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Hydrodynamical characteristics of a falling cylinder in water column

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Abstract

The hydrodynamic features of a falling cylinder into the water column is investigated experimentally. The experiment consisted of dropping three cylinders of various lengths into a pool where the trajectories were filmed from two angles. The controlled parameters were, cylinder parameters (length to diameter ratio, center of mass location), and initial conditions (initial velocity, and drop angle). Results indicate that center of mass position has the largest influence on the cylinder's trajectory and that accurate trajectory modeling requires the inclusion of both momentum and moment equations. A statistical-dynamic model has been established to predict the trajectories of the falling cylinders.

1 Introduction

Study on the movement of a rigid body in fluid has wide scientific significance and technical application. The theory of dynamics of a rigid body allows one to set up six nonlinear equations for the most general motion: three momentum equations and three moment of momentum equations. The scientific studies of the hydrodynamics of a rigid body in fluid involve the nonlinear dynamics, flight theory, body-fluid interaction, and instability theory.

The technical application of the hydrodynamics of a rigid body in fluid includes aeronautics and navigation. Recently, the scientific problem about the movement of a rigid body in water column drew attention to the Naval research. This is due to the threat of mine in the Naval operations. Within the past 15 years three U.S. ships, the USS Samuel B. Roberts (FFG-58), Tripoli (LPH-10) and Princeton (CG-59) have fallen victim to mines. Total ship damages were \$125 million while the mines cost approximately \$30 thousand (Boorda 1999). Mines have evolved over the years from the dumb "horned" contact mines that damaged the Tripoli and Roberts to ones that are relatively sophisticated - non-magnetic materials, irregular shapes, anechoic coatings, multiple sensors and ship count routines. Despite their increased sophistication, mines remain inexpensive and are relatively easy to manufacture, upkeep and place. Water mines are characterized by three factors: position in water (bottom, moored, rising, floating), method of delivery (aircraft, surface, subsurface) and method of actuation (acoustic and/or magnetic influence, pressure, contact, controlled).

Prediction of a falling rigid body in the water column is a key component in determining the impact speed and direction of mine on the sediment and in turn in determining its burial depth and orientation. In this study, a nonlinear dynamical system is established for the movement of a nonuniform (center of gravity not the same as the center of volume) rigid cylinder in the water column. A cylinder-drop experiment was conducted. The experimental results show the nonlinear characteristics of the trajectory pattern. The data collected from the experiment can be used for model development and verification.

2 Dynamics

2.1 Earth Coordinate System

Two coordinate systems are used to describe a cylinder falling through the water column: earth and relative coordinates. The earth coordinate system is fixed to the Earth surface with horizontal sides as x and y-axes (along the two sides of the pool), and vertical direction as z-axis (upward positive, Fig. 1). Suppose a cylinder falling into the water column. The cylinder rotates around its main axis (r_1) with an angle ψ_1 and an angular velocity of Ω. Its position is represented by the center of mass (COM), and its orientation is represented by two angles: ψ_2 and ψ_3 (Fig. 2). Here, ψ_2 is the angle between the r_1-axis and the horizontal plane; and ψ_3 is the angle between the projection of the main axis in the (x, y) plane and the x-axis. The angle, $\psi_2 + \pi/2$, is usually called attitude.

The relative coordinate is rigidly connected with the cylinder. The origin (O) of the relative coordinate system coincides with the center of mass (COM); the axis-r_1 is along the central line of the cylinder; the axis-r_2 is perpendicular to the plane constructed by axis-r_1 and axis-z (r_1-z plane); and the axis-r_3 lies in the (r_1-z) plane and is perpendicular to axis-r_1. The selection of axes (x, y, z) and (r_1, r_2, r_3) follows the right-hand rule. Let $\mathbf{V}^* \equiv (V_1^*,\ V_2^*,\ V_3^*)$ be the three components of the velocity of

COM, i.e., be the origin velocity of the coordinate system (r_1, r_2, r_3). The geometric center (GC) is located at $(\chi, 0, 0)$. For GC below COM, $\chi > 0$, and for GC above COM, $\chi < 0$. The relative coordinate system (r_1, r_2, r_3) is obtained through the translation and two rotations (ψ_2 and ψ_3) of the earth coordinate system. Let $\mathbf{P}$ be represented by $\mathbf{P}_E$ and $\mathbf{P}_B$ in the earth and relative coordinate systems, $\mathbf{P}_E$ and $\mathbf{P}_B$ are connected by

$$\mathbf{P}_E = \begin{bmatrix} \cos\psi_3 & -\sin\psi_3 & 0 \\ \sin\psi_3 & \cos\psi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi_2 & 0 & \sin\psi_2 \\ 0 & 1 & 0 \\ -\sin\psi_2 & 0 & \cos\psi_2 \end{bmatrix} \mathbf{P}_B + \begin{bmatrix} x_m^* \\ y_m^* \\ z_m^* \end{bmatrix} \tag{1}$$

where (x_m^*, y_m^*, z_m^*) are the location of COM in the earth coordinate system.

Figure 1: Earth coordinate system.

2.2 Nonlinear Dynamical Equations

Any solid object falling through a fluid (air and water) should obey two physical principles: (1) momentum balance and (2) moment of momentum balance. $\mathbf{V}_w^* \equiv (V_{w1}^*,\ V_{w2}^*,\ V_{w3}^*)$ be the water velocity, and $(\omega_1^*, \omega_2^*\ ,\ \omega_3^*)$ be the components of the angular velocity, referring to the direction of the relative coordinate system. The independent and dependent variables are made non-dimensional by:

$$t = \sqrt{\frac{g}{L}}t^*, \quad \omega = \sqrt{\frac{L}{g}}\omega^*, \quad \mathbf{V} = \frac{\mathbf{V}^*}{\sqrt{gL}},$$

where g is the gravitational acceleration, and L the length of the cylinder. The nondimensional momentum equations for COM are given by (Mises 1959)

Figure 2: Cylinder orientation and relative coordinate system.

$$\frac{dV_1}{dt} + \omega_2 V_3 - \omega_3 V_2 = \frac{\rho - \rho_w}{\rho} \sin\psi_2 + \frac{F_1^*}{\rho\Pi} \tag{2a}$$

$$\frac{dV_2}{dt} + \omega_3 V_1 = \frac{F_2^*}{\rho\Pi} \tag{2b}$$

$$\frac{dV_3}{dt} - \omega_2 V_1 = -\frac{\rho - \rho_w}{\rho} \cos\psi_2 + \frac{F_3^*}{\rho\Pi} \tag{2c}$$

where C_D is the drag coefficient, ρ the cylinder density, ρ_w the water density, g the gravitational acceleration, and L the length of the cylinder.

The non-dimensional equations of the moment of momentum for axial symmetric cylinder are

$$\frac{d\Omega}{dt} + \frac{J_3 - J_2}{J_1}\omega_2\omega_3 = \frac{LM_1^*}{gJ_1} \tag{3a}$$

$$\frac{d\omega_2}{dt} = \frac{\chi\Pi(\rho_w - \rho)L}{J_2} \cos\psi_2 + \frac{LM_2^*}{gJ_2} \tag{3b}$$

$$\frac{d\omega_3}{dt} = \frac{LM_3^*}{gJ_3} \tag{3c}$$

where g is the gravitational acceleration; χ^* is the distance between COM and geometric Center (GC); (F_1^*, F_2^*, F_3^*) and (M_1^*, M_2^*, M_3^*) are components of external force and moment due to the water drag; Π is the volume of the cylinder; J_1, J_2, and J_3 are the three moments of gyration,

$$J_1 = \int (r_2^2 + r_3^2) dm^*, \quad J_2 = \int (r_3^2 + r_1^2) dm^*, \quad J_3 = \int (r_1^2 + r_2^2) dm^*; \quad (4)$$

The orientation of the cylinder (ψ_2, ψ_3) is determined by

$$\frac{d\psi_2}{dt} = \omega_2, \quad \cos\psi_2 \frac{d\psi_3}{dt} = \omega_3. \quad (5)$$

The eight non-dimensional nonlinear equations (2a-c), (3a-c), and (5) are the basic system for determining the cylinder movement in the water column.

3 Model Cylinders

3.1 Description

Three model cylinders were used for the drop experiment at the Naval Postgraduate School swimming pool. All had a circular diameter of 4 cm, however the lengths were 15, 12 and 9 cm respectively. The bodies were constructed of rigid plastic with aluminum-capped ends. Inside each was a threaded bolt, running lengthwise across the cylinder, and an internal weight (Fig 3). The internal cylindrical weight made by copper was used to vary the cylinder's center of mass and could be adjusted fore or aft. The center of gravity of the model cylinder is the origin of the body fixed coordinate system.

The model is composed of six uniform cylindrical parts (Fig. 4): (a) a plastic hollow cylinder ($C^{(1)}$) with mass of m_1, outer and inner radii of R_1 and R_2, length of (L-2l_1), and the center of gravity for the part (COMP) to be at its geometric center located along the r_1-axis at $r_1 = \chi$; (b) an aluminum-capped left end (Fig. 3) solid cylinder ($C^{(2)}$) with mass of m_2, radius of R_1, length of l_1, and COMP located along the r_1-axis at $r_1 = L/2 - l_1/2 + \chi$; (c) an aluminum-capped right end solid cylinder ($C^{(3)}$) with mass of m_3, radius of R_1, length of l_1, and COMP located along the r_1-axis at $r_1 = L/2 - l_1/2 - \chi$; (d) a cylindrical thread ($C^{(4)}$) with mass of m_4, radius of R_3, length of (L - 2l_1), and COMP located along the r_1-axis at $r_1 = \chi$; (e) a cylindrical threaded bolt ($C^{(5)}$) with mass of m_5, out and inner radii of R_2 and R_3, length of l_2, and COMP located along the r_1-axis at $r_1 = L/2 - \chi - l_1 - l_2/2$; (f) an adjustable copper cylindrical weight ($C^{(6)}$) with mass of m_6, outer and inner radii of R_2 and R_3, length of l_3, and COMP located along the r_1-axis at $r_1 = \delta + \chi$, where δ is the distance between the COMP of the adjustable weight and the geometric center of the model cylinder.

Figure 3: Internal components of the model cylinder.

3.2 Moments of Gyration

Since the six parts (all cylinders) all have uniform mass distribution, the moments of gyration for these parts are:

$$J_1^{(1)} = \frac{m_1}{2}(R_1^2 + R_2^2), \quad J_1^{(2)} = \frac{m_2}{2}R_1^2, \quad J_1^{(3)} = \frac{m_3}{2}R_1^2$$

$$J_1^{(4)} = \frac{m_4}{2}R_3^2, \quad J_1^{(5)} = \frac{m_5}{2}R_2^2, \quad J_1^{(6)} = \frac{m_6}{2}(R_2^2 + R_3^2),$$

$$\begin{aligned}
J_2^{(1)} = J_3^{(1)} &= \frac{m_1}{12}[3R_1^2 + 3R_2^2 + (L - 2l_1)^2], \\
J_2^{(2)} = J_3^{(2)} &= \frac{m_2}{12}(3R_1^2 + l^2), \\
J_2^{(3)} = J_3^{(3)} &= \frac{m_3}{12}(3R_1^2 + l^2), \\
J_2^{(4)} = J_3^{(4)} &= \frac{m_4}{12}[3R_3^2 + (L - 2l)^2], \\
J_2^{(5)} = J_3^{(5)} &= \frac{m_5}{12}(3R_2^2 + 3R_3^2 + l_2^2), \\
J_2^{(6)} = J_3^{(6)} &= \frac{m_6}{12}(3R_2^2 + 3R_3^2 + l_3^2),
\end{aligned} \tag{6}$$

where the superscripts for the moments indicate the cylindrical parts. The resultant moments of gyration is computed by

$$J_1 = \sum_{j=1}^{6} J_1^{(j)},$$

$$J_2 = J_3 = \sum_{j=1}^{6} J_2^{(j)} + m_1\chi^2 + m_2(\frac{L - l_1}{2} - \chi)^2 + m_3(\frac{L - l_1}{2} + \chi)^2$$

Figure 4: Internal structure of the model cylinder.

$$+m_4\chi^2 + m_5(\frac{L}{2} - \chi - l_1 - \frac{l_2}{2})^2 + m_6(\delta + \chi)^2. \tag{7}$$

According to the definition of COM, the coordinate of GC (χ^*) is determined by

$$\chi^* = \frac{[m_5(L/2 - l_1 - l_2/2) - m_6\delta]}{\sum_{j=1}^{6} m_j} \tag{8}$$

which indicates how the adjustable weight determines the location of COM for the model cylinder.

3.3 Model Parameters

3.3.1 Length/Diameter and Density Ratios

Our goal was to choose a scale that was somewhat representative of the real world ratio of water depth to mine length, but at the same time would be large enough to film and would not damage the pool's bottom. The model cylinders were based on the realistic assumption that a 3 m mine is laid in water depths of 45 m, thus producing a 15:1 ratio. The depth of the pool is 2.4 m. From this ratio, the length (L) of the model cylinder is chosen as 15 cm. The addition of a 12 and 9 cm length allowed for later comparison of the sensitivity of water phase trajectory to the ratio of mine length over diameter. The outer radius of the model cylinder is 2 cm. Three length/diameter ratios (L/D: 15/4, 12/4, and 9/4) were used for the experiment. The corresponding density ratios ρ/ρ_w are 1.70, 1.68, and 1.88, respectively.

Figure 5:

3.3.2 χ-Values

In each of the three model cylinders, the location of the weight (i,e., the value of δ) is adjustable. Use of (7) location of the COM (χ-value) can be determined. During the experiment five χ-values (unit: cm) are used for each model cylinder (Table 1). The positive χ-values indicate that COM is below the geometric center, and the negative χ-values indicate that COM is above the geometric center.

Model	L/D	ρ/ρ_w	χ_1	χ_2	χ_0	χ_{-1}	χ_{-2}
1	15/4	1.70	1.85	3.69	0	-1.85	-3.69
2	12/4	1.68	1.21	2.43	0	-1.21	-2.43
3	9/4	1.88	0.68	1.37	0	-0.68	-1.37

Table 1. Model L/D and density ratios, and χ-values (unit: cm).

4 Cylinder Drop Experiment (CYDEX)

Cylinder Drop Experiment (CYDEX) was conducted at the NPS swim pool in June 2001. The purpose of the experiment is to collect data about cylinder's motion in the water column for various combinations of the model cylinder parameters. It basically consisted of dropping each of three solid cylinders into the water where each drop was recorded underwater from two viewpoints. Figure 5 depicts the overall setup. The controlled parameters for each drop were: L/D ratio, χ-value, initial velocity (V_{in}), and drop angle. The Earth's coordinate system is chosen with the origin at the corner of the swimming pool with the two sides as x- and y-axes and the vertical as z-axis. The initial injection of cylinder was in the (y, z) plane (Fig. 1).

4.1 Initial Velocity

Initial velocity ($V^{(in)}$) was calculated by using the voltage return of an infrared photo detector located at the base of the mine injector. The infrared sensor produced a square wave pulse when no light was detected due to blockage caused by the mine's passage. The length of the square wave pulse was converted into time by using a universal counter. Dividing the cylinder's length by the universal counter's time yielded $V^{(in)}$. The cylinders were dropped from several positions within the injector mechanism in order to produce a range of $V^{(in)}$. The method used to determine $V^{(in)}$ required that the infrared light sensor be located above the water's surface. This distance was held fixed throughout the experiment at 10 cm.

4.2 Drop Angle

The drop angle (initial $\psi_2^{(in)}$) was controlled using the drop angle device. Five screw positions marked the 15°, 30°, 45°, 60°, and 75°. The drop angles were determined from the lay of the pool walkway, which was assumed to be parallel to the water's surface. A range of drop angles was chosen to represent the various entry angles that air and surface laid cylinders exhibit. This range produced velocities whose horizontal and vertical components varied in magnitude. This allowed for comparison of cylinder trajectory sensitivity with the varying velocity components.

4.3 Methodology

For each run the cylinders were set to a χ-value. For positive χ-values, the cylinders were placed into the injector so that the COM was located below the geometric center. For negative χ-values, the COM was located above the geometric center to release. A series of drops were then conducted in order of decreasing cylinder length for each angle. Table 2 indicates number of drops conducted for different drop angles and χ-values for $L/D = 15/4$. Number of drops for other L/D ratios (12/4, 9/4) is comparable to that for L/D ratio of 15/4. All together there were 712 drops. Each video camera had a film time of approximately one hour. At the end of the day, the tapes were replayed in order to determine clarity and optimum camera position.

Drop Angle	15°	30°	45°	60°	75°
χ_2	13	15	15	15	12
χ_1	9	15	15	15	9
χ_0	12	14	15	18	6
χ_{-1}	0	6	6	6	0
χ_{-2}	2	6	6	0	0

Table 2. Number of drops conducted for different drop angles and χ-values for L/D = 15/4.

5 Data Retrieval and Analysis

5.1 Data Retrieval

Upon completion of the drop phase, the video from each camera was converted to digital format. The digital video for each view was then analyzed frame by frame (30 Hz) in order to determine the cylinder's position in the (x, z) and (y, z) planes. The cylinder's top and bottom positions were input into a MATLAB generated grid, similar to the ones within the pool. The first point to impact the water was always plotted first. This facilitated tracking of the initial entry point throughout the water column. The cameras were not time synchronized; thus, the first recorded position corresponded to when the full length of the mine was in view.

5.2 Source of Errors

There were several sources of error that hindered the determination of the cylinder's exact position within the water column. Locations above or below the camera's focal point were subjected to parallax distortion. Placing the cameras as far back as possible, while still being able to resolve the individual grid squares, minimized this error. Second, the background grids were located behind the cylinder's trajectory plane. This resulted in the cylinder appearing larger than normal. This error was minimized by not allowing the plotted points to exceed the particular cylinder's length. Third, an object injected into the water will generate an air cavity. This air cavity can greatly affect the initial motion, particularly at very high speeds (hydro ballistics). The air cavity effect was deemed to be minimal due to the low inject velocities used.

5.3 Data Analysis

The 2-D data provided by each camera was first used to produce raw 2-D plots of the cylinder's trajectory. Next, 2-D data from both cameras was then fused to produce a 3-D history. This 3-D history was then made non-dimensional in order to generalize the results. The non-dimensional data was used to generate impact scatter plots and was also used in multiple linear regression calculations.

6 Experimental Results

6.1 Trajectory Patterns

After analyzing the 3-D data set, seven trajectory patterns were found. The plots on the $(y,\ z)$ plane were chosen for trajectory analysis, as this plane was parallel to the direction of the cylinder drop. The generalized trajectory patterns are described in Table 3, and Figures 6-9. The water phase trajectory a cylinder experiences ultimately determines the impact orientation. In CYDEX, the categorizing of trajectories into general patterns served two purposes. Observed trajectories were found to be most sensitive to χ-value, drop angle and L/D ratio. As COM distance (χ-value) increased from GC the cylinder tended to follow a straight pattern. As COM was moved closer to the GC (decreasing χ-value) the cylinder's trajectory tended towards being more parallel with the pool's bottom. At steep drop angles, the cylinder experienced little lateral movement and tended towards a straight pattern. Additionally, as L/D ratio decreased more complex trajectory patterns developed. This included significant oscillation about the vertical axis and increased lateral movement.

Trajectory Pattern	Description
Straight (or Slant)	Little angular change about z-axis Almost parallel with z-axis, ψ_2 near ($90° \pm 15°$)
Spiral	Oscillating about z-axis (no rotation)
Flip	Initial water entry point rotated at least 180°
Flat	ψ_2 near $0°$ (no oscillation)
Seesaw	ψ_2 oscillates about $0°$.
Combination	Exhibited several of the above patterns

Table 3. Description of relative coordinate based trajectory patterns

6.2 Impact Attitude

The angle, $\psi_2 + \pi/2$, is the impact attitude at the bottom of the water column. The mine burial is largely determined from the impact attitude of the mine. Mines whose impact attitudes are perpendicular ($\psi_2 = 90°$) to the sediment interface will experience the largest degree of impact burial (Taber 1999). It is therefore important to analyze the relationship between impact attitude and the controlled parameters, drop angle, $V^{(in)}$, L/D, and χ. The experiment shows that both L/D and $V^{(in)}$ had little influence on impact attitude. The drop angle and χ, however, were the largest determinants of impact attitude.

From Figure 10 it is apparent that there are several peaks centered near 90°, 140°, and 180°. Further analysis reveals that these peaks correspond to the COM positions 0, 1 and 2 (corresponding to χ_0, χ_1, χ_2), respectively. COM positions -1 and -2 (corresponding to χ_{-1}, χ_{-2}) followed the same trend as their positive counterparts.

Although drop angle was not the most influential parameter, variations did induce changes in impact orientation. As drop angle increased, the likelihood of any lateral movement decreased. This allowed for impact angles

Figure 6: Trajetory patterns.

Figure 7: Trajectory patterns.

Figure 8: Trajectory patterns for the COM-2 position (i.e., χ_2).

Figure 9: Trajectory patterns for negative COM position (χ_{-1}, χ_{-2}). Here, COM is above GC when the cylinder enters the surface).

Figure 10: Relationship between COM position and impact attitude.

that were more vertically orientated. This is primarily due to the fact that the vertical components of velocity were greater than those at shallow angles. Thus, the time to bottom and time for trajectory alteration was less.

6.3 Cylinder Tumbling Not Observed

The current Navy's mine impact burial model (IMPACT25) was developed by the Coastal System Station (Arnone and Bowen 1980; Satkowiak 1987)); subsequent upgraded by the New Zealand Defense Establishment (Hurst 1992) and the Naval Research Laboratory (NRL). Some of the major input parameters to the model are environment (sedimentation, shear strength, water depth), mine characteristics (shape, center of mass, weight, and mine deployment parameters), deployment platform (ship, aircraft, submarine), speed of platform, angle of mine upon entering water, rotational velocity at time of deployment and others. The theoretical base of IMPACT25 is the momentum equations (2a)-(2c). The model assumes that COM coincides with GC ($\chi = 0$). Chu et al. (2001) reported the discrepancy between observed and model predicted (by IMPACT25) mine burial depth from a mine impact burial experiment performed near the Monterey beach, California in May 2000. The model describes two trajectory patterns (Figure 11): (a) without any orientation change, (b) with a constant tumbling. For the second pattern, user should input the tumbling rate (Chu et al. 1998; 2000a,b). In CYDEX, cylinder tumbling was never observed even for the COM-0 position ($\chi = 0$), as shown in Figure 12.

Figure 11: Two trajectory patterns predicted using the Navy's Mine Impact Burial Model (IMPACT25): (a) no tumbling, and (b) constant tumbling. The model was established based only on the momentum equations (2a)-(2c).

6.4 Impact Points

Using a methodology similar to that used in impact attitude analysis, impact point scatter plots were analyzed by the controlled parameters, drop angle, V_{in} and χ-value (i.e., COM position). For χ_2 and χ_0 cases fell near the drop point greater than 90% of the time while for χ_1 cases displayed the most variability. Additionally, the flip experienced in negative COM cases (χ_{-1} and χ_{-2}) induces a greater degree of lateral movement than in positive COM (χ_1 and χ_2) cases.

7 Statistical Prediction Model

Multivariate linear regression model was established from the CYDEX data (712 drops) to establish relationships between the input non-dimensional parameters; ψ_2, L/D ratio, $V^{(in)}$, and χ, and the output variables (temporally varying) such as position x_m, y_m, z_m, velocity u, v, w and attitude ψ_2 at time t. Let Y represent the output variables. The regression equation is given by

$$Y(t) = \beta_0(t) + \beta_1(t)\phi + \beta_2(t)L/D + \beta_3(t)V_{in} + \beta_4(t)\chi. \tag{9}$$

Figure shows the temporally varying regression coefficients for u, v, w, and ψ_2. For the cylinder's attitude (ψ_2) the coefficient $\beta_4(t)$ is much larger than

Figure 12: Trajectory patterns for the COM-0 position (i.e., COM coincidence with GC).

the other coefficients, which indicates that the cylinder's orientation (versus the vertical direction) is mainly determined by the location of COM.

To show the dependence of impact of cylinder on the bottom (i.e., horizontal location relative to the cylinder's surface entry point, orientation, velocity), the multivariate regression between the input non-dimensional parameters; $\psi_2^{(in)}$, L/D ratio, $V^{(in)}$, and χ, and the final state (i.e., impact on the bottom) variables such as the horizontal position of COM (x_m, y_m), the velocity of COM (u, v, w) and the attitude ψ_2. Let Z represent the output variables. The regression equation is given by

$$Z = \alpha_0 + \alpha_1 \psi_2^{(in)} + \alpha_2 L/D + \alpha_3 V^{(in)} + \alpha_4 \chi. \tag{10}$$

with the regression coefficients α_0, α_1, α_2, α_3, α_4 for (x_m, y_m, u, v, w, ψ_2).

	x_m	y_m	ψ_2	u	v	w
α_0	-0.0746	-0.0546	103	0.00401	-0.0135	-0.948
α_1	0.119	-0.828	-13.4	-0.00750	-0.0106	-0.108
α_2	-0.469	-0.0798	-0.501	-0.110	0.0005	0.0295
α_3	0.0372	0.0622	1.045	0.00250	0.00111	-0.221
α_4	0.237	0.433	472	-0.00901	0.0537	-1.25

Table 4. Regression coefficients of Eq.(10).

Figure 13: Temporal varying regression coefficients for u, v, w, and ψ_2.

8 Conclusions

(1) Moment of a falling cylinder in water column is a highly nonlinear process, which should be described by both the momentum and moment of momentum equations. If the moment of momentum equations are absent, the motion of falling cylinder cannot be completely simulated. The new mine impact burial prediction model should include the moment of momentum equations.

(2) Six different trajectory patterns (straight, spiral, flip, flat, seesaw, combination) were detected from CYDEX. No tumbling of cylinder was observed. The flip of cylinder occurs only once for negative χ-values (i.e., COM is above GC as the cylinder enters the water surface). The flat pattern occurs usually for $\chi = 0$ (i.e., COM coincides with GC). The transition between patterns depends on the initial conditions (drop angle $\psi_2^{(in)}$ and initial velocity $V^{(in)}$) and the internal structure of cylinder (such as L/D ratio, χ-value, etc.). The dynamics of trajectory pattern formation and transition is very complicated. It involves stability, nonlinear dynamics , and fluid-body interaction.

(3) CYDEX shows that both L/D and $V^{(in)}$ had little influence on cylinder's impact attitude on the bottom. The drop angle ($\psi_2^{(in)}$) and χ, however, were the determinants of impact attitude. For $\chi = 0$, the cylinder was almost parallel to the bottom. For χ_{-2} and χ_2 cases, the cylinder is almost vertical to the bottom.

(4) CYDEX provided nondimensional data of position x_m, y_m, z_m, velocity u, v, w and the attitude ψ_2 for each input including data $\psi_2^{(in)}$, L/D ratio, $V^{(in)}$, and χ. The data can be used for model development and validation.

(5) The observed trajectories were far more complex than those theorized by using only the momentum equations. Simply assigning a rotation rate into the model will not simulate the movement of falling cylinder in the water column. At a minimum, updates to the IMPACT 25 model should include the more realistic moment of momentum equations.

(6) Further research on mine hydrodynamics is needed. The research needs to expand beyond the simple cylindrical shaped mine to those that are irregularly shaped (Rockan and Manta types). Additionally, the utilization of scaled down versions should be explored. A smaller mine that can be modeled as accurately as its real counterpart will save time, money and will require less logistical support.

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