2004 IEEE Intelligent Transportation Systems Conference
Washington, D.C., USA, October 3-6, 2004

Trajectory reconstruction by integration of GPS and a swarm of MEMS accelerometers: model and analysis of observability

Christian Spagnol, Riccardo Muradore, Manolo Assom, Alessandro Beghi and Ruggero Frezza

Abstract— **In this paper, we consider the problem of trajectory reconstrucion for the mobile mapping system Davide. Davide is equipped with a custom GPS/INS system and a number of dead-reckoning sensors, such as odometers, inclinometers, etc. In this paper, we investigate the possibility of substituting the INS with a swarm of low cost MEMS accelerometers located in various positions on board the vehicle. An observability analysis shows the feasibility of the solution and the need of integrating the distributed MEMS sensing architecture with a positioning sensor such as the GPS.**

I. INTRODUCTION

GPS/INS integration is a paradigmatic application of state reconstruction with multi-sensor architectures. The integration is often based on the implementation of an extended Kalman filter estimating the state of a nonlinear dynamical system describing rigid body dynamics, sensors parameters and error moments. Relevant areas of research in this fields concern, on one hand customized dynamical models which guarantee better accuracy [4] and, on the other hand, the development and application of more advanced nonlinear filtering techniques such as Montecarlo filters [8]. In a previous paper [6], we combined the two afore mentioned approaches in order to maximize the accuracy of the reconstruction of the trajectory followed by a vehicle for mobile mapping applications, the Davide van. A dynamical nonholonomic model of the vehicle including load transfer in roll and pitch has been derived. The model presents nonlinearities which make challenging the state estimation problem. The nonholonomic constraints are directly included in the model, differently from other approaches presented in the literature [4] which model them as virtual observations. Such a detailed model is necessary to achieve satisfactory accuracies in the reconstruction when the GPS signal is not available for long time windows. In fact, all the other sensors, the INS system, the odometer etc. are dead-reckoning and the error is integrated in time. A refined dynamical model allows to control the drift of the sensors and maintain the accuracy of the reconstruction within tolerable values. The model is nonlinear which implies the use of nonlinear estimation techniques such as the standard extended Kalman filter.

In this paper, we investigate an alternative technology to the INS. In particular, we propose to use a swarm of low cost MEMS accelerometers located in different positions on board the vehicle. This idea is not entirely new. In [7] a similar sensing architecture has been proposed. What characterizes the research presented in this paper are: (a) the use of a simple, yet dynamically accurate, nonholonomic vehicle model to increase accuracy and (b) a full observability analysis of the state of the model based on the MEMS measurements. The analysis shows the need of integrating the MEMS sensors with positioning data generated, for example, by a GPS system. It also represents an innovative methodology to evaluate sensing architectures in terms of sensors locations, numbers, and type.

The paper is organized as follows. In the first Section we illustrate the vehicle dynamical model used for multi sensors integration. In the following Section, we present the observability analysis. Finally, we show the estimation procedure and we present some results obtained in a simulated environment. By the date of the conference we will be able to present results obtained on an experimental setting with a vehicle equipped with all the needed sensors [2].

II. PROBLEM STATEMENT

Davide is a van for mobile mapping developed, maintained and used by Giove S.r.l. a company based in Treviso, Italy. Davide is equipped with a number of digital and analog cameras and other kind of sensors. In order to register the sensor data geographically, Davide uses a custom GPS/INS system. To accurately register the location of sites

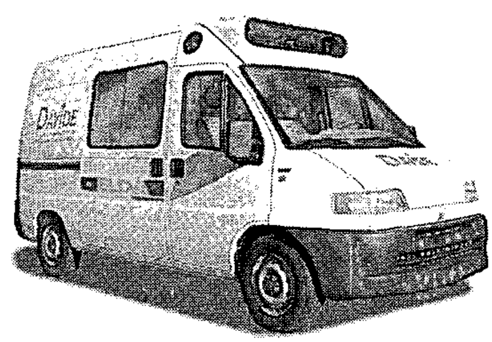

Fig. 1. The Davide van

Christian Spagnol is with the Department of Information Engineering, University of Padova, 35131 Padua, Italy akira28@libero.it
Manolo Assom is with Giove S.r.l., Treviso, Italy m.assom@giove.org
Alessandro Beghi is with the Department of Information Engineering, University of Padova, 35131 Padua, Italy beghi@dei.unipd.it
Ruggero Frezza is with the Department of Information Engineering, University of Padova, 35131 Padua, Italy frezza@dei.unipd.it
Riccardo Muradore is with the Department of Chemical Engineering, University of Padova, 35131 Padua, Italy muradore@dei.unipd.it

0-7803-8500-4/04/$20.00 ©2004 IEEE

of interest, it is fundamental to achieve the highest possible accuracy in the GPS/INS trajectory reconstruction. The problem we address in this paper is the study of alternative, low cost and reliable sensing architectures to the INS. As sensing elements, we used the LIS2L02AS MEMS accelerometers by ST Microelectronics. These devices are capable of measuring accelerations up to $\pm 6\,g$, and have a 4 kHz bandwidth. To remove aliasing effects, we further reduced the bandwidth by using a low pass second order Butterworth filter at 500 Hz.

In [7] a distributed MEMS accelerometers architecture has been proposed for a similar goal. Our work is innovative for two reasons: (a) the use of a particular dynamical model of the vehicle; (b) a full observability analysis on the model based on the MEMS measurements.

III. THE MATHEMATICAL MODEL OF THE VEHICLE

In our previous work [6], the integration of INS measurements with GPS measurements has been used to guarantee the accuracy whenever a *black* window in the GPS measurements occurs. Now we want to substitute the INS unit with a low cost set of MEMS accelerometers which are less expensive, easy to initialize and do not require the introduction in the mathematical model of state variables taking into account the bias effects.

As usual, two reference frames have to be introduced: the inertial frame (or navigation frame) $\Sigma_I = \{\mathbf{u}_X, \mathbf{u}_Y, \mathbf{u}_Z\}$, centered in O_I, moving with the Earth, and the body frame $\Sigma_b = \{\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z\}$, centered in O_b, fixed with the vehicle. From now on we use capital letters to identify point, velocity, etc. in the inertial frame, and small letters to identify the same quantities in the body frame (i.e. $\mathbf{P}$, $\mathbf{V}$, $\mathbf{A}$ in Σ_I and $\mathbf{p}$, $\mathbf{v}$, $\mathbf{a}$ in Σ_b). The orientation of Σ_b with respect to Σ_I is given by a transformation 321 based on the Euler angles, θ (pitch), ψ (yaw), ϕ (roll), which are shown in figure 2, [9]. The position of the vehicle X, Y, Z is assumed to be the coordinate of the origin O_b of the body frame in the inertial frame. O_b is located at the center of the rear axle, $\mathbf{u}_x$ defines the direction of motion, and the GPS receiver is assumed, without loss of generality, located in O_b.

A. The state equation

Following [6], the vehicle is approximated by a bicycle constrained to move without sliding along the road surface. Since such nonholonomic constraints imply that the velocity of the vehicle in the body frame is

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix}, \qquad (1)$$

the kinetic model describing the trajectory of the point (X, Y, Z) in the inertial frame is given by the following

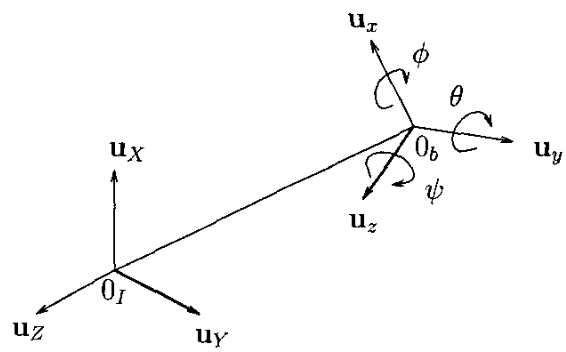

Fig. 2. Reference frame and body frame

nonlinear differential equations

$$\begin{aligned} \dot{X} &= v_x \cos(\theta)\cos(\psi) \\ \dot{Y} &= v_x \cos(\theta)\sin(\psi) \\ \dot{Z} &= v_x \sin(\theta). \end{aligned}$$

In order to describe the dynamic of the angular velocity, the rotation rates of the vehicle, and the approximation in the nonholonomic constraint (1), the above equations are completed with

$$\begin{aligned} \dot{\psi} &= \frac{\omega_y \sin(\phi) + \omega_z \cos(\phi)}{\cos(\theta)} \\ \dot{\theta} &= \omega_y \cos(\phi) - \omega_z \sin(\phi) \\ \dot{\phi} &= \omega_x + [\omega_y \sin(\phi) + \omega_z \cos(\phi)] \tan(\theta) \\ \dot{v}_x &= a_x \\ \dot{v}_y &= a_y + v_x \omega_z \\ \dot{v}_z &= a_z - v_x \omega_y \\ \dot{a}_x &= \nu_x^a \\ \dot{a}_y &= \nu_y^a \\ \dot{a}_z &= \nu_z^a \\ \dot{\omega}_x &= \alpha_x \\ \dot{\omega}_y &= \alpha_y \\ \dot{\omega}_z &= \alpha_z \\ \dot{\alpha}_x &= \nu_x^\omega \\ \dot{\alpha}_y &= \nu_y^\omega \\ \dot{\alpha}_z &= \nu_z^\omega \end{aligned}$$

where $\omega = [\omega_x \ \ \omega_y \ \ \omega_z]^T$ is the angular velocity in the body frame, $\mathbf{a} = [a_x \ \ a_y \ \ a_z]^T$ is the acceleration in the body frame, $\alpha = [\alpha_x \ \ \alpha_y \ \ \alpha_z]^T$ and ν_x^a, ν_y^a, ν_z^a, ν_x^ω, ν_y^ω, ν_z^ω are white uncorrelated noises. It is assumed that the angular velocity is approximated by a second order random walk. In order to increase the accuracy, it is useful to introduce the arclength coordinate s, [6], having the following dynamic

$$\dot{s} = v_x.$$

The nonlinear state equation can then be written in a compact form as

$$\dot{\mathbf{x}}(t) \quad = \quad f(\mathbf{x}(t)) + \mathbf{w}(t) \tag{2}$$

where $\mathbf{w}$ is the model error. It is assumed to be a zero-mean white process and its variance will be chosen in accordance with all the model assumptions.

B. Measurement equation

The low-cost measurement system is composed by a commercial GPS unit and an accelerometers data acquisition system.

The GPS unit gives the vehicle position in the inertial reference frame X^m, Y^m, Z^m. The accelerometers data acquisition system consists of a swarm of N MEMS accelerometers. The i-th accelerometer is located in point $\mathbf{p}_i$ (known and fixed in Σ_B) on the vehicle. It measures the acceleration $\mathbf{a}_i$ of $\mathbf{p}_i = [p_{xi} \quad p_{yi} \quad p_{zi}]^T$ along a specific direction $\mathbf{d}_i$, i.e. the sensing axis of the accelerometer. To relate the acceleration measurements to the variables in the model equations, a little bit of work is necessary. Let R and T be the rotation matrix and the translation $T = \overrightarrow{O_I O_b}$ relating the body frame Σ_b and the inertial frame Σ_I, respectively. Since the identity $\mathbf{P}_i = T + R\mathbf{p}_i$ holds, by time derivation we have

$$\begin{aligned} \mathbf{A}_i \quad = \quad & \mathbf{A} + R\omega \times \omega \times \mathbf{p}_i + +R\dot{\omega} \times \mathbf{p}_i + 2R\omega \times \dot{\mathbf{p}}_i + \\ & +R \times \omega \times \ddot{\mathbf{p}}_i + \mathbf{g} \end{aligned} \tag{3}$$

where $\mathbf{A}_i$ is the acceleration in the reference frame of the i-th accelerometer, $\mathbf{A}$ is the acceleration in the reference frame of the vehicle and $\mathbf{g}$ is the gravity. Using the shape invariance to rigid body motion, we have that the relative position of the MEMS accelerometer and the origin of the body frame O_b is fixed. Then the terms $\ddot{\mathbf{p}}_i$ and $\dot{\mathbf{p}}_i$ vanish. At the end the expression for each accelerometer in the body frame becomes, (see also [7])

$$\mathbf{a}_i = \mathbf{a} + \dot{\omega} \times \mathbf{p}_i + \omega \times \omega \times \mathbf{p}_i + R^T\mathbf{g}. \tag{4}$$

The above equation can be also rewritten as

$$\mathbf{a}_i = \mathbf{a} + \dot{\Omega}\mathbf{p}_i + \Omega^2\mathbf{p}_i + R^T\mathbf{g}. \tag{5}$$

where Ω is the skew-symmetric matrix

$$\Omega = \omega\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \tag{6}$$

The component of the acceleration $\mathbf{a}_i$ along the direction $\mathbf{d}_i$ is given by $a_i^m = \langle \mathbf{a}_i - R^T\mathbf{g}, \mathbf{d}_i \rangle = (\mathbf{a}_i - R^T\mathbf{g})^T\mathbf{d}_i$. Let $[d_{xi} \quad d_{yi} \quad d_{zi}]^T$ be the components of $\mathbf{d}_i$ in the body frame. Then the i-th measurement equation given by the accelerometers data acquisition system takes the form (7) (top of the next page).

All the measurements $\mathbf{y} = [X^m \quad Y^m \quad Z^m \quad a_1^m \quad \dots \quad a_N^m]^T$ can be rewritten in a more compact form as

$$\mathbf{y}(t) \quad = \quad h(\mathbf{x}(t)) + \mathbf{e}(t)$$

where $\mathbf{e}$ is the vector having as components the measurement errors. Its variance matrix depends on the different accuracies with which the measurements are obtained.

IV. OBSERVABILITY ANALYSIS

It is easy to understand that using only accelerometers it is not possible to determine the trajectory. In fact straight trajectories at constant velocity are non observable. In this section we perform an observability analysis in order to show mathematically such fact and to achieve a tool to study the optimal location of the N MEMS accelerometers. In order to keep the analysis easy and without loss of generality, we consider a reduced state equation (subscript a)

$$\dot{\mathbf{x}}_a = f_a(\mathbf{x}_a) + \mathbf{w}_a \tag{8}$$

given by

$$\begin{cases} \dot{a}_x & = & \nu_x^a \\ \dot{a}_y & = & \nu_y^a \\ \dot{a}_z & = & \nu_z^a \\ \dot{\omega}_x & = & \alpha_x \\ \dot{\alpha}_x & = & \nu_x^\omega \\ \dot{\omega}_y & = & \alpha_y \\ \dot{\alpha}_y & = & \nu_y^\omega \\ \dot{\omega}_z & = & \alpha_z \\ \dot{\alpha}_z & = & \nu_z^\omega \end{cases} \tag{9}$$

Model (8) contains all the variables of interest for the accelerometer measurements a_i, $i =, \dots, N$. Let $N = 9$ be the available MEMS, the reduced measurement equation

$$\mathbf{y}_a = h_a(\mathbf{x}_a) + \mathbf{e}_a \tag{10}$$

takes the form

$$\mathbf{y}_a = \begin{bmatrix} a_1^m \\ \vdots \\ a_9^m \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_9 \end{bmatrix} (\mathbf{x}_a) + \begin{bmatrix} e_1 \\ \vdots \\ e_9 \end{bmatrix}.$$

The goal is to study the observability of the reduced system (8), (10) by means of nonlinear system theory tools, [5]. The observability space is the smallest space containing

$$\mathcal{O} = \left[h_1, \dots, h_9, L_{f_a}^1 h_1, \dots, L_{f_a}^{n-1} h_1, \dots, L_{f_a}^1 h_9, \dots, L_{f_a}^{n-1} h_9\right]^T$$

where $n = \dim\{\mathbf{x}_a\}$ and $L_{f_a} h_i(\mathbf{x}_a)$ is the Lie differentiation (the derivative of h_i along f_a)

$$\begin{aligned} L_{f_a} h_i(\mathbf{x}_a) \quad & = \quad \frac{\partial h_i}{\partial \mathbf{x}_a} f_a \\ L_{f_a}^k h(\mathbf{x}_a) \quad & = \quad L_{f_a}(L_{f_a}^{k-1} h_i(\mathbf{x}_a)). \end{aligned}$$

Starting from the h_i defined in (7), it is possible to write the other Lie derivatives (11)–(19). The system is locally observable in $\mathbf{x}_a^*$ if the codistribution

$$\begin{aligned} d\mathcal{O} \quad & = \quad \text{span}\left\{\frac{\partial h_1}{\partial \mathbf{x}_a}, \frac{\partial h_2}{\partial \mathbf{x}_a}, \dots, \frac{\partial h_9}{\partial \mathbf{x}_a}, \frac{\partial L_f h_1}{\partial \mathbf{x}_a}, \dots\right\} \\ & = \quad \text{span}\{dh_1, dh_2, \cdots, dh_9, dL_f h_1, \cdots\} \\ & = \quad \text{span}\{dH : H \in \mathcal{O}\} \end{aligned}$$

$$
\begin{aligned}
a_i^m \;=\; & \left(a_x - \dot{\omega}_z p_{yi} + \dot{\omega}_y p_{zi} - \omega_z(\omega_z p_{xi} - \omega_x p_{zi}) + \omega_y(-\omega_y p_{xi} + \omega_x p_{yi})\right) d_{xi} + \\
& + \left(a_y + \dot{\omega}_z p_{xi} - \dot{\omega}_x p_{zi} + \omega_z(-\omega_z p_{yi} + \omega_y p_{zi}) - \omega_x(-\omega_y p_{xi} + \omega_x p_{yi})\right) d_{yi} + \\
& + \left(a_z - \dot{\omega}_y p_{xi} + \dot{\omega}_x p_{yi} - \omega_y(-\omega_z p_{yi} + \omega_y p_{zi}) + \omega_x(\omega_z p_{xi} - \omega_x p_{zi})\right) d_{zi}
\end{aligned} \tag{7}
$$

is such that

$$\dim d\mathcal{O}(\mathbf{x}_a^\star) = n$$

where n is the size of the reduced system (8). This is equivalent to require that the rank of the matrix

$$
\begin{bmatrix}
dh_1 \\ \vdots \\ dh_9 \\ dL^1_{f_a} h_1 \\ \vdots \\ dL^1_{f_a} h_9 \\ \vdots \\ dL^{n-1}_{f_a} h_1 \\ \vdots \\ dL^{n-1}_{f_a} h_9
\end{bmatrix}
$$

is equal to n in $\mathbf{x}_a^\star$. The above differentials can be easily computed starting from equations (11)–(19), obtaining (20)–(28). The rank analysis will be performed in the Result Section for a particular trajectory.

V. KALMAN SMOOTHER

Since the recorded data will be processed off-line, the reconstruction algorithm is based on a discrete-time Kalman smoother. Starting from the model derived in the previous Sections,

$$
\left\{
\begin{array}{rcl}
\dot{\mathbf{x}}(t) & = & f(\mathbf{x}(t)) + \mathbf{w}(t) \\
\mathbf{y}(t_k) & = & h(\mathbf{x}(t_k)) + \mathbf{e}(t_k)
\end{array}
\right. \tag{29}
$$

where the measurements are considered to be available at times t_k, we want to build an estimator for the state variable $\mathbf{x}(t)$ with $t \in T := [t_o, t_f]$. A Rauch-Tung-Striebel fixed-interval optimal smoother [3] is implemented and the smoothed estimate of $\mathbf{x}$ based on all the measurements between 0 to T is given by

$$
\begin{array}{l}
\hat{\mathbf{x}}(k|T) = \hat{\mathbf{x}}_f(k|k) + K(k)\left[\hat{\mathbf{x}}(k+1|T) - \hat{\mathbf{x}}_f(k+1|k)\right] \\
\hat{\mathbf{x}}(T|T) = \hat{\mathbf{x}}_f(T|T)
\end{array} \tag{30}
$$

where the smoothed error covariance matrices $P(k|T)$ satisfy the backward recursive matrix equation

$$
\begin{array}{l}
P(k|T) = P_f(k|k) + K(k)\left[P(k+1|T) - P_f(k+1|k)\right] \\
P(T|T) = P_f(T|T).
\end{array} \tag{31}
$$

The matrix gain $K(k)$ is given by

$$K(k) \;=\; P_f(k|k) \cdot \hat{\Psi}^T(k|k) \cdot P_f^{-1}(k+1|k) \tag{32}$$

where the matrix $\hat{\Psi}(k|k)$ is the Jacobian obtained by discretization and linearization of the state equation (2), (or (8) in the MEMS only case). $\hat{\mathbf{x}}_f(k|k)$ is the forward Kalman filter with error covariance matrix $P_f(k|k)$, as usual. The solution (30)-(31) is particularly useful when the estimated states and the covariance matrices have to be recorded for a long time and have large dimensions, as is the case in the situation at hand.

Since GPS measurements are not always available, the measurement equation is of variable structure, i.e. the nonlinear function $h(\cdot)$ is a *switching* function:

$$
h(\mathbf{x}(t_k)) = \left\{
\begin{array}{ll}
h_Y(\mathbf{x}(t_k)), & \text{with GPS measurements} \\
h_N(\mathbf{x}(t_k)), & \text{without GPS measurements.}
\end{array}
\right.
$$

Then, a switching forward Extended Kalman filter has to be implemented and the Jacobians of the nonlinear function $h(\cdot)$ are

$$
H(\hat{\mathbf{x}}(k|k)) = \left\{
\begin{array}{l}
H_Y(\hat{\mathbf{x}}(k|k)) = \left.\frac{\partial h_Y(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k|k)}, \\
\qquad\qquad \text{with GPS measurements} \\
H_N(\hat{\mathbf{x}}(k|k)) = \left.\frac{\partial h_N(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k|k)}, \\
\qquad\qquad \text{without GPS measurements.}
\end{array}
\right.
$$

The tuning of the variance matrix Q for the model error $\mathbf{w}$ in the forward Kalman filter is based on a scheduled tuning, see [6].

VI. EXPERIMENTAL RESULTS

The following figures show the reconstruction of a vehicle trajectory based on the distributed MEMS accelerometers with and without GPS. Clearly, in the absence of GPS data the estimate drifts away as some state variables are not observable since the accelerometers are dead reckoning. The observability analysis based on the smallest singular value of the observability matrix. This matrix norm is of particular significance in the design of optimal sensing architectures in term of positioning, number and type of sensors. In figure 5, the time behavior of the smallest singular value of $\mathcal{O}$ is reported clearly showing the loss of observability when the vehicle moves on constant speed on a straight path. By the time of the deadline for the submission of the full paper we expect to be able to show the influence of sensors location on such norm and experiments on a vehicle equipped with 9 MEMS accelerometers with the characteristics described in section III.

VII. CONCLUSIONS

In this paper,we proposed an innovative distributed architecture based on MEMS accelerometers to create a low cost, reliable alternative to INS systems for vehicle trajectory reconstruction. The use of MEMS accelerometers is interesting not only because of the low cost, but also because

$$
\begin{aligned}
h_i &= (a_x - \alpha_z p_{yi} + \alpha_y p_{zi} - \omega_z (\omega_z p_{xi} - \omega_x p_{zi}) + \omega_y (-\omega_y p_{xi} + \omega_x p_{yi}))\, d_{yi} + \\
&\quad + (a_y + \alpha_z p_{xi} - \alpha_x p_{zi} + \omega_z (-\omega_z p_{yi} + \omega_y p_{zi}) - \omega_x (-\omega_y p_{xi} + \omega_x p_{yi}))\, d_{xi} + \\
&\quad + (a_z - \alpha_y p_{xi} + \alpha_x p_{yi} - \omega_y (-\omega_z p_{yi} + \omega_y p_{zi}) + \omega_x (\omega_z p_{xi} - \omega_x p_{zi}))\, d_{zi} \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
L^1_{f_a} h_i &= ((\omega_z p_{zi} + \omega_y p_{yi}) d_{yi} + (\omega_y p_{xi} - 2\omega_x p_{yi}) d_{xi} + (\omega_z p_{xi} - 2\omega_x p_{zi}) d_{zi})\alpha_x + \\
&\quad + ((-2\omega_y p_{xi} + \omega_x p_{yi}) d_{yi} + (\omega_z p_{zi} + \omega_x p_{xi}) d_{xi} + (\omega_z p_{yi} - 2\omega_y p_{zi}) d_{zi})\alpha_y + \\
&\quad + ((-2\omega_z p_{xi} + \omega_x p_{zi}) d_{yi} + (-2\omega_z p_{yi} + \omega_y p_{zi}) d_{xi} + (\omega_y p_{yi} + \omega_x p_{xi}) d_{zi})\alpha_z \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
L^2_{f_a} h_i &= ((-2p_{yi} d_{xi} - 2p_{zi} d_{zi})\alpha_x + (p_{yi} d_{yi} + p_{xi} d_{xi})\alpha_y + \\
&\quad + (p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_z)\alpha_x + ((p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_x + \\
&\quad + (p_{zi} d_{xi} + p_{yi} d_{zi})\alpha_y + (-2p_{xi} d_{yi} - 2p_{yi} d_{xi})\alpha_z)\alpha_y + \\
&\quad + ((-2\omega_y p_{xi} + \omega_x p_{yi}) d_{yi} + (\omega_z p_{zi} + \omega_x p_{xi}) d_{xi} + (\omega_z p_{yi} - 2\omega_y p_{zi}) d_{zi})\alpha_z \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
L^3_{f_a} h_i &= 0 \qquad (14)\\
L^4_{f_a} h_i &= 0 \qquad (15)\\
L^5_{f_a} h_i &= 0 \qquad (16)\\
L^6_{f_a} h_i &= 0 \qquad (17)\\
L^7_{f_a} h_i &= 0 \qquad (18)\\
L^8_{f_a} h_i &= 0 \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
dh_i &= [d_{yi} \quad d_{xi} \quad d_{zi} \quad (\omega_z p_{zi} + \omega_y p_{yi}) d_{yi} + (\omega_y p_{xi} - 2\omega_x p_{yi}) d_{xi} + (\omega_z p_{xi} - 2\omega_x p_{zi}) d_{zi} \quad \cdots \\
&\cdots \quad -p_{zi} d_{xi} + p_{yi} d_{zi} \quad (-2\omega_y p_{xi} + \omega_x p_{yi}) d_{yi} + (\omega_z p_{zi} + \omega_x p_{xi}) d_{xi} + (\omega_z p_{yi} - 2\omega_y p_{zi}) d_{zi} \quad \cdots \\
&\cdots \quad p_{zi} d_{yi} - p_{xi} d_{zi} \quad (-2\omega_z p_{xi} + \omega_x p_{zi}) d_{yi} + (-2\omega_z p_{yi} + \omega_y p_{zi}) d_{xi} + (\omega_y p_{yi} + \omega_x p_{xi}) d_{zi} \quad \cdots \\
&\cdots \quad -p_{yi} d_{yi} + p_{xi} d_{xi}\,] \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
dL^1_{f_a} h_i &= [0 \quad 0 \quad 0 \quad (-2p_{yi} d_{xi} - 2p_{zi} d_{zi})\alpha_x + (p_{yi} d_{yi} + p_{xi} d_{xi})\alpha_y + (p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_z \quad \cdots \\
&\cdots \quad (\omega_z p_{zi} + \omega_y p_{yi}) d_{yi} + (\omega_y p_{xi} - 2\omega_x p_{yi}) d_{xi} + (\omega_z p_{xi} - 2\omega_x p_{zi}) d_{zi} \quad \cdots \\
&\cdots \quad (p_{yi} d_{yi} + p_{xi} d_{xi})\alpha_x + (-2p_{xi} d_{yi} - 2p_{zi} d_{zi})\alpha_y + (p_{zi} d_{xi} + p_{yi} d_{zi})\alpha_z \quad \cdots \\
&\cdots \quad (-2\omega_y p_{xi} + \omega_x p_{yi}) d_{yi} + (\omega_z p_{zi} + \omega_x p_{xi}) d_{xi} + (\omega_z p_{yi} - 2\omega_y p_{zi}) d_{zi} \quad \cdots \\
&\cdots \quad (p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_x + (p_{zi} d_{xi} + p_{yi} d_{zi})\alpha_y + (-2p_{xi} d_{yi} - 2p_{yi} d_{xi})\alpha_z \quad \cdots \\
&\cdots \quad (-2\omega_z p_{xi} + \omega_x p_{zi}) d_{yi} + (-2\omega_z p_{yi} + \omega_y p_{zi}) d_{xi} + (\omega_y p_{yi} + \omega_x p_{xi}) d_{zi}\,] \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
dL^2_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 2(-2p_{yi} d_{xi} - 2p_{zi} d_{zi})\alpha_x + 2(p_{yi} d_{yi} + p_{xi} d_{xi})\alpha_y + 2(p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_z \quad \cdots \\
&\cdots \quad 0 \quad 2 \quad (p_{yi} d_{yi} + p_{xi} d_{xi})\alpha_x + 2(-2p_{xi} d_{yi} - 2p_{zi} d_{zi})\alpha_y + 2(p_{zi} d_{xi} + p_{yi} d_{zi})\alpha_z \quad \cdots \\
&\cdots \quad 0 \quad 2(p_{zi} d_{yi} + p_{xi} d_{zi})\alpha_x + 2(p_{zi} d_{xi} + p_{yi} d_{zi})\alpha_y + 2(-2p_{xi} d_{yi} - 2p_{yi} d_{xi})\alpha_z\,] \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
dL^3_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (23)\\
dL^4_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (24)\\
dL^5_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (25)\\
dL^6_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (26)\\
dL^7_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (27)\\
dL^8_{f_a} h_i &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \qquad (28)
\end{aligned}
$$

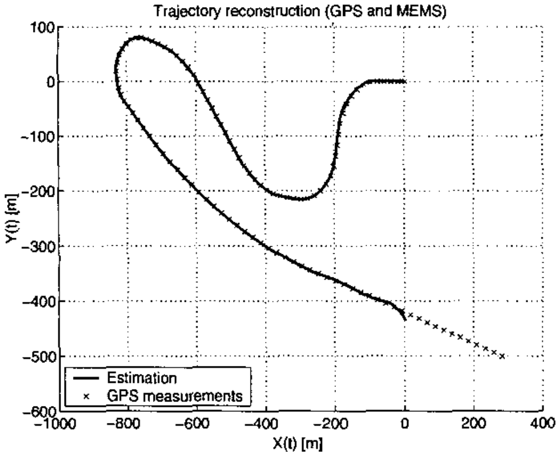

Fig. 3. Trajectory reconstruction using the GPS measurements and the MEMS accelerometer measurements

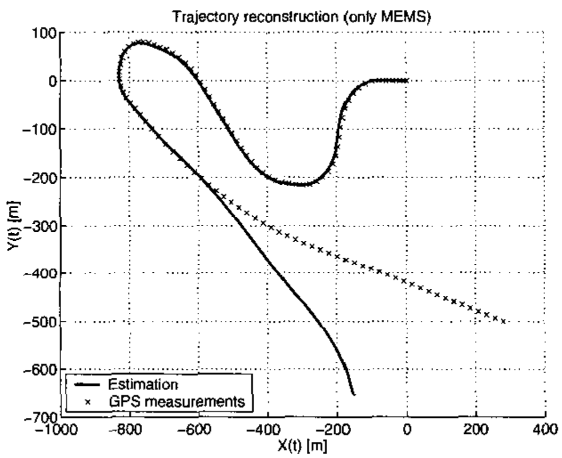

Fig. 4. Trajectory reconstruction using only the MEMS accelerometer measurements

it is easy to increase sensor redundancy by adding other units to the architecture at will. We have shown a full state observability analysis which, on one hand, demonstrates the need of a positioning sensors like a GPS and, on the other hand, gives a methodology to optimally locate the sensors on board the vehicle.

ACKNOWLEDGMENT

The authors thank Prof. Oboe for the interesting and motivating discussions and for the support in preparing the experimental setup.

REFERENCES

[1] Arulampalam, S., Maskell, S., Gordon, N. and Clapp, T., *A tutorial on particle filters for on-line non-linear/non-Gaussian Bayesian Tracking*, IEEE Transactions on Signal Processing, vol. 50(2), 2002.

[2] Gasbarro L., Beghi A., Frezza R., and Spagnol C., *Motorcycle Trajectory Reconstruction by Integration of Vision and MEMS Accelerometers*. submitted to IEEE CDC 2004.

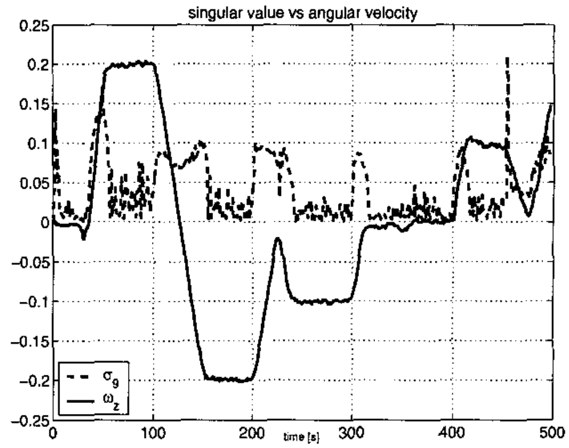

Fig. 5. Smallest singular value and angular velocity

[3] Gelb A., *Applied optimal estimation*, MIT Press, 1974.

[4] Dissanayake, G., Sukkarieh, S., Nebot, E., Durrant-Whyte, H., *The aiding of a low-cost strapdown inertial measurement unit using vehicle model constraints for land vehicle applications*, IEEE Transactions on Robotics and Automation, vol. 17(5), pp.731–747, 2001.

[5] Isidori A., *Nonlinear Control System*, Springer, 3rd edition, 1995.

[6] Spagnol C., Assom M., Beghi A., Frezza R., Muradore R. *Model based GPS/INS integration for high accuracy land vehicle applications*, submitted.

[7] Tan C.W., Park S., Mostov K., Varaya P., *Design of Gyroscope-Free Navigation Systems*, Proc. IEEE Intelligent Transportation Systems Conference, Oakland (CA), USA, Aug. 2001.

[8] Wan, E. A. and var der Merwe, R., *The unscented Kalman filter for nonlinear estimation*, Proceedings of Symposium on adaptive Systems for signal processing, comunications and control, (Lake Louise,Alberta,Canada, 2000).

[9] Wertz J.R., *Spacecraft Attitude Determination and Control*, Reidel Dordrecht, 1986.