A Finite Volume Coastal Ocean Model

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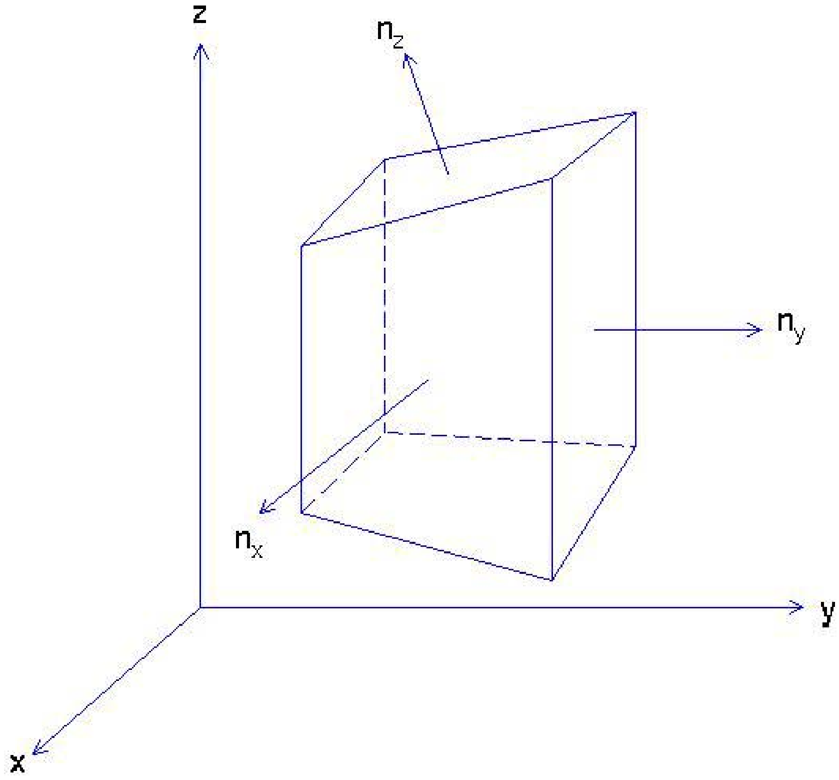
Four Types of Numerical Models

- Spectral Model (not suitable for oceans due to irregular lateral boundaries)
- Finite Difference (z-coordinate, sigma-coordinate, …)
- Finite Element
- Finite Volume

Finite Volume



Finite Volume Model

- Transform of PDE to Integral Equations
- Solving the Integral Equation for the Finite Volume
- Flux Conservation

Dynamic and Thermodynamic Equations

- Continuity

$$\nabla \cdot (\rho \mathbf{V}) = 0$$

- Momentum

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V}\mathbf{V}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{V}) + \mathbf{F}$$

- Thermodynamic

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{V}\phi) = \nabla \cdot (\kappa_\phi \nabla \phi) + F_\phi$$

Integral Equations for Finite Volume

- Continuity

$$\int_{\Omega} \nabla \cdot (\rho \mathbf{V})\, d\Omega = \oint_{\Gamma} \rho \mathbf{V} \cdot \mathbf{n} d\Gamma = \mathbf{0}$$

- Momentum

$$\int_{\Omega} \frac{\partial(\rho \mathbf{V})}{\partial t} d\Omega + \oint_{\Gamma} \rho \mathbf{V}\mathbf{V} \cdot \mathbf{n} d\Gamma = -\oint_{\Gamma} p d\Gamma + \oint_{\Gamma} \mu \nabla \mathbf{V} \cdot \mathbf{n} d\Gamma + \int_{\Omega} \mathbf{F} d\Omega$$

- Thermodynamic

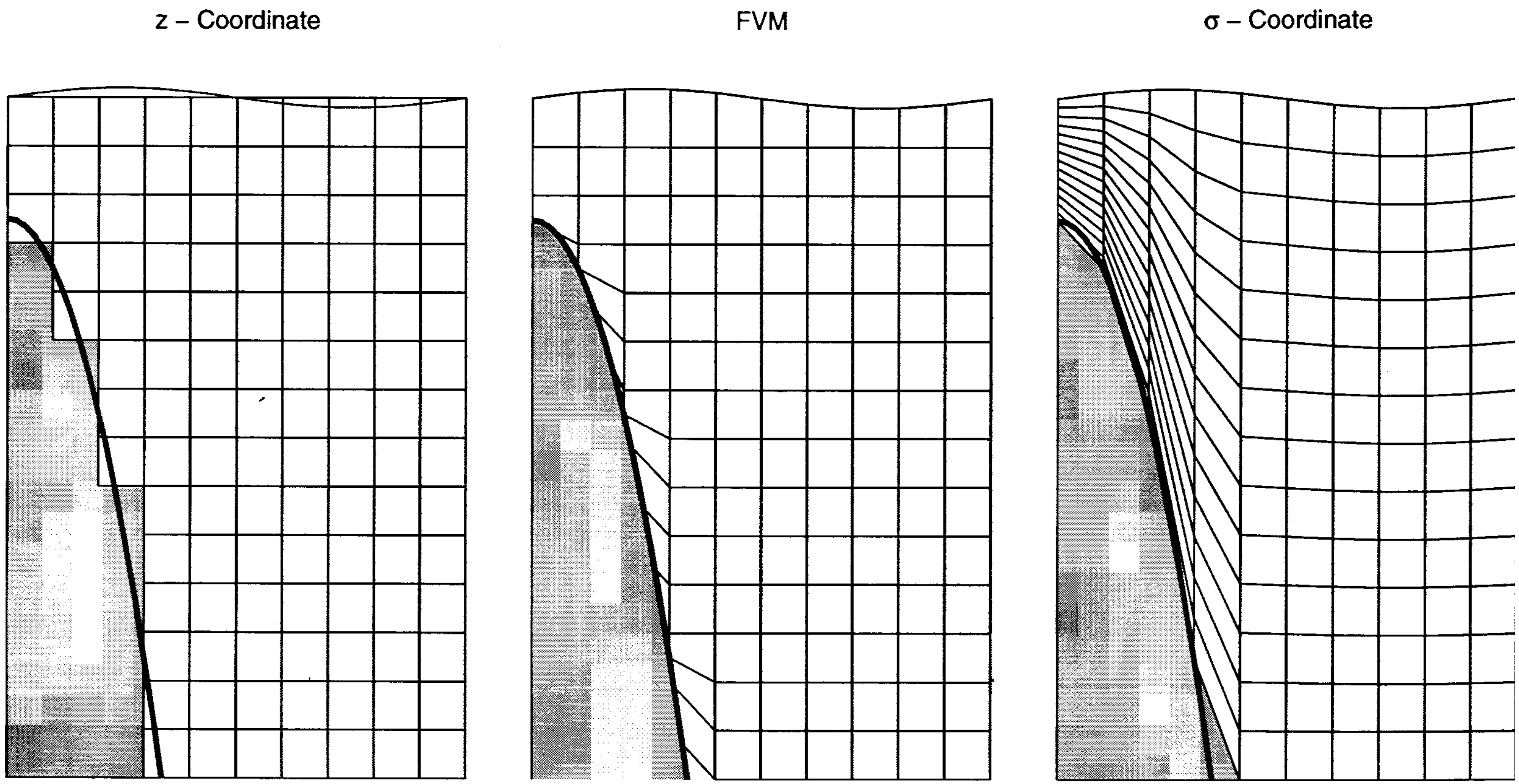
$$\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega + \oint_{\Gamma} \phi \mathbf{V} \cdot \mathbf{n} d\Gamma = \oint_{\Gamma} \kappa_{\phi} \nabla \phi \cdot \mathbf{n} d\Gamma + \int_{\Omega} F_{\phi} d\Omega$$

Time Integration of Phi-Equation

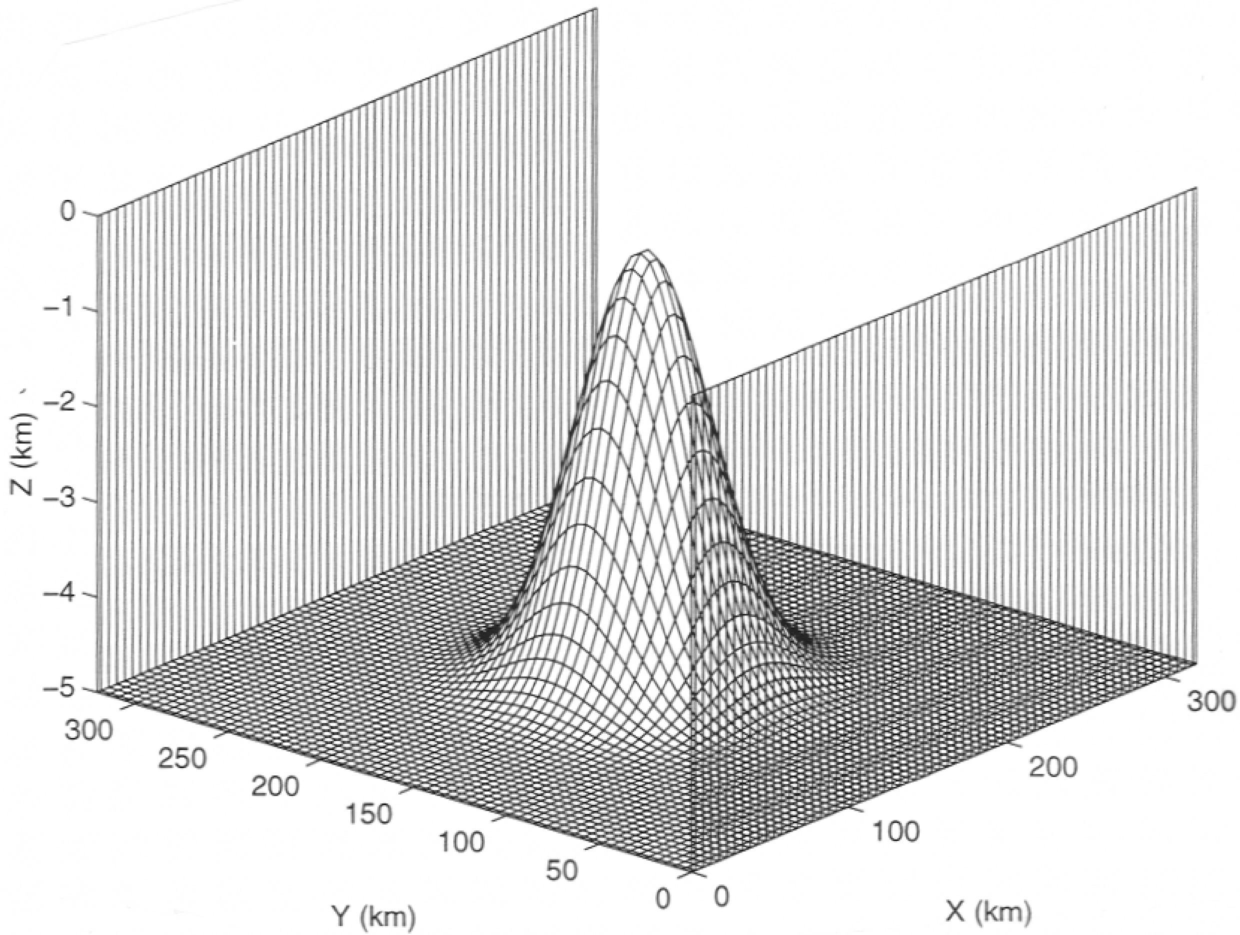
$$\int_{\Omega} \phi(t_2) d\Omega - \int_{\Omega} \phi(t_1) d\Omega = - \Delta t \oint_{\Gamma} \phi(t^*) \mathbf{V} \cdot \mathbf{n} d\Gamma$$

$$+ \Delta t \oint_{\Gamma} \kappa_{\phi} \nabla \phi(t^*) \cdot \mathbf{n} d\Gamma + \Delta t \int_{\Omega} F_{\phi}(t^*) d\Omega \text{ -}$$

Comparison Between Finite Difference (z- and sigma-coordinates) and Finite Volume Schemes



Seamount Test Case



Initial Conditions

- $V = 0$
- $S = 35$ ppt

$$T(z) = 5 + 15\exp(\frac{z}{H_T}) \qquad \text{(unit: °C)}$$

- $H_T = 1000$ m

Known Solution

- $V = 0$
- Horizontal Pressure Gradient = 0

Evaluation

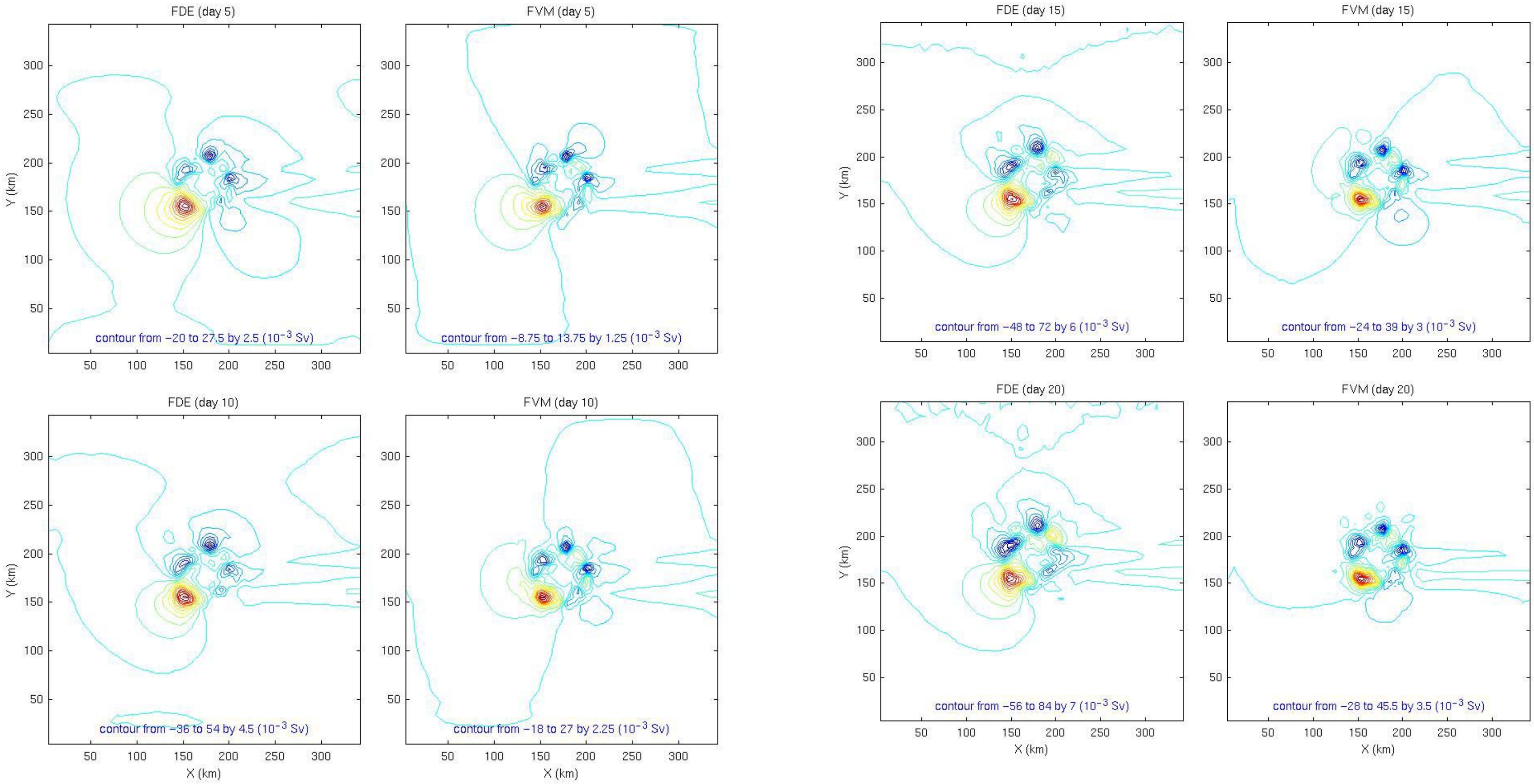
- Princeton Ocean Model
- Seamount Test Case

- Horizontal Pressure Gradient (Finite Difference and Finite Volume)

Numerics and Parameterization

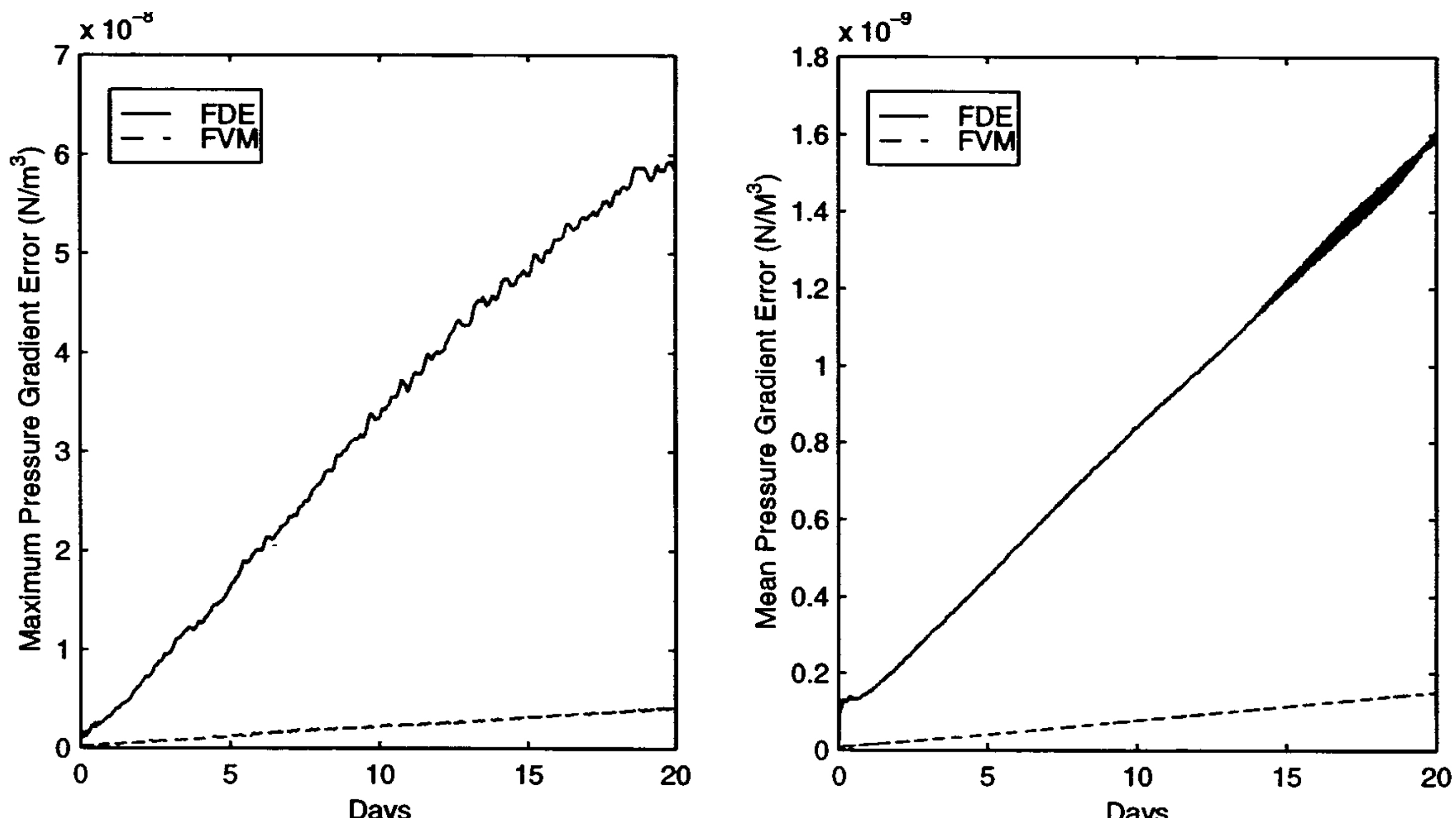
- Barotropic Time Step: 6 s
- Baroclinic Time Step: 180 s
- Delta x = Delta y = 8 km
- Vertical Eddy Viscosity: Mellor-Yamada Scheme
- Horizontal Diffusion: Samagrinsky Scheme with the coefficient of 0.2

Error Volume Transport Streamfunction



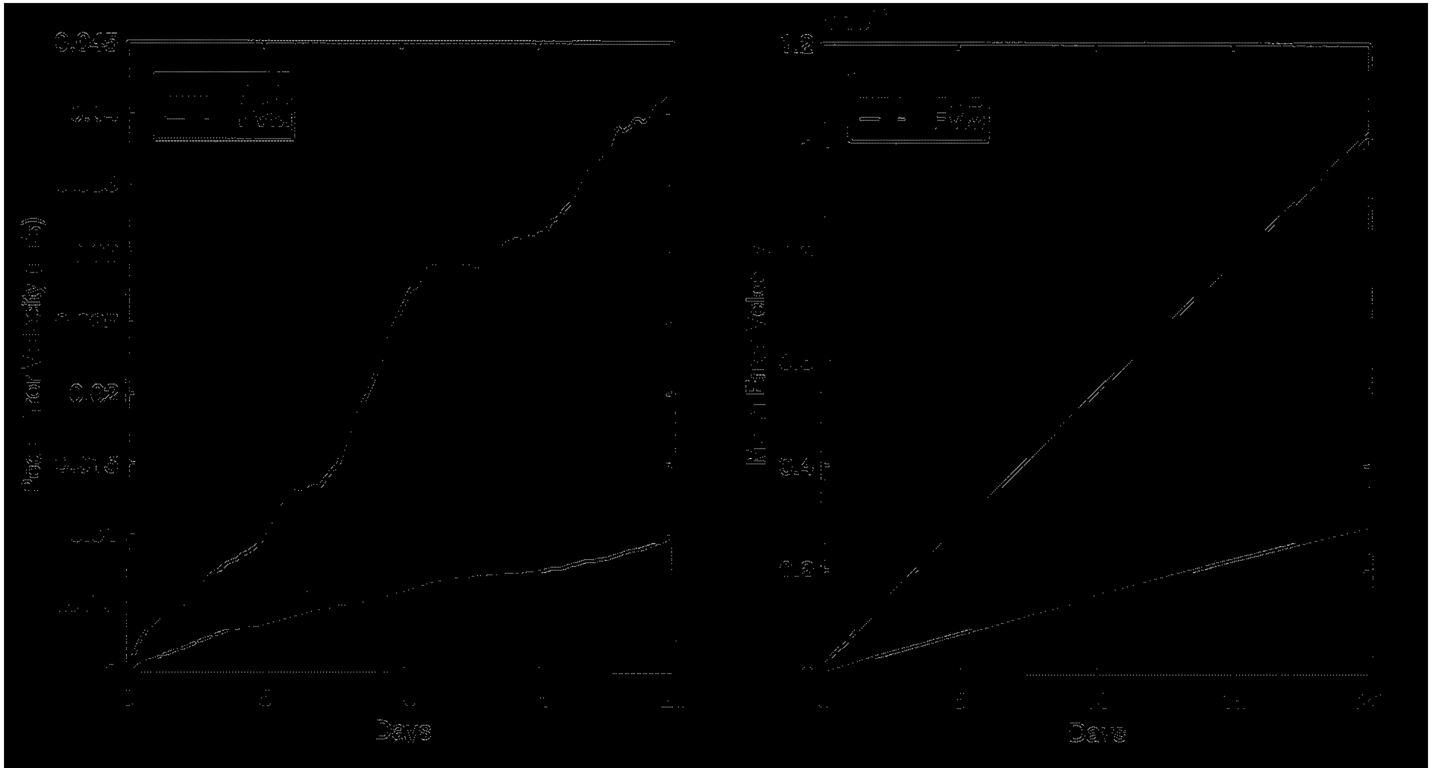
Temporally Varying Horizontal Gradient Error

- The error reduction by a factor of 14 using the finite volume scheme.



Temporally Varying Error Velocity

- The error velocity reduction by a factor of 4 using the finite volume scheme.



Conclusions

- Use of the finite volume model has the following benefit:

  (1) Computation is as simple as the finite difference scheme.

  (2) Conservation on any finite volume.

  (3) Easy to incorporate high-order schemes

  (4) Upwind scheme