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Model based GPS/INS integration for high accuracy land vehicle applications: calibration of a swarm of MEMS sensors

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Abstract— **We consider the problem of reconstructing the trajectory of a mobile mapping system based on a mid-size van. Mobile mapping requires, of course, high accuracy. Usually this is achieved by resorting to costly GPS/INS integrated systems. The INS, in particular, must guarantee high performance when the GPS signal is occluded. This paper concerns the possibility of using, in alternative, a swarm of low cost MEMS accelerometers mounted in random positions and orientations. In order to be able to reconstruct the trajectory, the relative position and orientation of each accelerometer should be known. Here, we propose a method for an automatic calibration of the cloud of MEMS sensors and an algorithm for trajectory reconstruction by GPS and MEMS accelerometers integration.**

I. Introduction

GPS/INS integration is a paradigmatic application of state reconstruction with multi-sensor architectures. The integration is often based on the implementation of an extended Kalman filter estimating the state of a nonlinear dynamical system describing rigid body dynamics, sensors parameters and error moments. Relevant areas of research in this field concern, on one hand customized dynamical models which guarantee better accuracy [4] and, on the other hand, the development and application of more advanced nonlinear filtering techniques such as Montecarlo filters [5].The two afore mentioned approaches have been combined in the paper [7] in order to maximize the accuracy of the reconstruction of the trajectory followed by a vehicle for mobile mapping applications. A dynamical nonholonomic model of the vehicle including load transfer in roll and pitch was derived. The model presents nonlinearities which make the state estimation problem challenging. For example, from GPS measurements only, the angular velocity is linearly non observable. The nonholonomic constraints are directly included in the model, differently from other approaches presented in the literature [4] which model them as virtual observations. Such a detailed model is necessary to achieve satisfactory accuracies in the reconstruction when the GPS signal is not available for long time windows. Infact, all the other sensors, the INS system, the odometer etc. are dead-reckoning and the error is integrated in time. A refined dynamical model allows to control the drift of the sensors and maintain the accuracy of the reconstruction within tolerable values.

In this paper, we propose to use a swarm of low cost, miniaturized MEMS accellerometers instead of an INS. The accellerometers are spread in random positions and orientations on the vehicle. Inspired by [6], in [8] and [9], we proposed an algorithm for trajectory reconstruction based on these measurements. We exploit the accuracy of the vehicle model described above and the sensor redundancy to improve the characteristics of each MEMS sensor. The integrated system should reach the performance of a INS, but cost much less. The problem of this approach is that the relative position and orientation of each accelerometer should be known. Unfortunately, these are not easy to measure. In [9], where we applied the algorithm for the reconstruction of the trajectory followed by a motorcycle, we used a high accuracy motion capture system that is available to our laboratory to measure the position and orientation of the accelerometers.

In this paper, we propose an automatic calibration algorithm. We first acquire the accelerometers data together with high accuracy DGPS/INS data. Knowing the vehicle trajectory, we calibrate the MEMS accelerometers by applying standard system identification techniques. From then on, the trajectory is reconstructed using the GPS and the swarm of on-board MEMS. In this way a fleet of mobile mapping vehicles would require only one high cost GPS/INS system for calibration and operate with low cost systems instead.

II. Model based sensor integration

The integration of internal information (MEMS measurements) with external information (GPS measurements) is necessary to guarantee the accuracy bound required by mobile mapping applications, whenever the GPS signal is occluded. Standard GPS positioning algorithms are sufficient to determine the vehicle position with the required accuracy, but the use of a INS unit is essential whenever the GPS signal is absent.

The predominant error sources of dead-reckoning sensors are biases. To reduce such errors, we proposed [7] to design the reconstruction algorithm on the basis of an accurate model of the vehicle. Two reference frames are considered: the inertial frame (or navigation frame) $\Sigma_I = \{\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z\}$ moving with the Earth, and the body frame $\Sigma_B = \{\mathbf{u}_{bx}, \mathbf{u}_{by}, \mathbf{u}_{bz}\}$, fixed with the vehicle. The orientation of Σ_B with respect to Σ_I is given by a transformation based on the Euler angles, θ, ψ, ϕ which are shown in figure 1. In this paper the order of rotation is around $\mathbf{u}_{bz}$, followed by $\mathbf{u}_{by}$ and $\mathbf{u}_{bx}$. Such rotation is called 321 in [1]. The position of the vehicle x, y, z is assumed to be the coordinate of the origin O_B of the body frame in the inertial frame. O_B is located at the center of the rear axle

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with $\mathbf{u}_x$ defining the direction of motion. Without loss of generality and in order to increase readability, we assume that the GPS receiver and the INS unit are located in O_B.

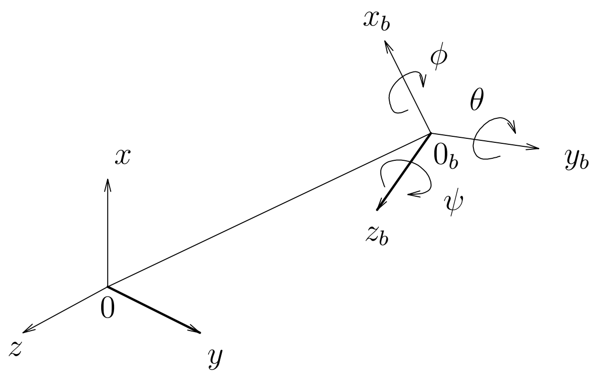

Fig. 1. Reference frame and body frame

The state-space model describing the vehicle trajectory is kept simple by approximating the vehicle with a one-track model constrained to move without sliding along the road surface. The nonholonomic constraints imply that the velocity of the vehicle is

$$\mathbf{v}_b = \begin{bmatrix} v_{bx} \\ v_{by} \\ v_{bz} \end{bmatrix} = \begin{bmatrix} v_{bx} \\ 0 \\ 0 \end{bmatrix}, \tag{1}$$

i.e. there is no sliding along $\mathbf{u}_{by}$, nor vertical motion. Of course, such conditions are never satisfied in practice and in [7], it has been shown how to take into account the fact that $v_{by} \neq 0$, and $v_{bz} \neq 0$.

The kinematic model describing the trajectory of the point (x, y, z) in the inertial frame is given by the following nonlinear differential equations

$$\begin{aligned} \dot{x} &= v_{bx}\cos(\theta)\cos(\psi) \\ \dot{y} &= v_{bx}\cos(\theta)\sin(\psi) \\ \dot{z} &= v_{bx}\sin(\theta) \end{aligned}$$

where v_{bx} is the velocity in the body frame, (under assumption (1)), and θ and ψ are the pitch and yaw angles, respectively. In addition, the rotation rates of the vehicle are introduced in the model as follows

$$\begin{aligned} \dot{\psi} &= \frac{\omega_{by}\sin(\phi) + \omega_{bz}\cos(\phi)}{\cos(\theta)} \qquad (2) \\ \dot{\theta} &= \omega_{by}\cos(\phi) - \omega_{bz}\sin(\phi) \\ \dot{\phi} &= \omega_{bx} + [\omega_{by}\sin(\phi) + \omega_{bz}\cos(\phi)]\tan(\theta) \end{aligned}$$

where ω_{bx}, ω_{by}, ω_{bz} are the angular velocities in the body frame. It is important to highlight that configurations with $\theta = \pm\frac{\pi}{2}$ introducing singularities in (2) cannot occur with land vehicles. The assumption $\mathbf{v}_b = [\, v_{bx} \quad 0 \quad 0\,]^T$ can be relaxed introducing dynamic equations for v_{by} and v_{bz}, i.e., considering the accelerations a_{bx}, a_{by}, a_{bz} along the body axes,

$$\begin{aligned} \dot{v}_{bx} &= a_{bx} \\ \dot{v}_{by} &= a_{by} + v_{bx}\omega_{bz} \\ \dot{v}_{bz} &= a_{bz} - v_{bx}\omega_{by} \\ \dot{a}_{bx} &= \nu_{bx}^a \\ \dot{a}_{by} &= \nu_{by}^a \\ \dot{a}_{bz} &= \nu_{bz}^a \end{aligned}$$

where $\nu_{bx}^a, \nu_{by}^a, \nu_{bz}^a$ are white uncorrelated noises. Assuming a similar description of the angular velocities dynamics, the complete model becomes

$$\left\{ \begin{aligned} \dot{x} &= v_{bx}\cos(\theta)\cos(\psi) \\ \dot{y} &= v_{bx}\cos(\theta)\sin(\psi) \\ \dot{z} &= v_{bx}\sin(\theta) \\ \dot{s} &= v_{bx} \\ \dot{\psi} &= \tfrac{\omega_{by}\sin(\phi)+\omega_{bz}\cos(\phi)}{\cos(\theta)} \\ \dot{\theta} &= \omega_{by}\cos(\phi) - \omega_{bz}\sin(\phi) \\ \dot{\phi} &= \omega_{bx} + [\omega_{by}\sin(\phi) + \omega_{bz}\cos(\phi)]\tan(\theta) \\ \dot{v}_{bx} &= a_{bx} \\ \dot{a}_{bx} &= \nu_{bx}^a \\ \dot{v}_{by} &= a_{by} + v_{bx}\omega_{bz} \\ \dot{a}_{by} &= \nu_{by}^a \\ \dot{v}_{bz} &= a_{bz} - v_{bx}\omega_{by} \\ \dot{a}_{bz} &= \nu_{bz}^a \\ \dot{\omega}_{bx} &= \nu_{bx}^\omega \\ \dot{\omega}_{by} &= \nu_{by}^\omega \\ \dot{\omega}_{bz} &= \nu_{bz}^\omega \end{aligned} \right. \tag{3}$$

where $\nu_{bx}^\omega, \nu_{by}^\omega, \nu_{bz}^\omega$ are white noises.

A remark is now at order. In the model, there is an equation giving the dynamics of the arclength coordinate s. Such equation seems to be redundant, since $\dot{s}^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$. However, it turns out that the explicit inclusion of such equation in the model helps to increase accuracy. In order to consider the biases of the dead-reckoning sensors, additional state variables are considered and modeled as random walks.

A. Accelerometers measurement equation

The derivation of the algorithm for trajectory reconstruction based on a swarm of accelerometers is very similar to [6]. Here, however, we expresss the measurement w.r.t. the vehicle body frame Σ_B in order to include the non holonomic constraint. A MEMS accelerometer is an integrated device which measures acceleration along a specific direction. Let there be N MEMS accelerometers and β_i be the sensing axis of accelerometer i with respect to the body frame Σ_B. Let the accelerometer i be located in point p_i on the vehicle. Let $g_{IB} \in SE(3)$ be the rigid body transformation that takes the body frame Σ_B in Σ_I. The spatial velocity of the vehicle, is

$$V_{IB}^I = \dot{g}_{IB}g_{IB}^{-1}$$

V_{IB}^I belongs to $se(3)$ the algebra of $SE(3)$, the special Euclidean group of rototranslation matrices, and, in particular, it is

$$V_{IB}^I = \left[\begin{array}{cc} \Omega_{IB}^I & -\Omega_{IB}^I p_{IB} + \dot{p}_{IB} \\ 0_{1x3} & 0 \end{array} \right]. \quad (4)$$

The first 3x3 principal minor Ω_{IB}^I is an element of $so(3)$ the algebra of $SO(3)$ the group of rotation matrices and it is the angular velocity of the body expressed in the spatial frame Σ_I. The velocity of the point p_i in inertial frame coordinates is

$$\dot{p}_i^I = V_{IB}^I p_i^I$$

where p_i has been written as an homogeneous point, i.e. adding a fourth component equal to 1. The acceleration of point p_i in the inertial frame is, therefore,

$$\ddot{p}_i^I = \dot{V}_{IB}^I p_i^I + (V_{IB}^I)^2 p_i^I.$$

The acceleration in body frame coordinates is

$$\ddot{p}_i^B = g_{IB}^{-1} \ddot{p}_i^I \quad (5)$$

and the accelerometer measurement is

$$\xi_i = (\ddot{p}_i^B)^T \beta_i. \quad (6)$$

It is convenient to write the measurements writing the velocities in the instantaneous body frame

$$\begin{aligned} \xi_i &= (g_{IB}^{-1} (\dot{V}_{IB}^I g_{IB} p_i^B + (V_{IB}^I)^2 g_{IB} p_i^B)^T \beta_i \\ &= (\dot{V}_{IB}^B p_i^B + (V_{IB}^B)^2 p_i^B)^T \beta_i. \end{aligned} \quad (7)$$

The body velocity V_{IB}^B is

$$V_{IB}^B = \left[\begin{array}{cc} \Omega_{IB}^B & R_{IB}^T \dot{p}_{IB} \\ 0_{1x3} & 0 \end{array} \right]$$

where R_{IB}^T is the rotation matrix between frame Σ_B and Σ_I and

$$\Omega_{IB}^B = \left[\begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right] \wedge .$$

The GPS/MEMS integration algorithm is based upon the measurements equations (**??**) and the vehicle dynamical model described above. Equation (**??**) requires the knowledge of the position p_i and orientation β_i of each accelerometer. Errors in these data reduce the accuracy of the reconstruction even if, thanks to the redundancy of sensors if $N > 6$, with appropriate statistical techniques such as RANSAC one can determine and eliminate outliers.

We propose an automatic calibration technique based on the above model used in a reverse fashion. We assume the motion to be known and we estimate the parameters of each MEMS sensor accordingly.

III. Identification of the accellerometers positions and orientations

The identification algorithm is based on an extended Kalman filter. The vehicle motion g_{IB}, V_{IB} is assumed to be known. It is measured with an accurate GPS/INS integrated system. During calibration MEMS sensors data are acquired together with those of a high performance GPS/INS integrated system. Applying the technique described in [7] we reconstruct the trajectory of the vehicle not using the MEMS accelerometers. We calibrate the sensors and we use them for reconstructiong the trajectory from then on. The costly high accuracy GPS/INS system is used only for calibration and it can serve, therefore, a whole fleet of mobile mapping vehicles. We use half of the data of the calibration campaign for identifying the sensor parameters and the other half for validating it. Equation (**??**) is seen as an observation equation for the new state which is composed by the orientations β_i and the positions p_i^B. These, being parameters, are dynamically modeled as random walks. Identifiability is easily proven since changing the orientation or the position of one accelerometer would change its measurements. Clearly, we assume that the trajectory excites the dynamics and produces accelerations that span the whole parameter space.

IV. Experimental results

The main limitation to the system performances is the accuracy of the reconstruction in the calibration phase. The better the MEMS accelerometers are calibrated the better the ensuing reconstructions are. To evaluate the uncertainty affecting trajectory reconstruction, the following tests have been carried out:

a.) Evaluation of the behaviour of the algorithm with the vehicle moving along different types of roads.

b.) Analysis of trajectory uncertainty as resulting from simulated long length GPS-outages.

A. Trajectory reconstruction results

To secure good behaviour in every circumstance, the algorithm has been tested by simulating the data that can arise from the sensors mounted on the van driving along two different roads, one on a plain (see Fig. 2 and 3, the other in a mountainous region. Both roads trajectories have been obtained by interpolating real data. The simulated sensors' data has been obtained by considering the characteristics specified in the relative data-sheets.

Applying the algorithm on the simulated data has been useful for a first coarse choice of the covariance state matrix, for testing the correct behaviour in both the situations and realize what is the maximum accuracy that it can achieve with the sensors in use.

The algorithm has been tested on fifty different realizations of the sensors noise to obtain a proper characterization of its accuracy. In Tab. I we reported the mean over the fifty realizations of the maximum error between the algorithm

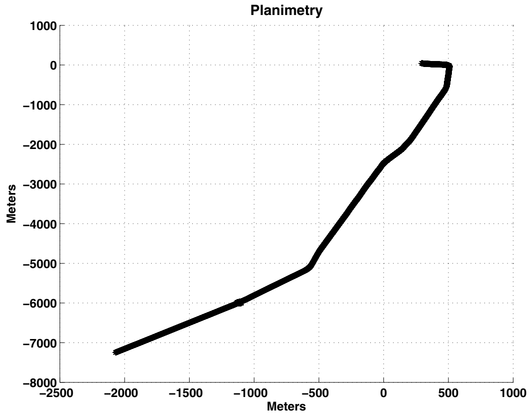

Fig. 2. Characteristic of the road on a plain (planimetry)

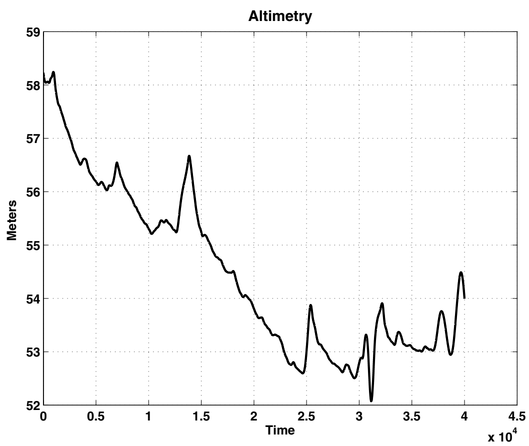

Fig. 3. Characteristic of the road on a plain (altimetry)

results and the real data, computed by simulating 5 minutes of missing GPS.

Fig. 4 shows a realization of the real error obtained by simulating a 5 minute GPS-outage within and without the smoother phase. The graph refres to the road on the plain, but the result is the same of the mountain case.

Tab. I and Fig. 4 show that the effect of the smoothing phase is particularly relevant and that after this step the accuracy of the filtered data is out by less than one meter, even with very-long GPS-outage.

TABLE I

SIMULATIONS RESULTS

	Plan	Altim	Pitch	Roll
Mean	0.638	0.065	0.0067	0.0034
Standard deviation	0.257	0.016	0.0003	0.0002

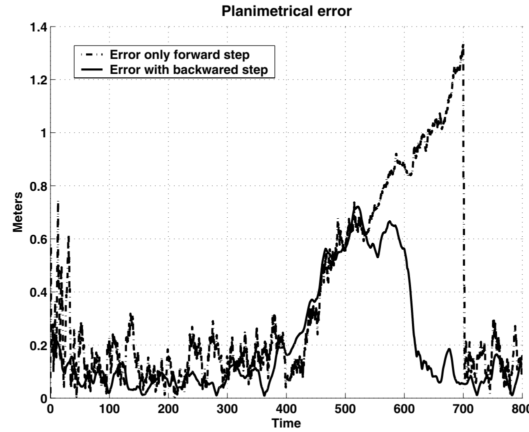

Fig. 4. Planimetrical errors simulating a 5 min GPS-outage in the on the plain route

B. Identification results

During the calibration tests, even if the vehicle was moving on a horizontal path, due to bumps in the road, the vehicle dynamics were nonetheless excited also along the vertical axis. The convergence of the filter is shown in Figure 5.

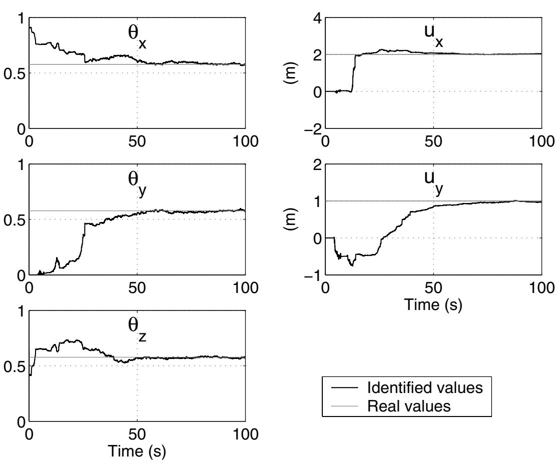

Fig. 5. Convergenge of accelerometer parameter estimates

The algorithm reveals a good insensibility to possible misalignments of the accelerometer with respect to the horizontal plane, allowing a correct identification of the parameters even if the sensor is mounted on the vehicle with a wrong (but sufficiently small) angle. For instance in Figure 6 we present the result of a simulation in which the component β_z is assumed to be zero, but actually the sensor has a wrong inclination with respect to the horizontal plane of about 6 degrees; the identification is anyway correct.

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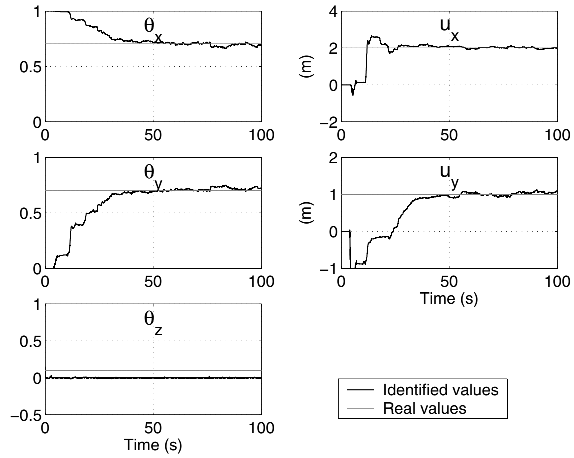

Fig. 6. Horizontal misalignment

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