Determination of Inter-Hemispheric Water Exchange Using the Ekman-Munk Model

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Existing Diagnostic Methods for Determination of Volume Transport

- From (T, S) Data – Total Geostrophic Flow (Reid 1985, 1994, …)

- Top 500 m Transport From (T, S) and Wind Data – Sverdrup Model (Godfrey 1989)
• Reid (1994)
Three Components of Ekman-Munk Model

- Ekman Model for Extra-Equatorial Regions
- Munk Model for the Equatorial Region
- Stokes Theorem for Determining $\Psi$ for Islands
Steady-State Large-Scale Dynamics

\[- f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2} + A_h \nabla^2 u \]

\[f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2} + A_h \nabla^2 v \]

\[\frac{\partial p}{\partial z} = -\rho g \]

\[u_g = -\frac{1}{f \rho_0} \frac{\partial p}{\partial y}, \quad v_g = -\frac{1}{f \rho_0} \frac{\partial p}{\partial x} \]

\[\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]
(1) Ekman Number (Mid-Latitude)

\[ E = \frac{O(|A_h \nabla^2 V|)}{O(|fV|)} = \frac{A_h}{|f| L^2} \]

- \( L \approx 20 \times 10^5 m \)

\( A_h = 5 \times 10^5 \text{ m}^2\text{s}^{-1} \)

\[ E \approx \frac{5 \times 10^5 \text{ m}^2\text{s}^{-1}}{(10^{-4}\text{s}^{-1}) \times (2 \times 10^5 \text{m})^2} = 0.125 \]

- Horizontal diffusion can be neglected
Ekman Model for Mid-Latitudes

\[- f(v - v_g) = A_z \frac{\partial^2 u}{\partial z^2}\]

\[f(u - u_g) = A_z \frac{\partial^2 v}{\partial z^2}\]
Vertically Integrated Ekman Model

\[ -f(V - V_g) = A_z \frac{\partial u}{\partial z} |_{z=\eta} - A_z \frac{\partial u}{\partial z} |_{z=-H} \]

\[ f(\ddot{U} - U_g) = A_z \frac{\partial v}{\partial z} |_{z=\eta} - A_z \frac{\partial v}{\partial z} |_{z=-H} \]
Vertically Integrated Velocity

\[ U = \hat{U}_g + U_r + \frac{\tau_y}{f \rho_0}, \quad V = \hat{V}_g + V_r - \frac{\tau_x}{f \rho_0} \]
Vertically Integrated Velocity

\[(\tilde{U}_g, \tilde{V}_g) = \frac{g}{f \rho_0} \left( \int_{-H}^{\eta} \int_{-H}^{z} \frac{\partial \rho}{\partial y} \, dz' \, dz, - \int_{-H}^{\eta} \int_{-H}^{z} \frac{\partial \rho}{\partial x} \, dz' \, dz \right). \]

\[U_r = H u_{-H} - \frac{C_D}{f} \sqrt{u_{-H}^2 + v_{-H}^2 u_{-H}}, \]

\[V_r = H v_{-H} + \frac{C_D}{f} \sqrt{u_{-H}^2 + v_{-H}^2 v_{-H}}. \]

- \( CD \sim \) Bottom Drag Coefficient \((0.0025)\)
Volume Transport Stream Function

\[ U = -\frac{\partial \Psi}{\partial y}, \quad V = \frac{\partial \Psi}{\partial x} \]
Poisson-Ψ Equation

\[ \nabla^2 \Psi = \Pi, \quad \Pi = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \]

\[ \Pi = \Pi_1 + \Pi_2 + \Pi_3 \]
Forcing Terms in The Poisson-$\Psi$ Equation

- $T$, $S$

- $T$, $S$ (P-vector Inverse Method)

- Wind Stress

\[
\Pi_1 = \left( \frac{\partial \hat{V}_g}{\partial x} - \frac{\partial \hat{U}_g}{\partial y} \right)
\]

\[
\Pi_2 = \left( \frac{\partial V_r}{\partial x} - \frac{\partial U_r}{\partial y} \right)
\]

\[
\Pi_3 = -\left[ \frac{\partial}{\partial x} \left( \frac{\tau_x}{f \rho_0} \right) + \frac{\partial}{\partial y} \left( \frac{\tau_y}{f \rho_0} \right) \right].
\]
Datasets

• Topography: DBDB5, 5’ resolution

• Annual and Monthly Mean T, S: NODC (Levitus et al. 1998)

• Annual and Monthly Mean Wind Stress: NCEP
Annual Mean Π Values
Ekman Number (Low-Latitude)
8° S – 8° N

- Horizontal diffusion cannot be neglected

\[ E \geq \frac{5 \times 10^5 \text{ m}^2\text{s}^{-1}}{(0.2 \times 10^{-4}\text{s}^{-1}) \times (2 \times 10^5 \text{m})^2} = 0.5 \]
(2) Munk Model (Equatorial Region)

- Vertically Integrated Vorticity Equation

\[ \nabla^2 \Pi = \frac{\beta}{A_h} V - \frac{1}{A_h} \left[ \frac{\partial}{\partial x} \left( \frac{\tau_y}{\rho_0} \right) - \frac{\partial}{\partial y} \left( \frac{\tau_x}{\rho_0} \right) \right]. \]

- Boundary Conditions:

\( \Pi \) Values at \( 8^\circ \) S – \( 8^\circ \) N
Integration of Munk Model

• Obtaining $\Pi$ Values in the tropics
(3) Solving Poisson $\psi$-Equation

$$\nabla^2 \psi = \Pi,$$

- With known forcing term for the globe
- We need to know $\Psi$-Values at islands.
Boundary Conditions for $\psi$

$\psi = C_2$

Cyclic

Cyclic

$\psi = C_1$
Multi-Connected Domain
Stokes Theorem

\[-\oint_{\partial \Omega_j} \mathbf{V} \cdot d\mathbf{s} + \oint_{\partial \omega_j} \mathbf{V} \cdot d\mathbf{s} = \iint_{C_j} \mathbf{k} \cdot (\nabla \times \mathbf{V}) \, dx \, dy\]

\[\oint_{\partial \Omega_j} \nabla \Psi \cdot \mathbf{n} \, ds = \oint_{\partial \omega_j} \mathbf{V} \cdot d\mathbf{s} - \iint_{C_j} \mathbf{k} \cdot (\nabla \times \mathbf{V}) \, dx \, dy\]
Minimum Circuit Method

\[ \int_{\delta \Omega_j} \nabla \Psi \cdot n \, ds \rightarrow \Gamma_j \quad \text{as} \quad C_j \rightarrow 0 \]

\[ \Gamma_j \equiv \int_{\delta \omega_j} V \cdot ds \]
Minimum Circuit Method
Ψ-Values at Islands (Annual Mean)
Global Volume Transport Streamfunction
• Transport Streamfunction
Malvinas Confluence
Conclusions

• Ekman-Munk model has capability to diagnose the volume transport from wind and hydrographic data

• Minimum circuit method is effective for determining streamfunction at islands

• Annual and monthly mean global volume transport data are useful for coastal modeling