Determination of Inter-Hemispheric Water Exchange Using the Ekman-Munk Model

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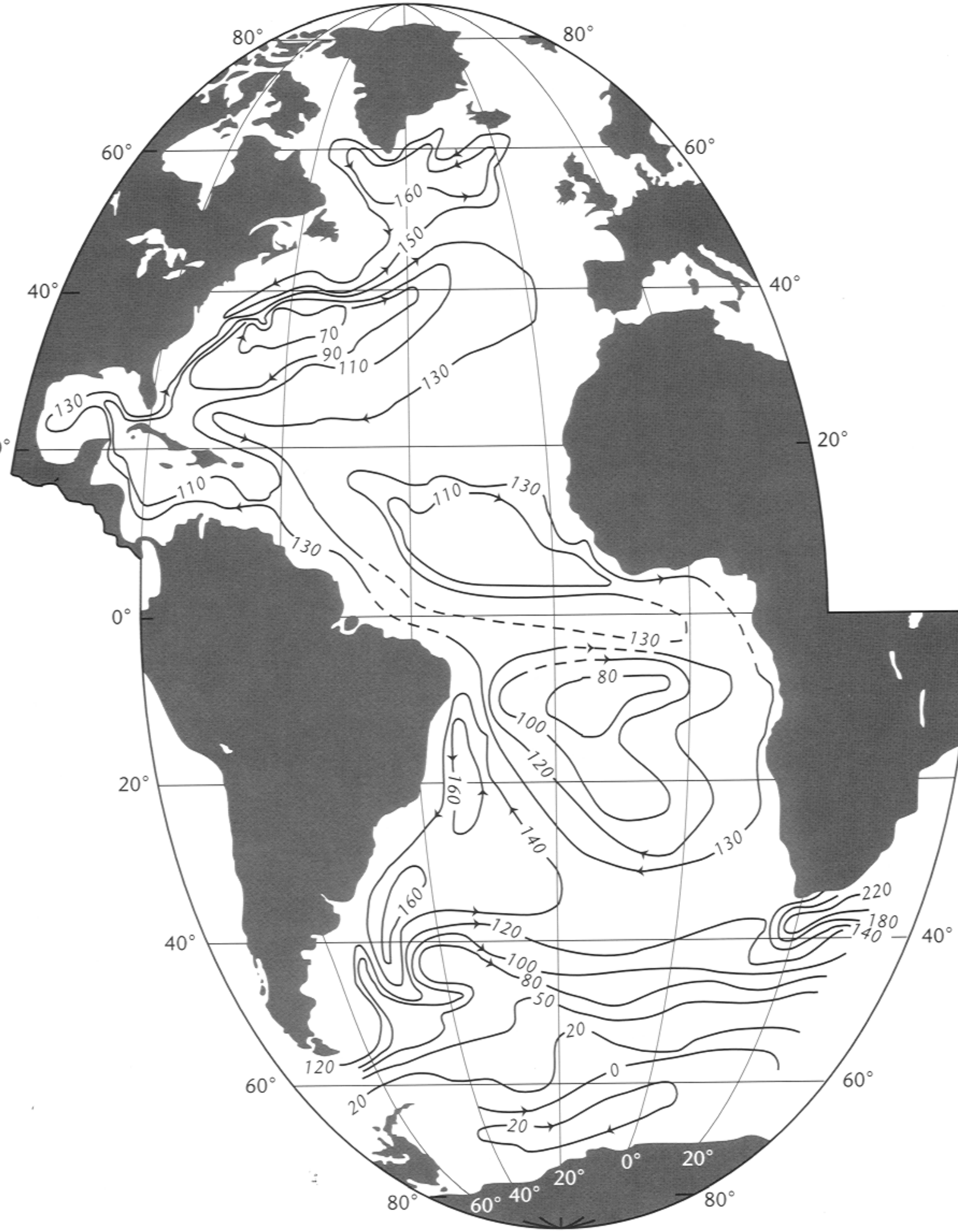
IAPSO IA03- Interhemispheric Water Exchange in the Atlantic Ocean

Existing Diagnostic Methods for Determination of Volume Transport

- From (T, S) Data – Total Geostrophic Flow (Reid 1985, 1994, …)

- Top 500 m Transport From (T, S) and Wind Data – Sverdrup Model (Godfrey 1989)

• Reid (1994)



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Three Components of Ekman-Munk Model

- Ekman Model for Extra-Equatorial Regions
- Munk Model for the Equatorial Region
- Stokes Theorem for Determining Ψ for Islands

Steady-State Large-Scale Dynamics

$$-f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2} + A_h \nabla^2 u$$

$$f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2} + A_h \nabla^2 v$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$u_g = -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}, \quad v_g = -\frac{1}{f\rho_0}\frac{\partial p}{\partial x}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

(1) Ekman Number (Mid-Latitude)

$$E = \frac{O(|A_h \nabla^2 \mathbf{V}|)}{O(|f\mathbf{V}|)} = \frac{A_h}{|f|\, L^2}$$

- *L ~ 2 10⁵m* $A_h = 5 \times 10^5 \ \mathrm{m^2 s^{-1}}$

$$E \simeq \frac{5 \times 10^5 \ \mathrm{m^2 s^{-1}}}{(10^{-4}\mathrm{s^{-1}}) \times (2 \times 10^5 \mathrm{m})^2} = 0.125$$

- Horizontal diffusion can be neglected

Ekman Model for Mid-Latitudes

$$-f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2}$$

$$f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2}$$

Vertically Integrated Ekman Model

$$-f(V - V_g) = A_z \frac{\partial u}{\partial z}\Big|_{z=\eta} - A_z \frac{\partial u}{\partial z}\Big|_{z=-H}$$

$$f(U - U_g) = A_z \frac{\partial v}{\partial z}\Big|_{z=\eta} - A_z \frac{\partial v}{\partial z}\Big|_{z=-H}$$

Vertically Integrated Velocity

$$U = \widehat{U}_g + U_r + \frac{\tau_y}{f\rho_0}, \quad V = \widehat{V}_g + V_r - \frac{\tau_x}{f\rho_0}$$

Vertically Integrated Velocity

$$(\widehat{U}_g, \widehat{V}_g) = \frac{g}{f\rho_0}\left(\int_{-H}^{\eta}\int_{-H}^{z}\frac{\partial\rho}{\partial y}dz'dz, -\int_{-H}^{\eta}\int_{-H}^{z}\frac{\partial\rho}{\partial x}dz'dz\right).$$

$$U_r = Hu_{-H} - \frac{C_D}{f}\sqrt{u^2_{-H} + v^2_{-H}}u_{-H},$$

$$V_r = Hv_{-H} + \frac{C_D}{f}\sqrt{u^2_{-H} + v^2_{-H}}v_{-H}.$$

- CD ~ Bottom Drag Coefficient (0.0025)

Volume Transport Stream Function

$$U = -\frac{\partial \Psi}{\partial y}, \quad V = \frac{\partial \Psi}{\partial x}$$

Poisson-Ψ Equation

$$\nabla^2\Psi = \Pi, \quad \Pi = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$$

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3$$

Forcing Terms in The Poisson-Ψ Equation

- T, S

$$\Pi_1 = (\frac{\partial \widehat{V}_g}{\partial x} - \frac{\partial \widehat{U}_g}{\partial y})$$

- T, S (P-vector Inverse Method)

$$\Pi_2 = (\frac{\partial V_r}{\partial x} - \frac{\partial U_r}{\partial y})$$

- Wind Stress

$$\Pi_3 = -[\frac{\partial}{\partial x}(\frac{\tau_x}{f\rho_0}) + \frac{\partial}{\partial y}(\frac{\tau_y}{f\rho_0})].$$

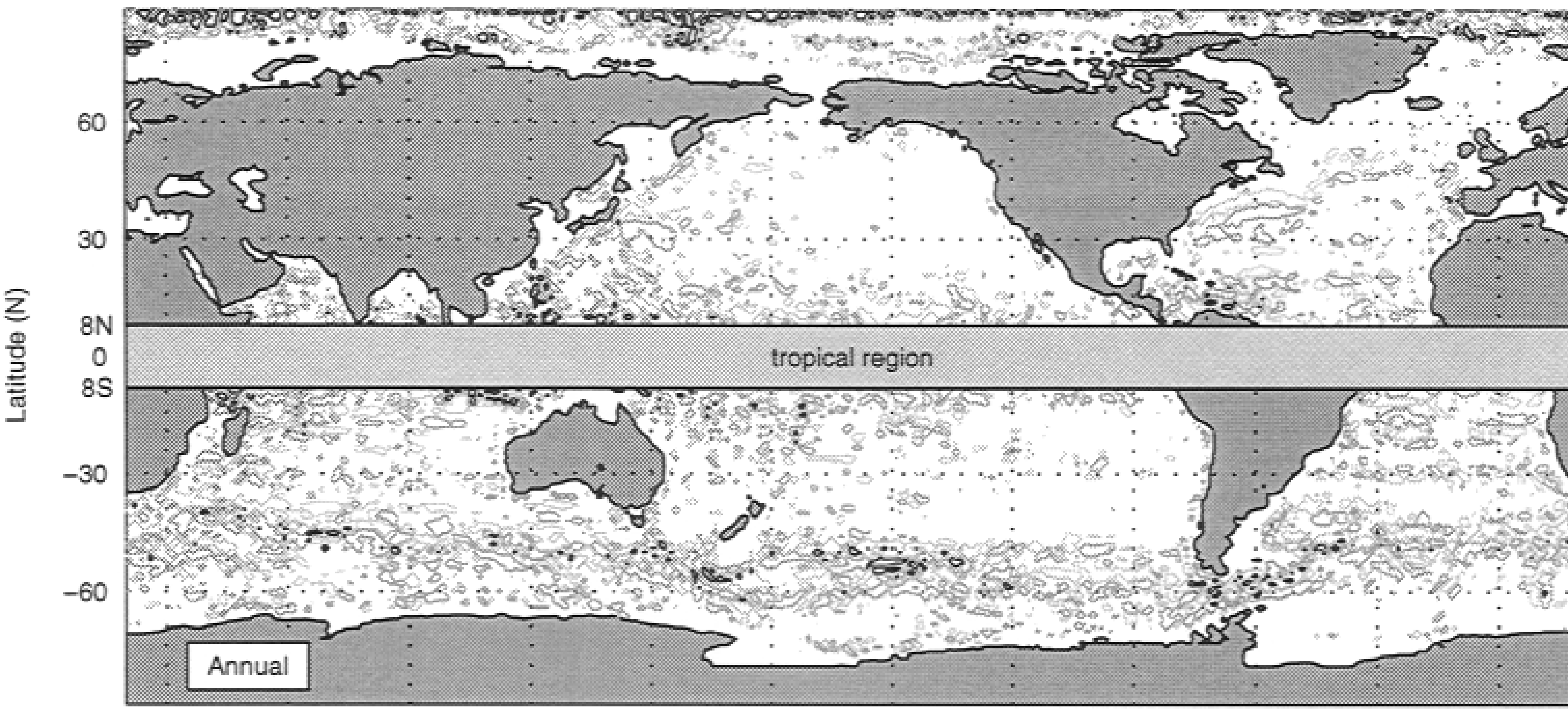
Datasets

- Topography: DBDB5, 5’ resolution

- Annual and Monthly Mean T, S: NODC (Levitus et al. 1998)

- Annual and Monthly Mean Wind Stress: NCEP

Annual Mean Π Values



Ekman Number (Low-Latitude) 8^o S – 8^o N

- Horizontal diffusion cannot be neglected

$$E \geq \frac{5 \times 10^5 \ \mathrm{m^2 s^{-1}}}{(0.2 \times 10^{-4} \mathrm{s^{-1}}) \times (2 \times 10^5 \mathrm{m})^2} = 0.5$$

(2) Munk Model (Equatorial Region)

- Vertically Integrated Vorticity Equation

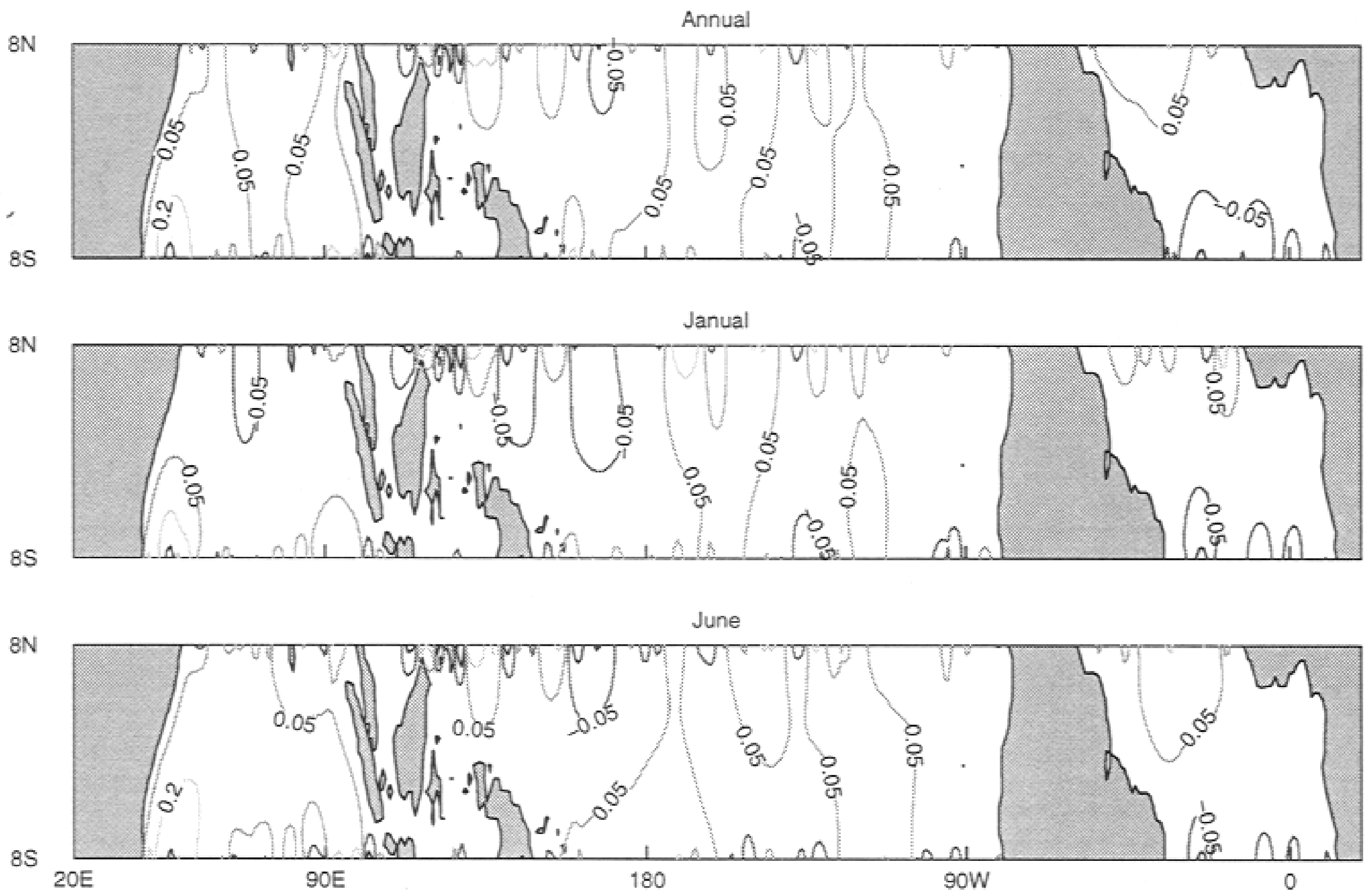
$$\nabla^2 \Pi = \frac{\beta}{A_h} V - \frac{1}{A_h}\left[\frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0}\right)\right].$$

- Boundary Conditions:

Π Values at 8° S – 8° N

Integration of Munk Model

- Obtaining Π Values in the tropics

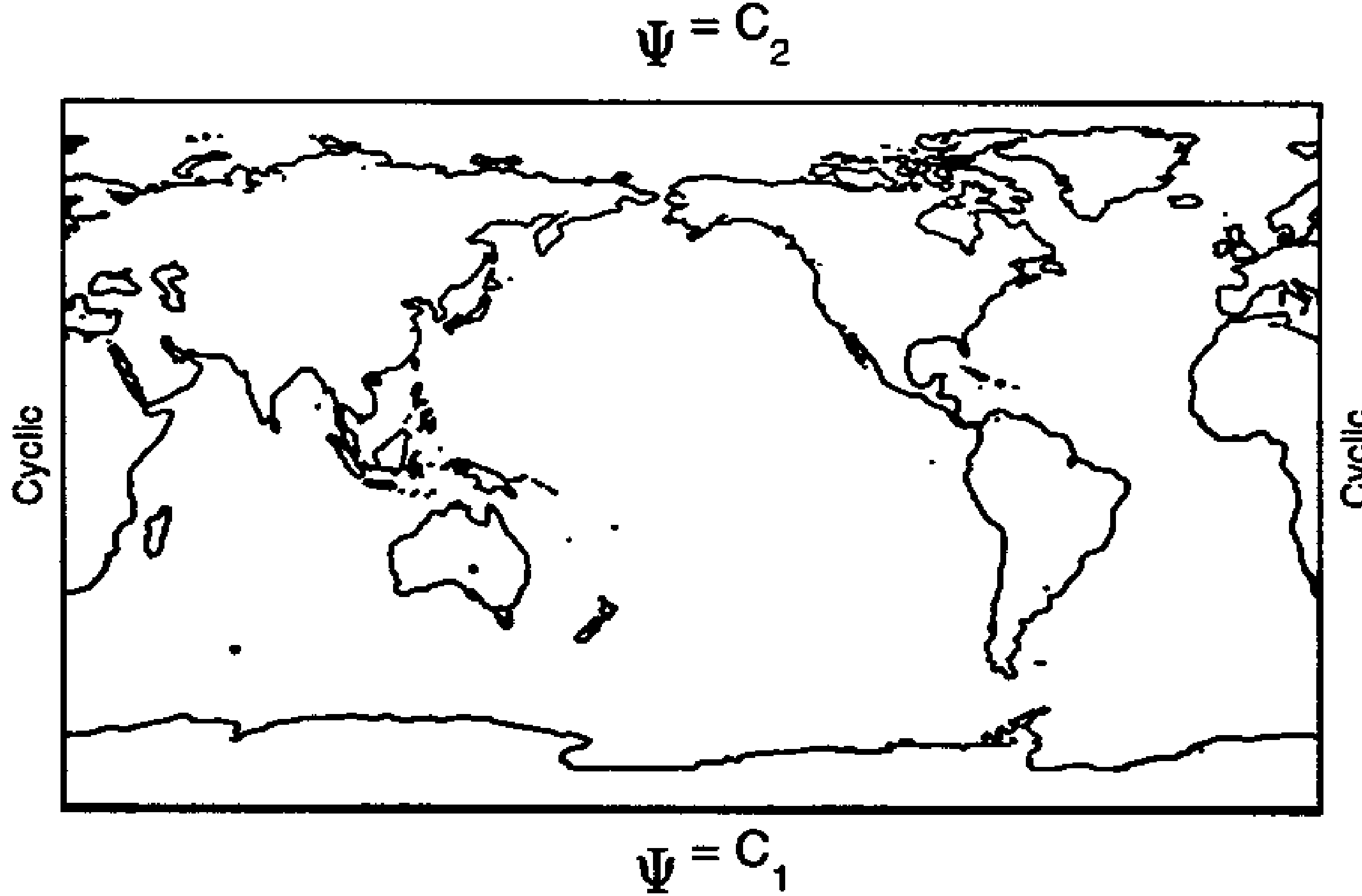


(3) Solving Poisson ψ-Equation

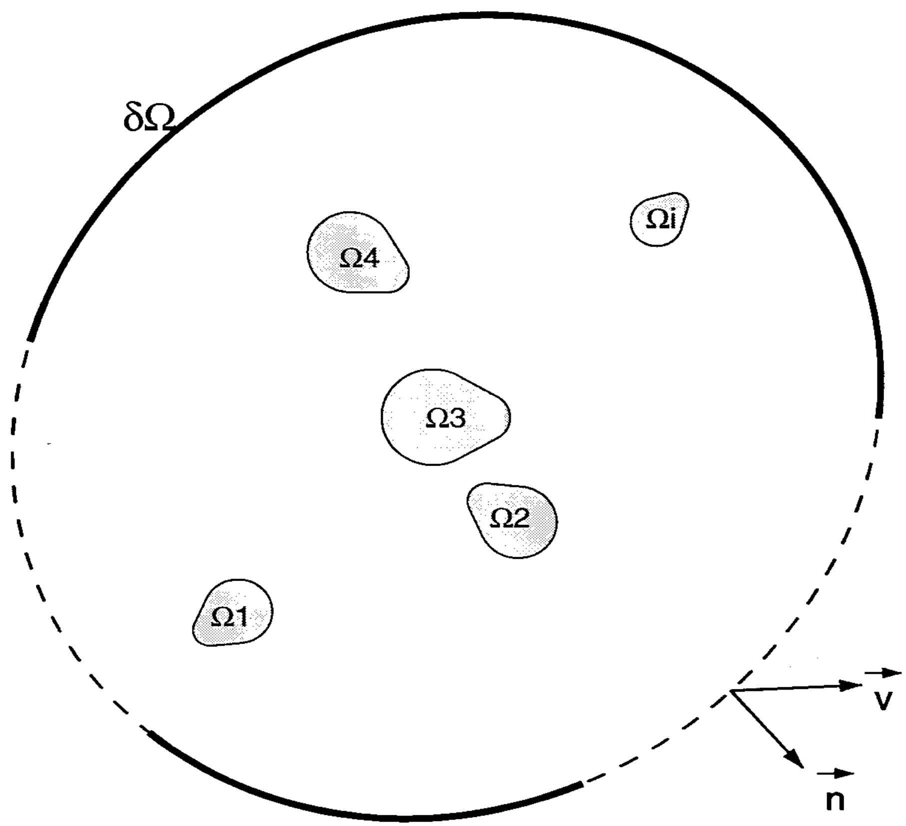
$$\nabla^2\Psi = \Pi,$$

- With known forcing term for the globe
- We need to know Ψ-Values at islands.

Boundary Conditions for ψ



Multi-Connected Domain



Stokes Theorem

$$-\oint_{\delta\Omega_j} \mathbf{V}\bullet d\mathbf{s} + \oint_{\delta\omega_j} \mathbf{V}\bullet d\mathbf{s} = \iint\limits_{C_j} \mathbf{k} \bullet (\boldsymbol{\nabla} \times \mathbf{V}) dx dy$$

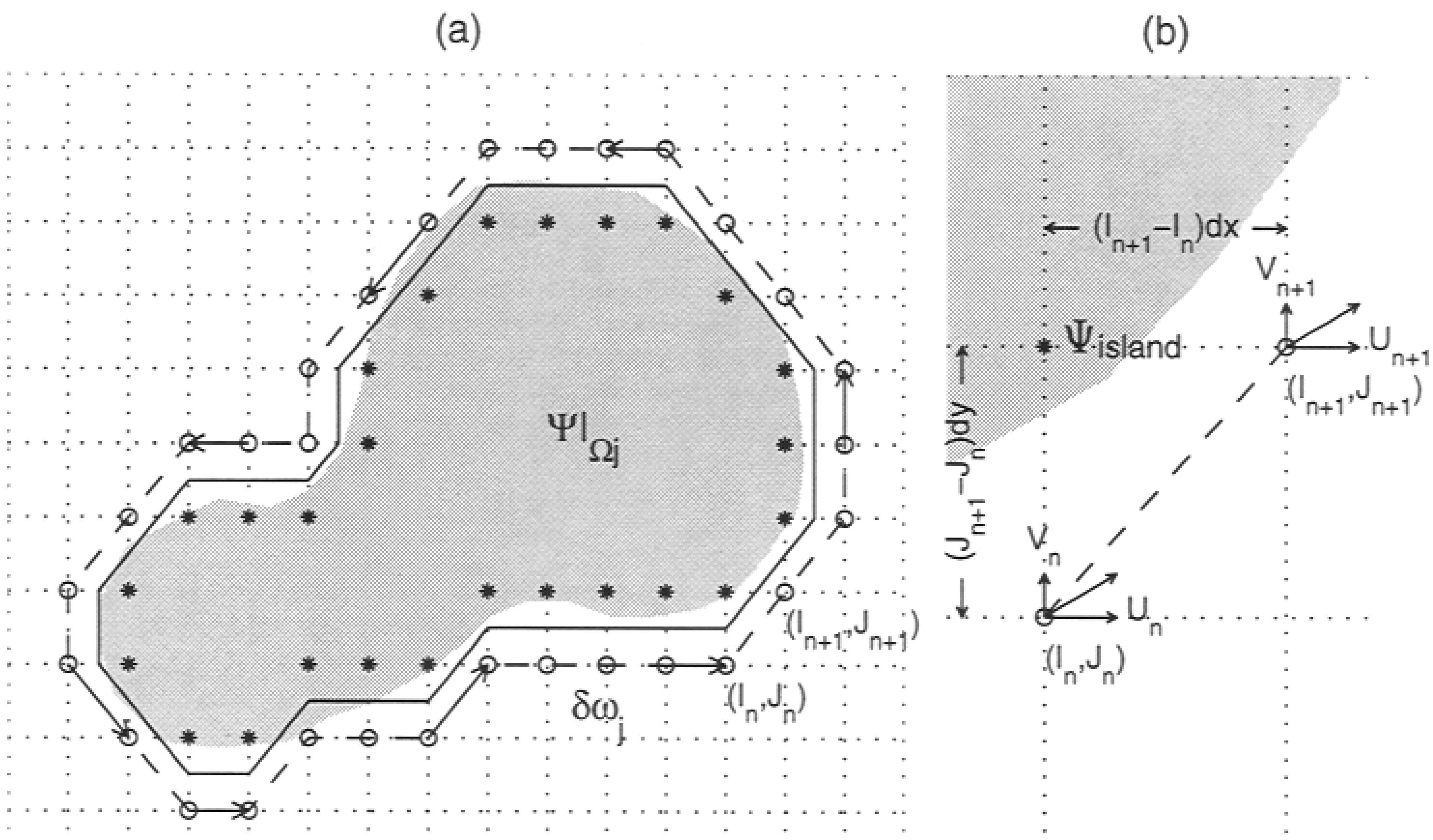
$$\oint_{\delta\Omega_j} \boldsymbol{\nabla}\boldsymbol{\Psi} \bullet \mathbf{n} ds = \oint_{\delta\omega_j} \mathbf{V}\bullet d\mathbf{s} - \iint\limits_{C_j} \mathbf{k} \bullet (\boldsymbol{\nabla} \times \mathbf{V}) dx dy$$

Minimum Circuit Method

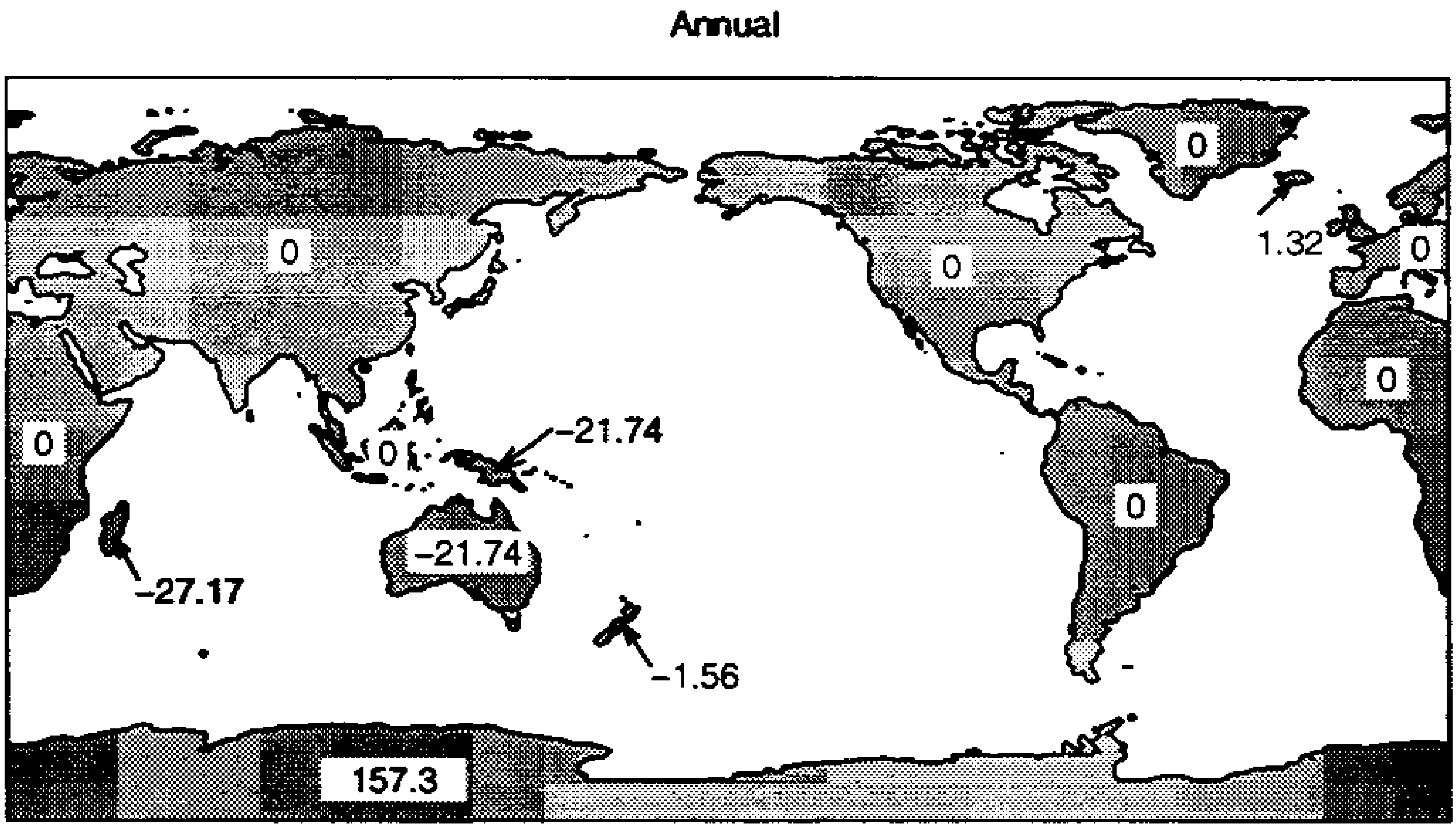
$$\oint_{\delta\Omega_j} \nabla\Psi \bullet \mathbf{n} ds \to \Gamma_j \quad \text{as } C_j \to 0$$

$$\Gamma_j \equiv \oint_{\delta\omega_j} \mathbf{V} \bullet d\mathbf{s}$$

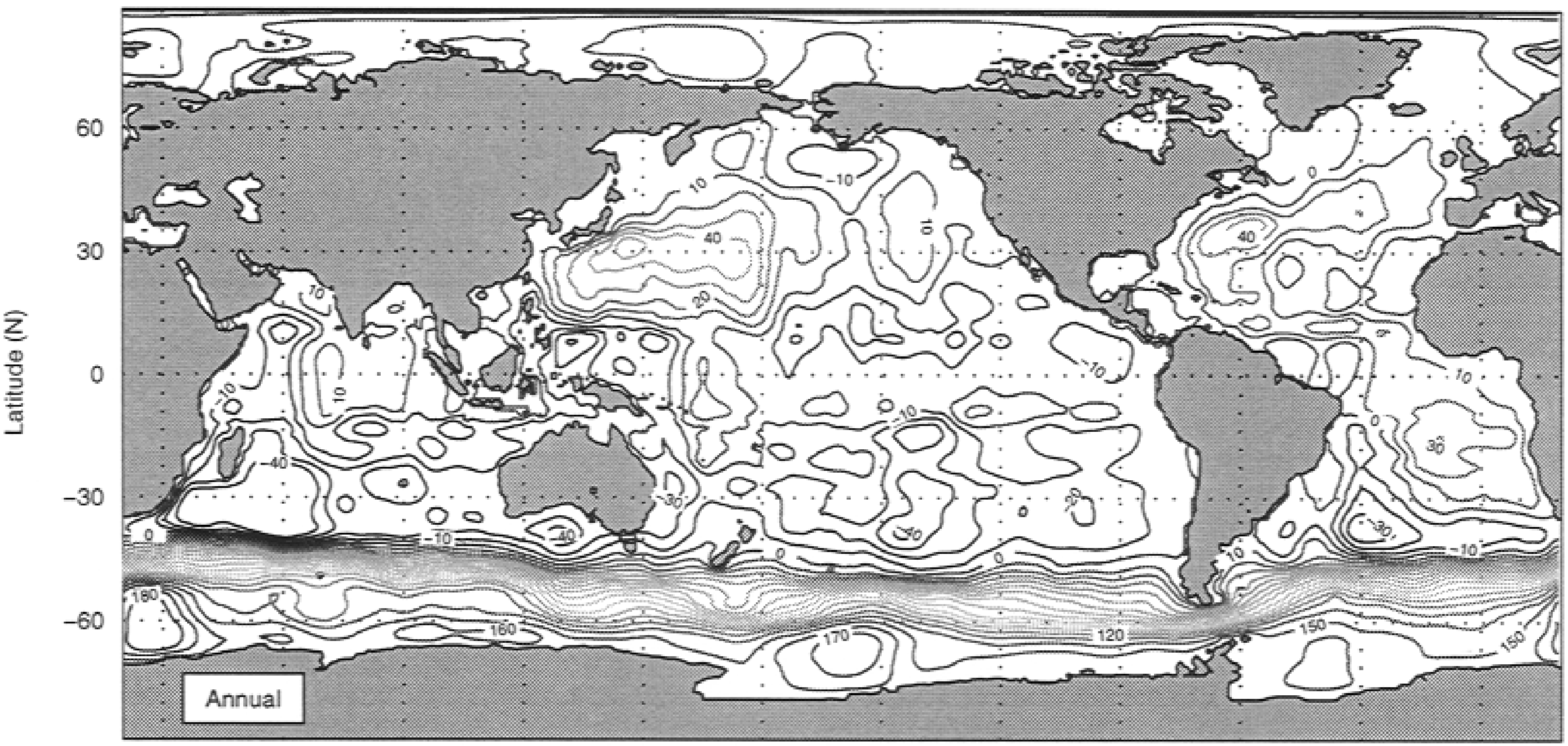
Minimum Circuit Method



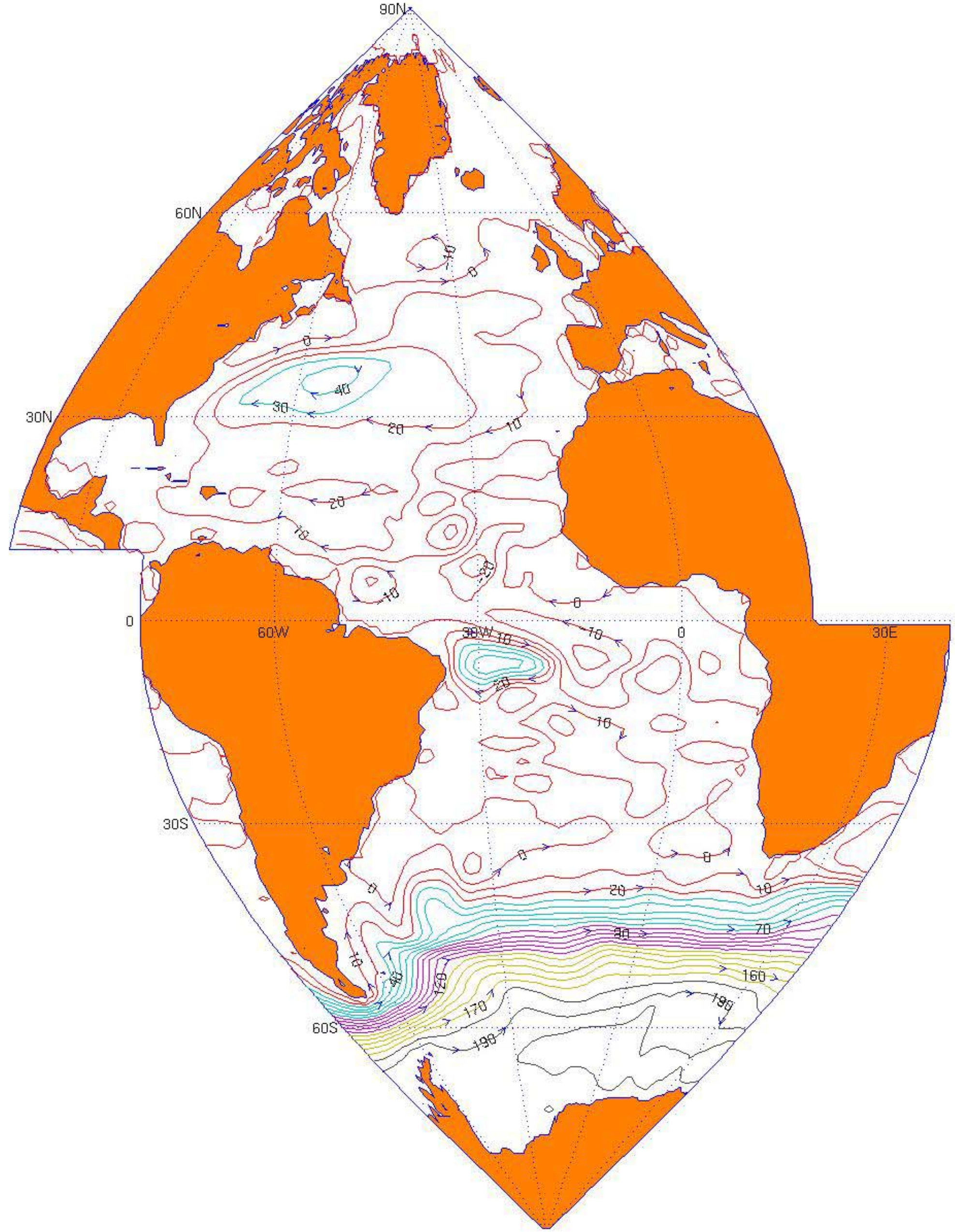
Ψ-Values at Islands (Annual Mean)



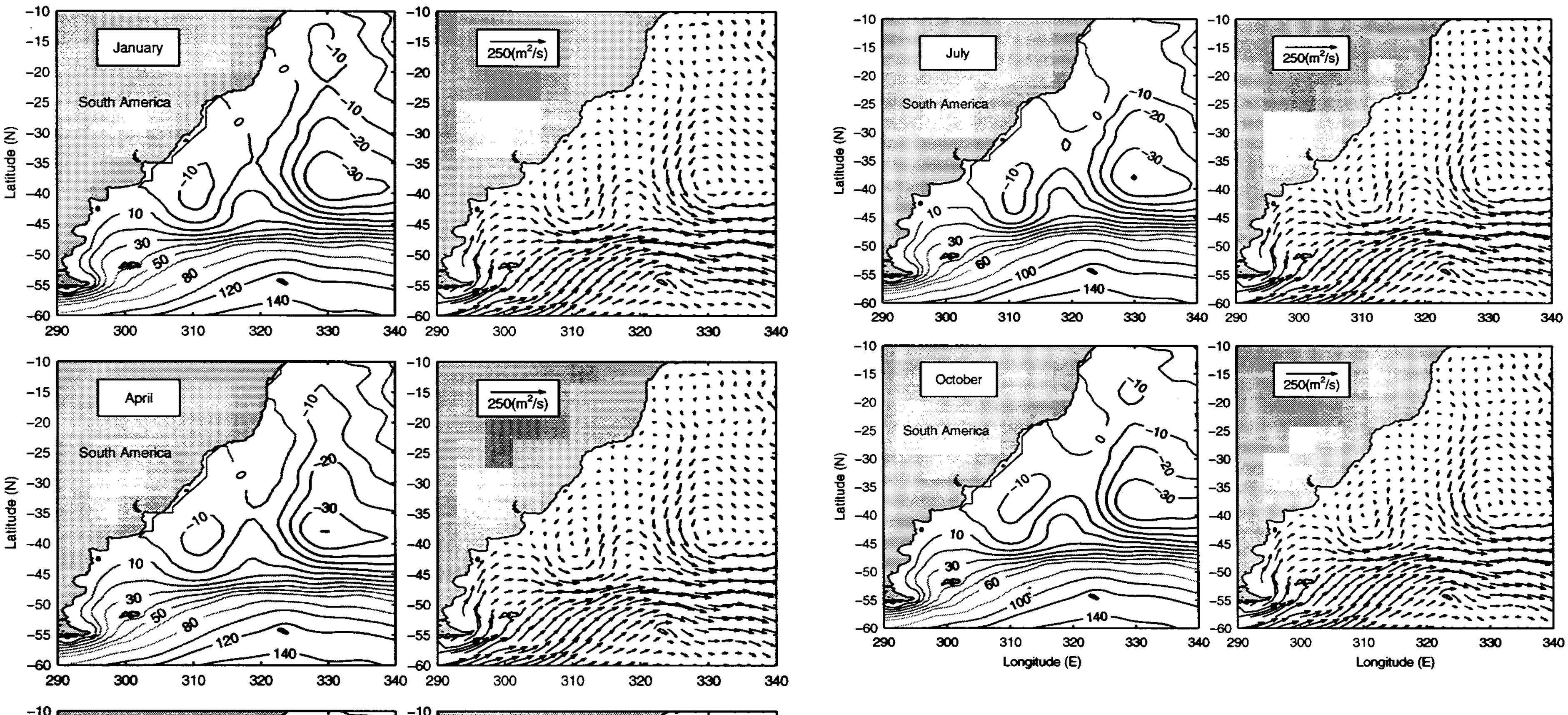
Global Volume Transport Streamfunction



• Transport Streamfunction



Malvinas Confluence



Conclusions

- Ekman-Munk model has capability to diagnose the volume transport from wind and hydrographic data
- Minimum circuit method is effective for determining streamfunction at islands
- Annual and monthly mean global volume transport data are useful for coastal modeling