: 15º, 30º, 45º, 60º, 75º.
2. Release Velocity V_{init}



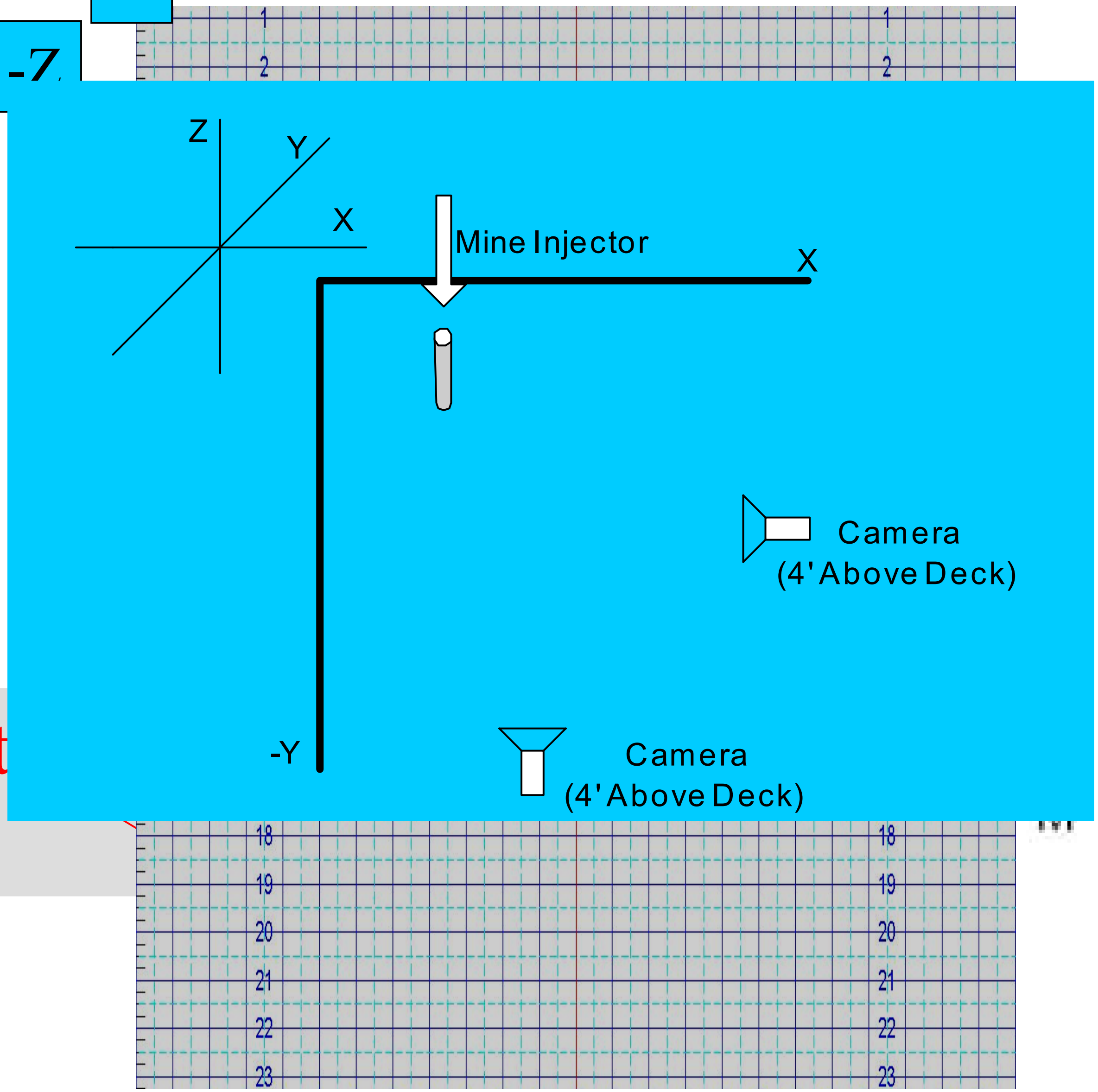
Mine Injector

Mine Shapes:
Length: 15, 12, 9 cm
Diameter: 4 cm

Coordinate System

X

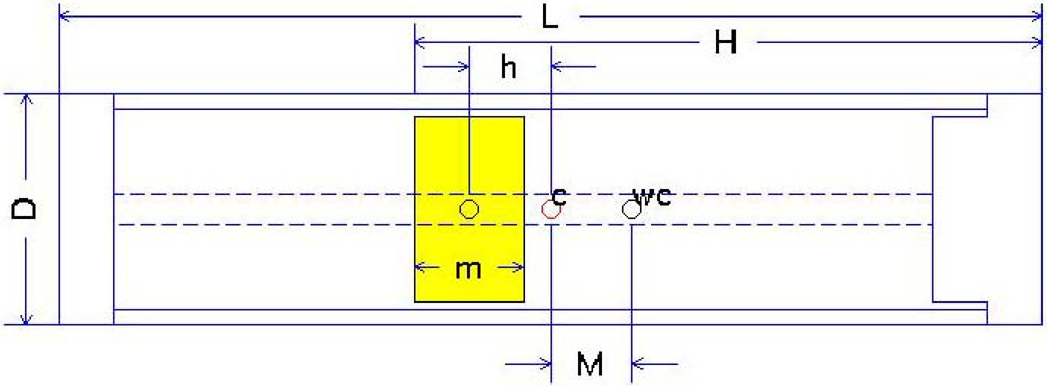
-Z



Mine Attit

(Psi)

Center of Mass



MODEL # 1

L=15.1359cm D=4cm m=2.7cm

Weight=322.5 g Volume=190.2028 cm^3 Density=1.6956 g/cm^3

H:	10.380	8.052	5.725	cm
h:	-1.462	0.866	3.193	cm
M:	0.000	18.468	36.935	mm

MODEL # 2

L=12.0726cm D=4cm m=1.7cm

Weight=254.2 g Volume=151.709 cm^3 Density=1.6756 g/cm^3

H:	8.450	6.609	4.768	cm
h:	-1.564	0.277	2.119	cm
M:	0.000	12.145	24.290	mm

MODEL # 3

L=9.1199cm D=4cm m=1.47cm

Weight=215.3 g Volume=114.6037 cm^3 Density=1.8786 g/cm^3

H:	6.662	5.592	4.521	cm
h:	-1.368	-0.297	0.774	cm
M:	0.000	6.847	13.694	mm

Defined COM position as:

2 or -2: Farthest from volumetric center

1or -1

0: Coincides with volumetric center

Hydrodynamic Theory

- Solid Body Falling Through Fluid Should Obey 2 Physical Principles:

1. Momentum Balance

$$\int (dV^* / dt^*) dm^* = W^* + F_b^* + F_d^*$$

2. Moment of Momentum Balance

$$\int [r^* \times (dV^* / dt^*)] dm^* = M^*$$

* Denotes dimensional variables

V^* → Velocity

W^* → gravity

F_b^* → buoyancy force

F_d^* → drag force

M^* → resultant moment

Data Analysis

1. Video converted to digital format.
2. Digital video from each camera analyzed frame by frame (30Hz) using video editing program.
3. Mine's top and bottom position determined using background x-z and y-z grids. Positions manually entered into MATLAB for storage and later processing.
4. Analyzed 2-D data to obtain mine's x,y and z center positions, attitude (angle with respect to z axis) and u,v, and w components.

Non-dimensional Conversions

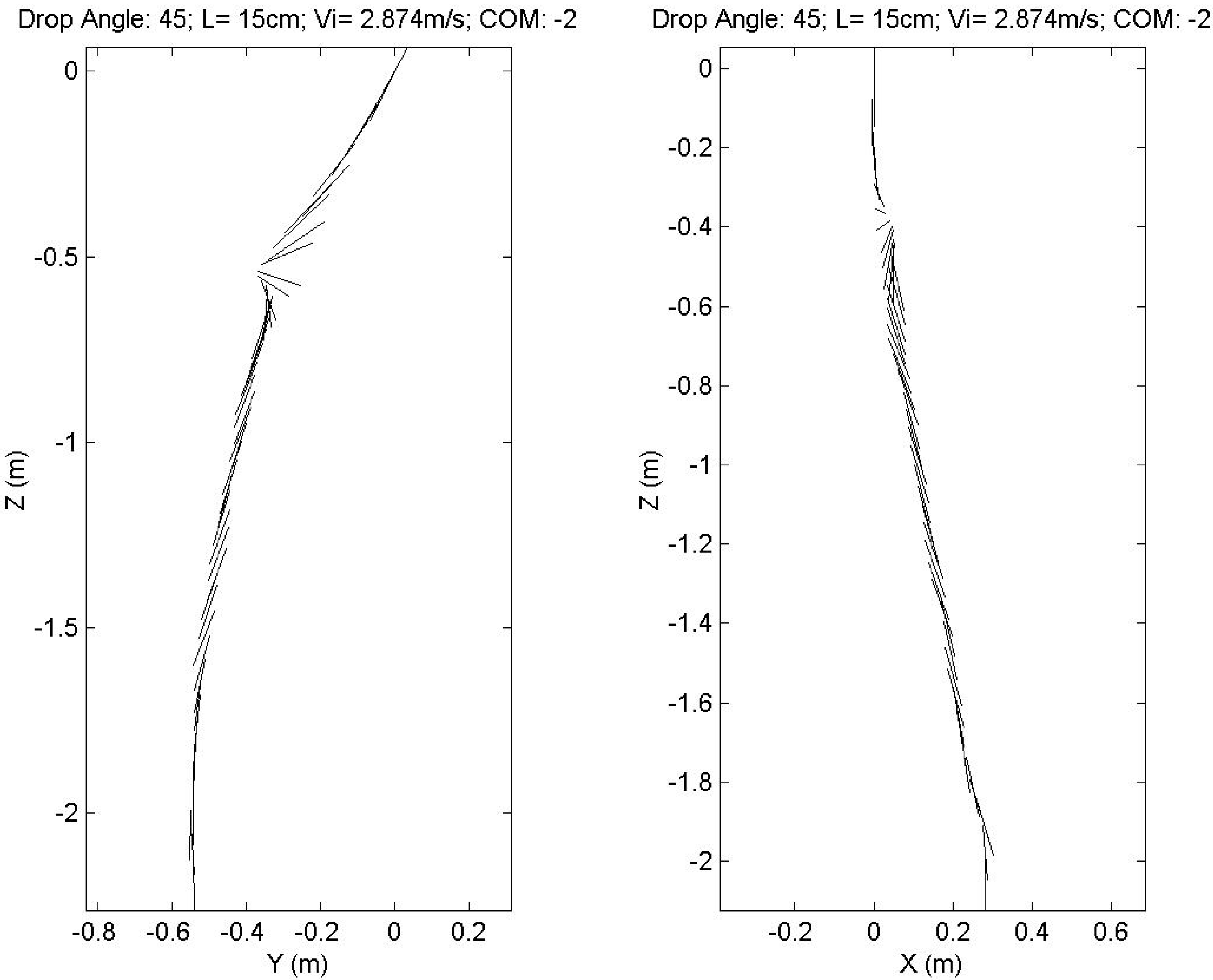
- In order to generalize results, data was converted to non-dimensional numbers.

Sources of Error

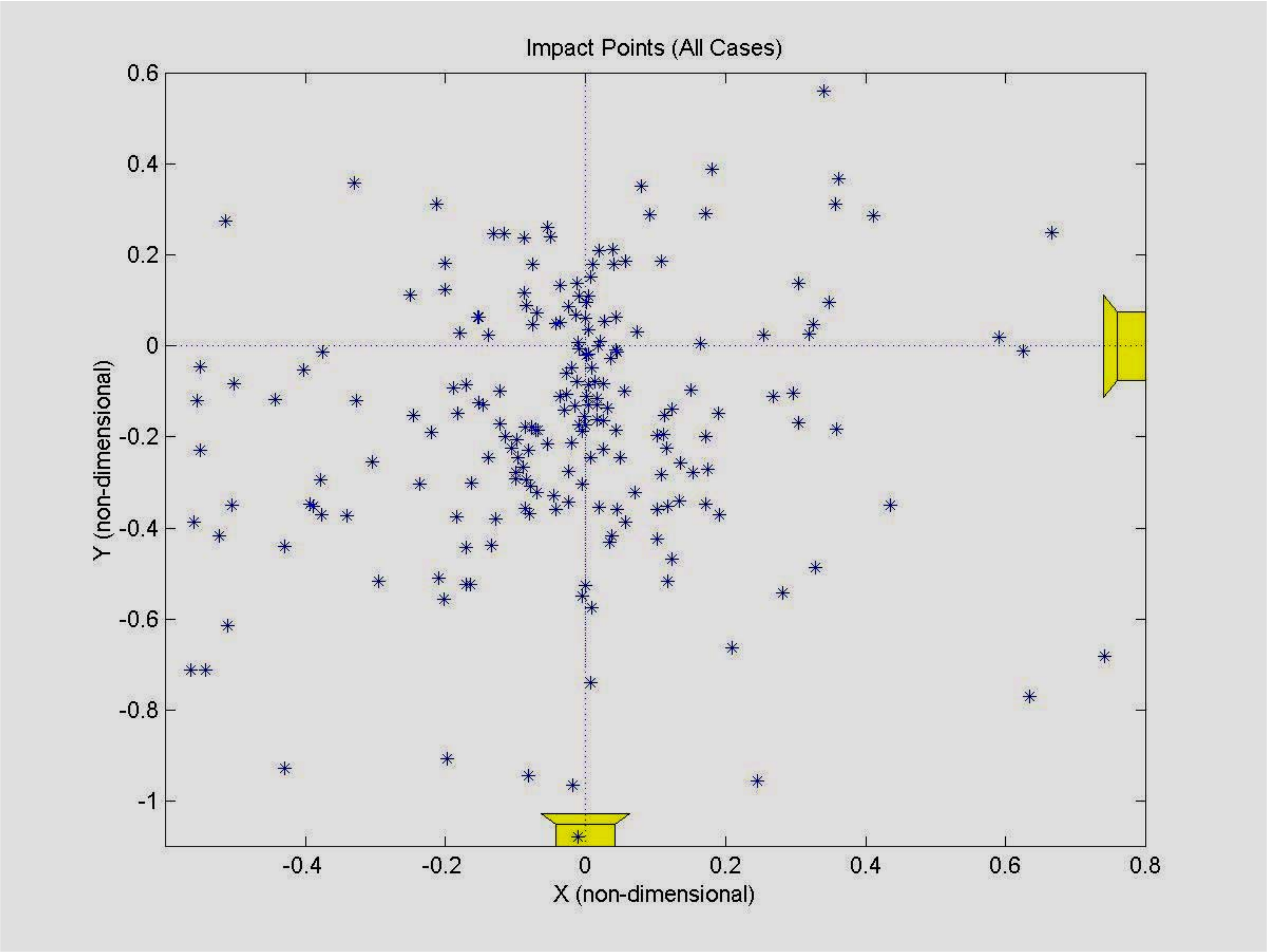
1. Grid plane behind mine trajectory plane. Results in mine appearing larger than normal.
2. Position data affected by parallax distortion and binocular disparity.
3. Air cavity affects on mine motion not considered in calculations.
4. Camera plane not parallel to x-y plane due to pool slope.

Underwater Video Clip

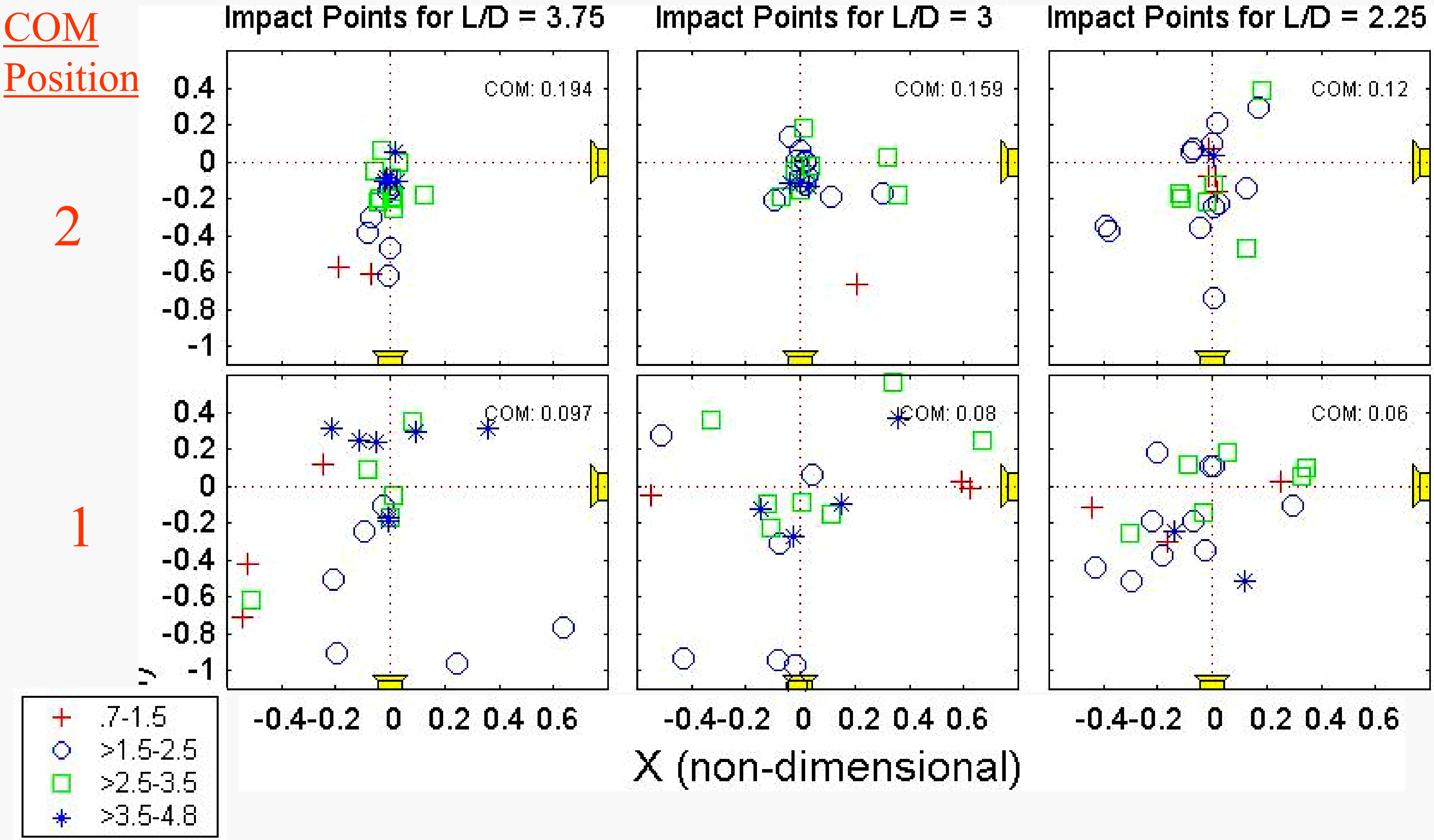
Center of Mass: Position 2

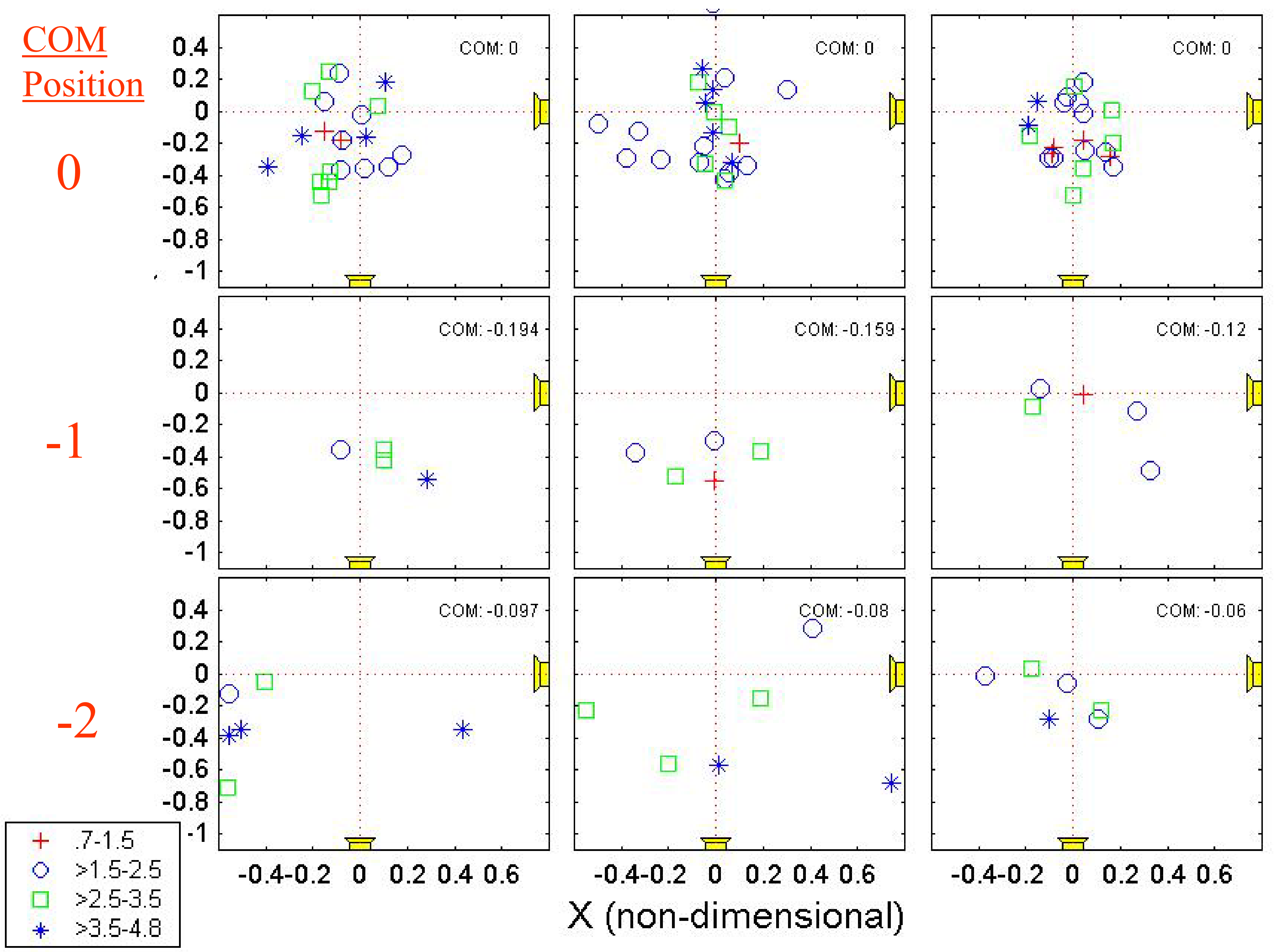

Drop Angle: 45; L= 15cm; Vi= 2.874m/s; COM: -2
Z (m)
Y (m)
0
-0.5
-1
-1.5
-2
-0.8
-0.6
-0.4
-0.2
0
0.2
Drop Angle: 45; L= 15cm; Vi= 2.874m/s; COM: -2
Z (m)
X (m)
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
-0.2
0
0.2
0.4
0.6

Impact Point (All Cases)



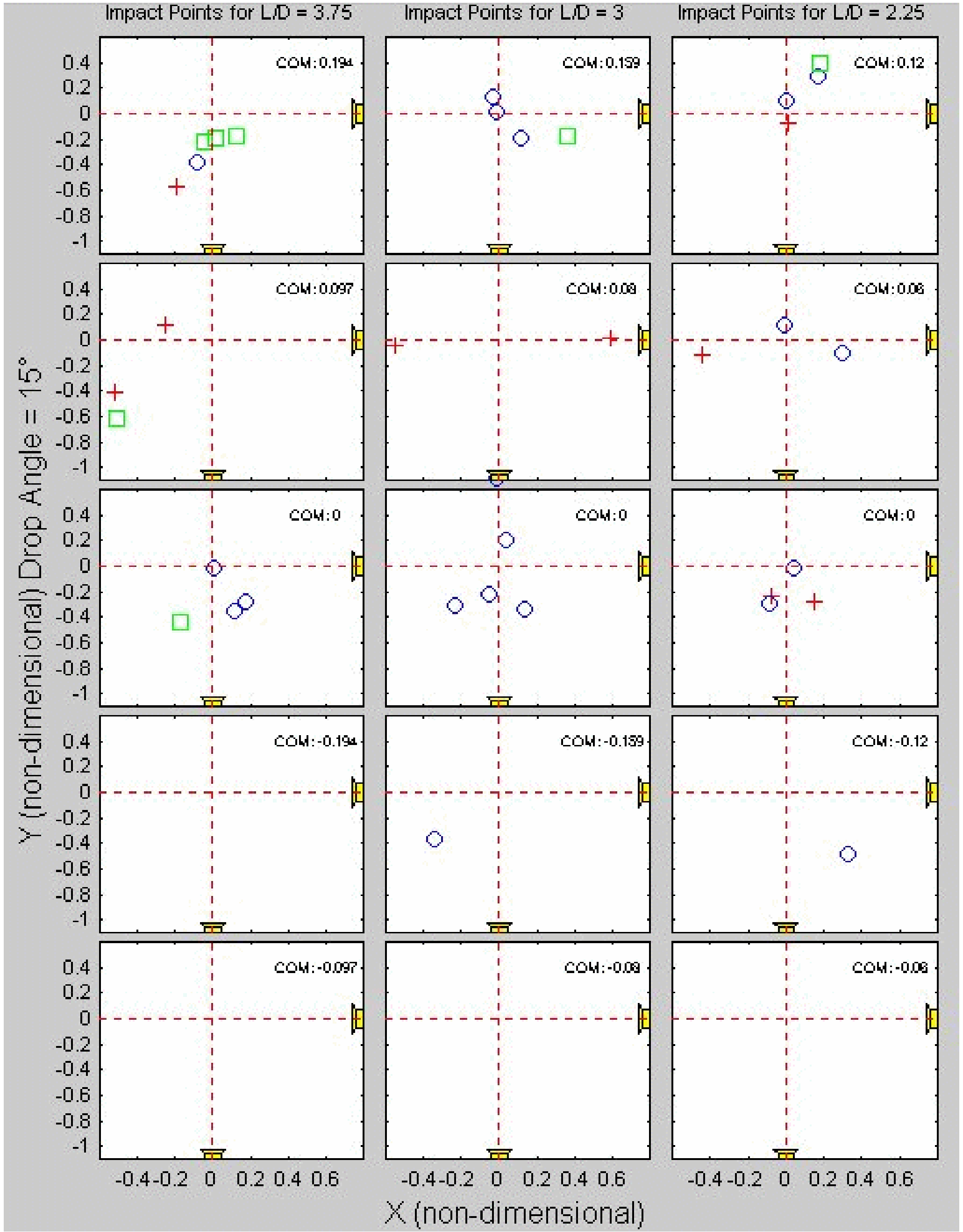
Impact Point (All Drop Angles)

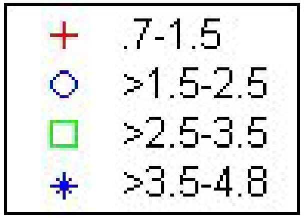




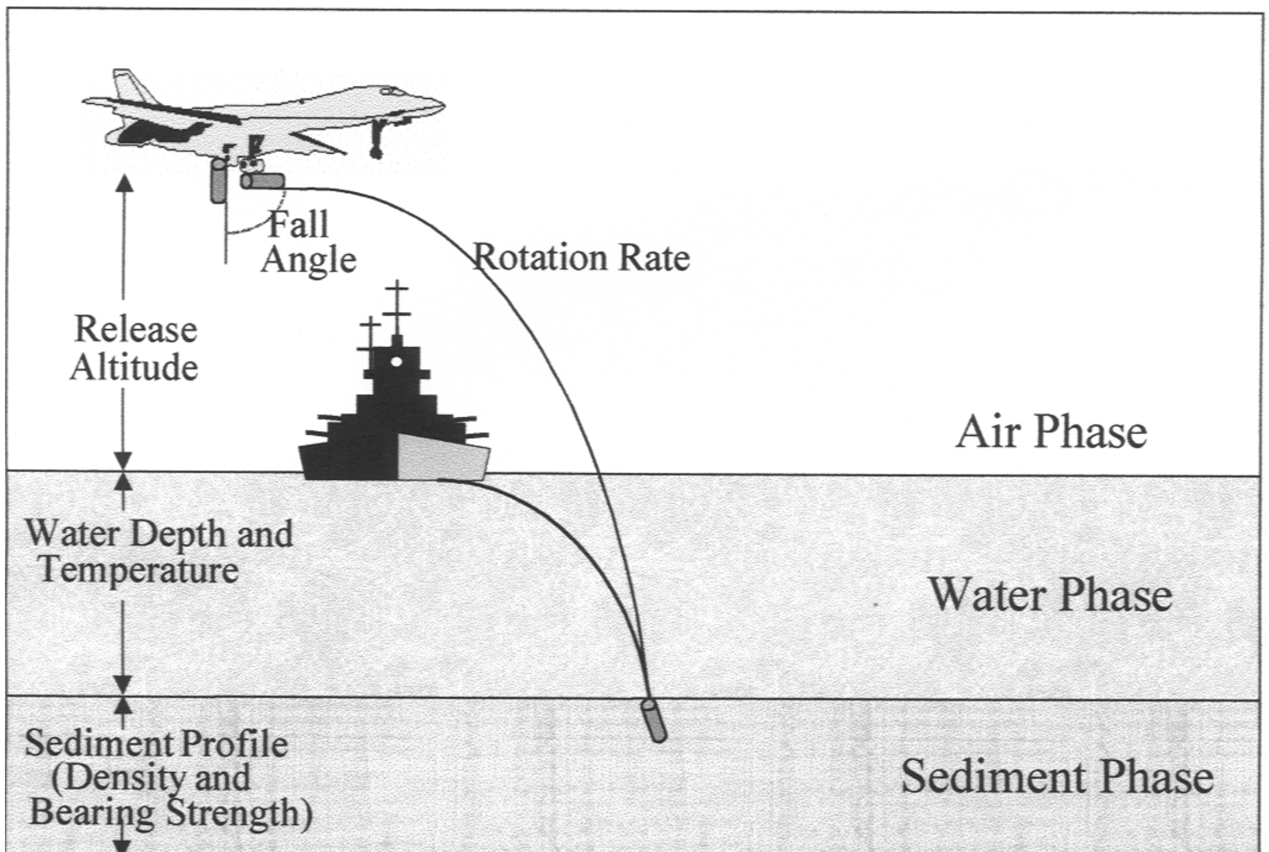
COM Position
0
-1
-2
COM: 0
COM: 0
COM: 0
COM: -0.194
COM: -0.159
COM: -0.12
COM: -0.097
COM: -0.08
COM: -0.06
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.4 -0.2 0 0.2 0.4 0.6
X (non-dimensional)
.7-1.5
>1.5-2.5
>2.5-3.5
>3.5-4.8

Impact Point (By Angle)





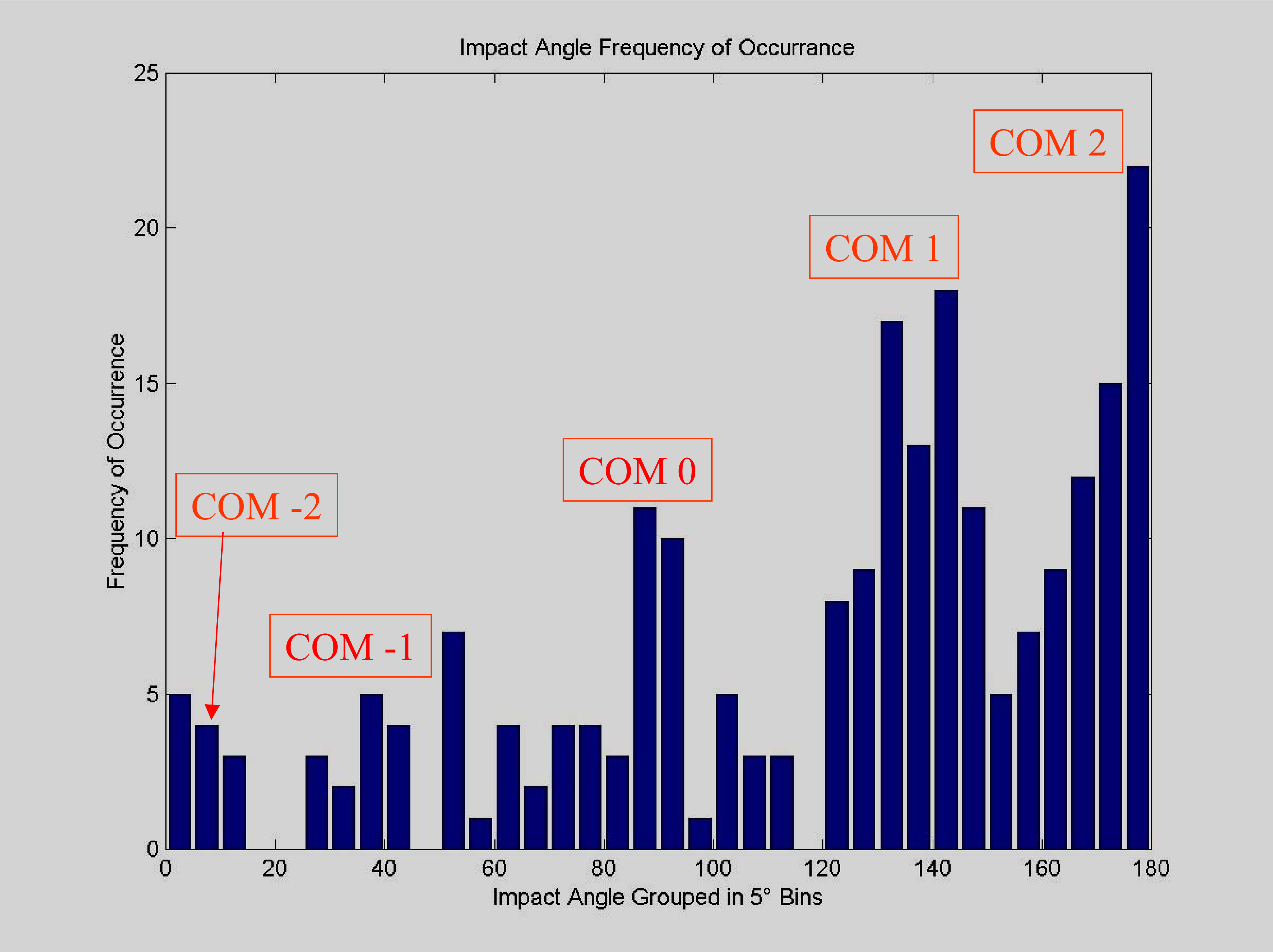
Impact Angle (Falling Angle Relative to Vertical Axis)



Impact Angle

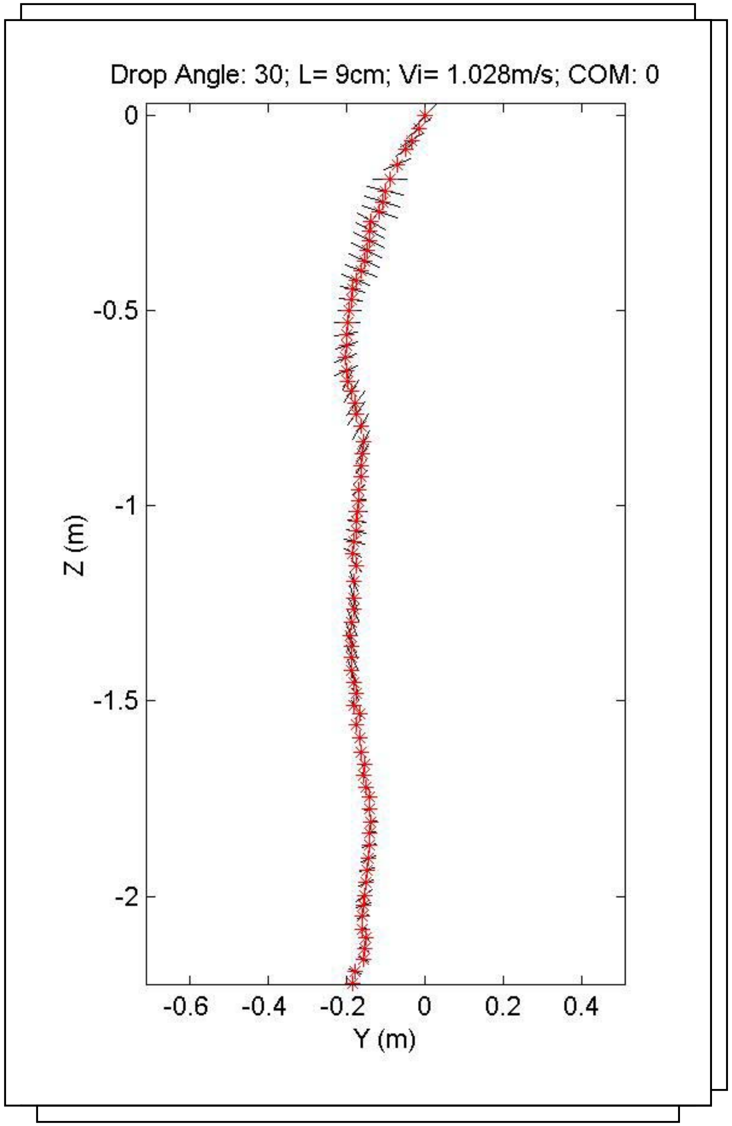
- Vertical (0^o or 180^o)
- Horizontal (90^o)

Impact Angle Frequency of Occurrence



Trajectory Patterns

1. Straight
2. Slant
3. Spiral
4. Flip
5. Flat
6. See Saw
7. Combination



Multiple Linear Regression

- General Multiple Linear Regression Equation:

$$f_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \varepsilon_i$$

- Used least squares solution to determine correlation coefficients.
- Input: cos(drop angle); L/D; V_{ind}; COM_{nd}
- Output: (x_m, y_m, z_m, Psi, u, v, w)

Multiple Regression Results

	x_m	y_m	Psi	u	v	w
β_0	-.0746	-.0546	102.5691	.0040	-.0135	-.9481
β_1	.1190	-.0828	-13.3508	-.0075	-.0106	-.1080
β_2	-.0469	-.0798	-.5009	-.0011	.0005	.0295
β_3	.0372	.0622	1.0437	.0025	.0011	-.0221
β_4	.2369	.4330	472.2135	-.0090	.0537	-1.2467

- Most important parameter for impact prediction is Psi (impact angle).

Check of regression equation:

Determine Psi for case where:

L=15cm, V_i = 3m/s, COM = 2, Drop Angle = 15°

Yields: Psi = 181.2°

For COM = 1: Psi = 136.1°

For COM = 0: Psi = 90.4°

Conclusion

- COM position is the most influential parameter for predicting a mine's impact position and angle.
- Final velocities were lowest for COM 0 cases due to the increased effect of hydrodynamic drag.
- Trajectories became more complex as L/D decreased (9 cm mine rotated about z-axis).
- Observed trajectory patterns were more complex than those assumed by IMPACT 25/28. Accurate representation of a mine's water phase motion requires both momentum and moment of momentum equations.