Evaluating the Effectiveness of Shoot-Look-Shoot Tactics in the Presence of Incomplete Damage Information

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ABSTRACT

The advent of new technologies, and in particular the development of efficient detection sensors, have produced advanced long-range and accurate weapon systems.

One of the major requirements for operating these weapon systems effectively and efficiently is to properly assess the damage that was inflicted upon the targets by previously delivered rounds.

This assessment process is commonly called Bomb Damage Assessment (BDA).

This paper presents a number of shoot-look-shoot (SLS) tactics and evaluates their efficiency in situations where the availability of damage information is not certain. The evaluations are performed with respect to the expected number of kills criterion.

It is shown that certain SLS tactics are superior to others and that a certain simple tactic may be the best choice in terms of fire efficiency and operational convenience.

Key Words: *Damage Assessment, Shoot-Look-Shoot, Fire Efficiency*

APPLICATION AREAS: Land Warfare, Air Warfare

OR METHODOLOGY: Stochastic Processes, Dynamic Programming

INTRODUCTION

The advent of new technologies, and in particular the development of effective detection sensors and efficient fire-control devices, have resulted in advanced long-range and accurate weapon systems. HELLFIRE, MILAN, SLAM and GBU-15 (see, e.g. [7], [9] and [11]) are a few examples which represent the wide range of these types of weapon systems. In general, these systems usually seek to engage clusters of point targets such as tanks, artillery pieces and missile launchers. Once such a cluster has been detected, a single target is then acquired out of this cluster and a round of ammunition is delivered towards it. Such munitions, which are usually referred to as Precision Guided Munitions (PGM), are accurate, lethal and very expensive.

In combat there are usually two assets that are in short supply with respect to these types of weapon systems: one is ammunition and the other is time. Because of the high cost of precision-guided munitions on one hand, and weight limitations on the carrying platform (e.g. helicopters) on the other, the number of munitions available to a particular system, such as AT missiles, may be limited. The shooter may have to carefully consider the effectiveness or utility of each delivered round.

Time is a major factor to consider in situations where certain missions are contingent on others or when the targets are maneuverable and an extended engagement may provide them time to hide. In such cases the time frame allotted for completing a mission may be constrained. Furthermore, staying at the same position for an extended period of time may cause the weapon system to become vulnerable to the opponent's fire. Therefore, the objective in such instances is to complete the mission as fast as possible.

To operate efficiently both in terms of ammunition expenditure and time, one of the important tasks of a shooter is to properly assess the damage that was inflicted on the targets by previously delivered rounds. Such an assessment may save both ammunition and time by preventing the engagement of previously killed targets. Hence, damage assessment is a major issue to consider when evaluating the effectiveness of PGM systems. The importance of damage assessment was notable in the Gulf War ([3], [4], [5], [6]) where costly munitions were wasted because of poor damage assessment that led to incidences of multikill on one hand, and uncompleted missions on the other.

Shoot-Look-Shoot (SLS) is a firing tactic which comprises both fire and damage assessment. In the presence of only a single shooter, this tactic represents a sequential engagement where the shooter may occasionally assess the damage inflicted on a certain target before acquiring and shooting at other targets. A deterministic analysis of some general SLS tactics is given in [1] and [2].

The purpose of this paper is to introduce several SLS tactics for a single shooter. We define effectiveness criteria for these tactics and construct probabilistic models that represent them. These selected SLS tactics are analyzed with respect to these criteria and their relative effectiveness is evaluated using typical scenarios.

It is shown that the performance of a certain variant of the Persistent Shooting tactic (which is discussed in detail in Sec-

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tion 4) appears to be very close to optimality. According to this tactic, which is called HBPS, once a shooter ceases to engage a certain target it will never return to engage it again. The number of rounds expended at a certain target is bounded by a number that is updated as the engagement progresses. This tactic minimizes the number of instances of fire redirection and it is relatively simple to implement. Therefore it is recommended as a practical and efficient tactic even though it is not optimal in the mathematical sense [10].

In the next section we describe the basic combat situation, introduce notation and define the criteria according to which the SLS tactics are evaluated. In Section 3 the basic model with no damage assessment provisions is discussed. Section 4 describes and evaluates the four variants of the Persistent Shooting Tactic and the operational advantages of this family of shooting strategies are discussed. The performance of these tactics are compared to the results of an optimal strategy in Section 5. Section 6 contains concluding remarks and summary.

2. Description, Notation and Criteria

We consider a situation where a single shooter is engaging a cluster of m homogeneous point-targets. During the firing process we assume that the targets do not fire back at the shooter and therefore the latter is invulnerable. We also assume that the cluster of targets remains unchanged throughout the engagement, that is, no targets disappear from the engagement area and no new target enters into it.

The engagement process comprises three stages: acquisition, firing, and damage assessment. When a specific target has been detected by the weapon system and the conditions are such that immediate shooting towards it is possible, then it is said that the target has been **acquired.** The acquisition stage is usually time consuming and therefore it is a major factor in evaluating the effectiveness of the weapon. A common phenomenon is that of **false acquisition** when either a non-target or a previously killed target is mistakenly acquired. Throughout the paper we will consider false acquisitions of one type only, namely, acquiring killed targets. Once a target has been acquired, a round of ammunition is fired upon it. The result of the shooting can be either a hit or a miss. If the round hits the target it may either kill it or cause no damage. In other words, we assume that the shooting process causes no cumulative damage to targets.

A killed target may appear alive to the shooter if there are not enough signs to indicate otherwise. A killed target which also appears as killed to the shooter is called an **evidently killed (EK) target.** In the third stage of the engagement process the damage that was caused by the shooting is assessed. We assume that a previously killed target that was not EK at the time of kill may not appear to the shooter as killed later on unless it is shot upon again and is "killed" once more. In other words, there is no "record" for previously killed targets and therefore a necessary condition for detecting a kill, following a round of fire, is that the particular round was potentially lethal.

The damage assessment is designed to evaluate the impact of the engagement on one hand, and to minimize the number of incidences of false acquisitions on the other. Damage assessment is performed by inspecting the targets. We assume that there is no "type II" error, that is, a live target will never appear to the shooter as killed. This is a reasonable assumption since a kill indication is usually obtained under very rigid conditions.

In conclusion, following delivery of a round, a target may be in one of the three possible states (i) undamaged, (ii) killed with no signs to that effect (K) and (iii) evidently killed (EK).

Denote the single-round kill probability of a target by p_k and let $p_{s/k}$ denote the conditional probability that a killed target will be recognized as such by the shooter. Thus, $p_s = p_{s/k}\, p_k$ is the probability that a target is evidently killed by a single-shot. The random variables X_n and Y_n represent the number of killed (K) targets and evidently killed (EK) targets, respectively, following the delivery of n rounds. We define T_i as the random variable that counts the number of rounds needed to kill i targets.

The basic criterion according to which the various SLS tactics are evaluated is $E(X_n)$ - the expected number of killed targets following the delivery of n rounds.

However, the models that are presented in this paper may be suitable also for evaluation of other effectiveness measures such as: $M_i(q)$ - the number of munitions needed to obtain a kill level of i targets with probability of at least q; the probability for a minimum number (r) of killed targets by a given number (n) of rounds ($Pr[X_n \geq r]$); and the expected number of rounds needed to obtain damage level r.

In this paper we focus on four SLS tactics: (1) Basic Persistent Shooter (BPS), (2) Fixed Bound Persistent Shooter (FBPS), (3) Dynamic Bound Persistent Shooter (DBPS) and (4) Heuristics for Bounded Persistent Shooter (HBPS). These tactics, which are described in Section 4, are compared to each other and are evaluated against two benchmarks: (a) the basic tactic that takes into consideration no damage assessment, a tactic that is usually described by Urn Models [8], and (b) The Greedy Shooting (GS) tactic - which is a globally optimal tactic under some reasonable assumptions [10]. We start off, in the next section, by describing the shooting tactic without damage assessment.

3. The Basic Model - No Damage Assessment

We consider a situation where n rounds are randomly and independently delivered towards m targets. The result of each shot is unknown to the shooter and therefore $p_{s/k}=0$. In other words, for each round the shooter randomly selects a target out of the cluster of m targets. His choices are independent of each other and in particular, the decision to engage a certain target does not depend upon its state (killed or alive).

Let $X = \{X_n; n=1,2,3,..\}$ be a stochastic process with a state space $\{0,1,..m\}$. X_n indicates the number of killed targets following the shooting of n rounds. Clearly, this process is a Markov Chain with an initial state $X_0 = 0$. Note that since no provisions for damage assessment are made, the random variable Y, which represents the EK targets, has no meaning here.

The transition probabilities for the Markov Chain are given by

$$P_{ij} = \begin{cases} p_k \dfrac{m-i}{m} & \text{if } j = i+1,\ 0 \leq i < m \\ 1 - p_k \dfrac{m-i}{m} & \text{if } j = i,\ 0 \leq i \leq m \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

The probability, $P_n(x)$, that x targets are killed by n rounds satisfies:

$$P_n(x) = P_{n-1}(x) \cdot \left(1 - p_k \frac{m-x}{m}\right) + P_{n-1}(x-1) \cdot p_k \frac{m-x+1}{m}. \tag{3.2}$$

It is easily seen [8] that the expected number of killed targets is:

$$E[X_n] = m\left[1 - \left(1 - \frac{p_k}{m}\right)^n\right]. \tag{3.3}$$

Table 3.1 presents the expected values $E[X_n]$ as a function of the number of rounds n and for the case where the number of targets m=10. The expected number of kills is computed for 3 representative values of the kill probability: p_k = 0.2, 0.5, 0.8 and 1. The effect of multiple kills is best noted for the case where p_k=1. Even though the kill is certain for each round, the expected number of kills, for n=10 rounds, is only about 6.5. That is, there are in this case, on average, 3.5 incidences of multiple kills.

Next consider the number of rounds needed to obtain a certain level of damage.

Let M_i, i=0,..,m-1, be a random variable which represents the number of rounds needed

Table 3.1 Expected Number of Killed Targets—Basic Model m = 10

n	p_k = 0.2	p_k = 0.5	p_k = 0.8	p_k = 1.0
5	0.96	2.26	3.41	4.10
10	1.83	4.01	5.66	6.51
20	3.32	6.42	8.11	8.78
30	4.55	7.85	9.18	9.58
40	5.54	8.72	9.64	9.85
50	6.36	9.23	9.85	9.95

to kill the (i+1)-th target, given that i targets have already been killed. Clearly,

$$E[M_i] = \frac{m}{p_k(m-i)}. \tag{3.4}$$

The expected number, $E[T_i]$, of rounds needed to kill i targets is given by

$$E[T_i] = E[M_0] + E[M_1] + \cdots + E[M_{i-1}], \tag{3.5}$$

or

$$E[T_i] = \frac{m}{p_k}\sum_{j=0}^{i-1}\frac{1}{m-j}. \tag{3.6}$$

Since $E[M_i]$ is monotone increasing in i, it follows that $E[T_i]$ is a convex function of i. The operational interpretation of this property is that the marginal expected number of munitions needed is increasing as the damage level requirement (number of killed targets, i) gets higher.

The simple Urn Model presented in this section represents a situation where the entire engagement is random and no decisions are made throughout it. Such shooting tactics may result in incidences of multiple kills, a phenomenon which may cause waste in both munitions and time. This elementary shooting tactic has been described as a back-drop for the SLS tactics which are discussed in detail in the next sections. These tactics incorporate damage assessment and decision making.

4. The "Persistent Shooter"

4.1 The Basic Persistent Model (BPS). The persistent shooter selects the first target to engage out of a cluster of m targets at random and keeps engaging this target (delivering rounds on it) as long as the target is not evidently killed (EK). Once the target is EK, the shooter selects, at random, a new live target to engage. Evidently, the efficiency of this shooting tactic strongly depends on the conditional probability $p_{s/k}$ that a kill is evident. For example, if $p_{s/k}$ = 0 then the maximum possible number of killed targets, under this tactic, is 1.

As in Section 3, let X_n be a random variable that counts the number of killed targets by n rounds. Define Y_n as the number of EK targets after shooting n rounds. Under the assumption that $p_{s/k}$ is constant throughout the engagement, Y_n is a Markov Chain. Clearly, the number of EK targets Y_n can never exceed the number of killed (K) targets X_n. For the Persistent Shooter tactic, X_n can exceed Y_n by at most one target. Hence

$$X_n - 1 \le Y_n \le X_n$$

It can be easily shown that the random variable Y_n has a truncated binomial distribution. That is,

$$P(Y_n = i) = \begin{cases} \binom{n}{i} p_s^i (1-p_s)^{n-i} & \text{if } n \le m \text{ or} \\ & \text{if } (n > m \text{ and } i \le m) \\ \sum_{j=m}^{n} \binom{n}{j} p_s^j (1-p_s)^{n-j} & \\ & \text{if } n > m \text{ and } i = m, \end{cases} \tag{4.1}$$

where $p_s = p_k p_{s/k}$.

However, the parameter that is most relevant to the shooter is the number of killed targets, X_n. Here, the process $\{X_n\}$ is not a Markov Chain; the transition probability from i killed targets to i+1 kills depends on the status (K or EK) of the i-th killed target. The probability distribution function of X_n is derived directly in the following way.

There are three possible states out of which one can arrive at the situation where X_n=x.

State a: Y_{n-1} = x-1 and X_{n-1} = x-1, in which case the transition to X_n = x is with probability p_k.

State b: Y_{n-1} = x-1 and X_{n-1} = x, in which case the transition is with probability 1.

State c: $Y_{n-1} = X_{n-1}$ = x, in which case the transition is with probability (1-p_k).

Therefore,

$$\begin{aligned} P(X_n = x) &= p_k \cdot P(X_{n-1} = x-1, Y_{n-1} = x-1) \\ &\quad + P(X_{n-1} = x, Y_{n-1} = x-1) \\ &\quad + (1-p_k) \cdot P(X_{n-1} = x, Y_{n-1} = x). \end{aligned} \tag{4.2}$$

Equation (4.2) applies for the case where $x \leq m-1$. Since $x = m$ is an absorbing state a slight modification of (4.2) is needed to account for that property. For the sake of brevity, this modification is omitted here.

We can obtain now a recursive formula for the probability distribution of the number of killed targets X_n following the firing of n rounds.

$$P(X_n = x) = \begin{cases} (1-p_k)^n & \text{if } x = 0 \\ (1-p_k)\cdot P(X_{n-1}=x) \\ \quad + pk \cdot P(Y_{n-1} = x-1) & \text{if } 1 \leq x \leq m-1 \\ (1-p_x)\cdot P(X_{n-1}=x) \\ \quad + p_k \cdot [P(Y_{n-1} = m-1) + P(Y_{n-1}=m)] & \text{if } x = m. \end{cases} \quad (4.3)$$

Since the probability distribution function (pdf) of Y_n is known from (4.1), the pdf of X_n in (4.3) can now be easily computed.

In particular, the expected number of killed targets is given by

$$E[X_n] = (1-p_k)\cdot E[X_{n-1}] + p_k \cdot E[Y_{n-1}] + p_k \cdot [1 - P(Y_{n-1} = m)]. \quad (4.4)$$

If $n \leq m$ (in which case $E[Y_{n-1}]=(n-1)p_s$ and $P[Y_{n-1}=m]=0$), then it can be shown that,

$$E[X_n] = np_s + (1-p_{s/k})\cdot[1-(1-p_k)^n]. \quad (4.5)$$

Table 4.1 shows the expected number of targets killed, in a cluster of m=10 targets, by a persistent shooter. The ranges of kill probabilities and kill-recognition probabilities are p_k= 0.2, 0.5 and 0.8 and $p_{s/k}$ = 0.1, 0.5 and 0.9, respectively.

Notice the tradeoffs between p_k and $p_{s/k}$. From the practical point of view, the realistic range of p_k in actual combat is 0.2 - 0.8. Kill detection however may have a larger range of probabilities; an advanced sensor may have a kill detection rate as high as 90% while the unassisted eye of a shooter may detect no more than 10% of the killed targets. Under these bounding assumptions, the advantage of high $p_{s/k}$ - low p_k over the reverse is evident as n increases. For example, if n=50 then the expected number of killed targets is 8.41 if p_k = 0.2 and $p_{s/k}$ = 0.9. This value is only 4.89 if p_k = 0.8 and $p_{s/k}$ = 0.1.

Table 4.2 shows the number of rounds to be delivered in order to obtain a 90% chance for a specified minimum level of damage. Here, p_k=0.5 while $p_{s/k}$ assumes two values 0.5 and 1.0. For example, if the mission's objective damage level is to kill 5 out of the 10 targets in the cluster, then if $p_{s/k}$=0.5, the number of rounds required is $M_5(0.9)$ = 27, while if $p_{s/k}$=1.0, then $M_5(0.9)$ = 14.

These two values are compared with the number of rounds, $M_5(0.9)$ = 19, needed in the Urn Model for this damage level objective and the same kill probability p_k=0.5.

From Table 4.2 it can be seen that, for the case discussed here, the choice between the random Urn Model and the Basic Persistent Shooting tactic depends on the required damage level and on the effectiveness of the damage assessment process (measured by $p_{s/k}$); for small values of required damage level the Urn Model is superior even for relatively high capabilities of damage assessment. However, if

Table 4.1 Expected Number of Killed Targets for BPS Tactics m = 10

n	p_k = 0.2			p_k = 0.5			p_k = 0.8		
	$p_{s/k}$ = 0.1	$p_{s/k}$ = 0.5	$p_{s/k}$ = 0.9	$p_{s/k}$ = 0.1	$p_{s/k}$ = 0.5	$p_{s/k}$ = 0.9	$p_{s/k}$ = 0.1	$p_{s/k}$ = 0.5	$p_{s/k}$ = 0.9
5	0.71	0.84	0.97	1.12	1.73	2.35	1.30	2.50	3.70
10	1.00	1.45	1.89	1.40	3.00	4.60	1.70	4.50	7.30
20	1.29	2.49	3.70	1.90	5.49	8.61	2.50	8.20	9.99
30	1.50	3.50	5.48	2.40	7.74	9.89	3.30	9.77	10.00
40	1.70	4.50	7.12	2.90	9.17	10.00	4.10	9.99	10.00
50	1.90	5.48	8.41	3.40	9.77	10.00	4.89	10.00	10.00

Table 4.2 Number of Rounds Required for BPS to Obtain a Certain Damage Level with 90% Confidence

Damage level	$p_{s/k} = 0.5$	$p_{s/k} = 1.0$	Urn Model
1	4	4	4
2	11	7	7
3	17	9	11
4	22	12	15
5	27	14	19
6	32	17	25
7	37	19	32
8	42	21	41
9	47	24	56
10	52	26	89

the required damage level is high (9 or 10 targets) then the Persistent Shooter tactic is superior, even for less than moderate damage assessment capabilities. In the latter case the effect of the "random" multi-kills in the Urn Model overrides the effect of the "single target" multi-kill that may occur in a persistent shooting.

4.2 Fixed Bound on the Number of Munitions (FBPS). A notable operational property of the Persistent Shooting tactic is the relatively small number of instances where fire is to be redirected between targets; there are at most m-1 such instances. This property is important in cases where the transition from engaging one target to engaging another is demanding both in terms of time (e.g. acquisition time) and resources (e.g. wasted ammunition in the fire adjustment process). However, as was shown in Table 4.1 above, this tactic may be of little use if the damage assessment effectiveness, $p_{s/k}$, is low, or even moderate. One way to reduce the possibility of wasting ammunition due to poor damage assessment is to limit the number of rounds per target. This may be done by setting an upper bound on the number of rounds shot at each target. The problem of determining this bound is an optimization problem where the tradeoff is between possible high rates of multi-kills - in the case where the bound is high - and terminating the engagement of a certain target prematurely, while it is still alive - in the case where the bound is low.

Let n and v be the number of rounds available and the number of targets that have not been engaged yet, respectively. Define $R_u(v,n)$ as the expected number of targets killed by a persistent shooter with an upper bound u, and denote by $u^*(v,n)$ the optimal upper bound which maximizes $R_u(v,n)$. Clearly, for the case of a single target we have:

$$u^*(1, n) = n; \quad R_n(1, n) = 1 - (1 - p_k)^n. \tag{4.6}$$

Define:

A_i - The target is declared EK right after the i-th shot, i = 1,..,u,

A^c - The target is not EK after firing upon it all of its u rounds, where the upper bound u is a number between 1 and n.

Let $P(K/A^c)$ denote the probability that a target is killed (K) after being shot upon by u rounds, even though it is not evidently killed (EK). It can be shown that:

$$P(A_i) = (1 - p_s)^i p_s, \tag{4.7}$$

$$P(A^c) = (1 - p_s)^u, \tag{4.8}$$

$$P(K|A^c) = 1 - \left(\frac{1 - p_k}{1 - p_s}\right)^u. \tag{4.9}$$

The expected kill $R_u(v,n)$ can now be derived recursively by:

$$R_u(v, n) \text{-} \sum_{i=1}^{\bar{u}}(1 - p)_s^{i-1} p_s[1 + R_u(v - 1, n - i)]$$
$$+ (1 - p_s)^{\bar{u}}\left[1 - \left(\frac{1 - p_k}{1 - p_s}\right)^{\bar{u}} + R_u(v - 1, n - \bar{u})\right], \tag{4.10}$$

where $\bar{u} = \min(n, u)$

The optimal value of u, denoted by u^*, may be obtained by solving

$$\max_{u=1,\ldots,n} \{R_u(m, n)\} \tag{4.11}$$

The following two properties are fairly obvious:

Property 4.1

$$u^*(v, n) \geq \left[\frac{n}{v}\right], \tag{4.12}$$

where [x] is the largest integer not greater than x.

Proof. Clearly the upper bound u cannot be smaller than [n/v] since otherwise at least v rounds are redundant and therefore the engagement is not executed in its maximal possible effectiveness. However, this bound may be equal to [n/v] (with a possible "waste" of n - v[n/v] rounds) when, for instance, p_k is high but $p_{s/k}$ is low.

Property 4.2. The FBPS tactic is superior to the BPS tactic.

Proof. This follows directly from the observation that

$$\max_{u=1,\ldots,n} \{R_u(m, n)\} \leq R_n(m, n). \tag{4.13}$$

Table 4.3 presents optimal upper bounds for the FBPS tactic when both the kill probability p_k and the kill-detection probability $p_{s/k}$ are equal to 0.5. Next to each such bound u we indicate, in parentheses, the expected number of killed targets, $R_u(m,n)$

It can be seen, in the above example, that for large values of n, the optimal upper bound may be strictly larger than the average (# of rounds)/(# of targets). Also notice that when comparing the expected number of kills using FBPS to those using BPS (as presented in Table 4.4 below), an improvement is evident. For example, in the case of n=20 and $p_k = p_{s/k} = 0.5$ these expected values are 5.49 and 7.68 for the BPS and the FBPS tactics, respectively, which represents a 40% improvement. For low values of $p_{s/k}$ the improvement is much higher. For example, if n=40, p_k= 0.5 and $p_{s/k}$= 0.1 the expected values are 2.90 and 9.41 for BPS and FBPS, respectively (225% improvement!).

Table 4.4, which is similar to Table 4.1, presents the expected values for FBPS.

Table 4.3 Optimal Upper Bounds for the FBPS Tactic ($p_k = p_{s/k} = 0.5$)

m / n	1	3	5	10
5	5 (0.97)	2 (2.12)	1 (2.50)	1 (2.50)
10	10 (1.00)	4 (2.79)	2 (3.85)	1 (5.00)
15	15 (1.00)	6 (2.95)	4 (4.57)	2 (6.46)
20	20 (1.00)	9 (2.99)	5 (4.84)	2 (7.68)

4.3 Dynamic Bound on the Number of Rounds (DBPS). Suppose now that the upper bound on the number of rounds allocated to a given target may be updated and changed as the shooter proceeds from one target to another. Clearly, this relaxed condition can only improve the performance of the shooter, as compared to the FBPS tactics where this upper bound remains fixed. This improvement is due to the ability to respond to **actual** outcomes in the engagement process and to optimize accordingly.

Determining the upper bound is a multistage problem which is once again solved by means of dynamic programming. As before, let $R^*(v,n)$ denote the expected number of kills under the optimal DBPS tactics when the shooter has n rounds and v targets have not been engaged yet.

The dynamic programming problem is stated as follows

$$R^*(v, n) = \max_{u=1,\ldots,n} \left\{ \sum_{i=1}^{u} (1 - p_s)^{i-1} p_s [1 + R^*(v-1, n-i)] + (1 - p_s)^u \left[1 - \left(\frac{1 - p_k}{1 - p_s} \right)^u + R^*(v-1, n-u) \right] \right\} \tag{4.14}$$

with $R^*(1,n) = 1 - (1-p_k)^n$.

Table 4.3 presents the optimal (dynamic) upper bounds on the number of rounds to be allocated to the **first** out of v remaining targets. The figures in parenthesis, next to the upper bound values, are the optimal expected number of kills - $R^*(v,n)$. To interpret the figures in Table 4.3 suppose that the shooter has yet to engage n=10 targets with n=20 remaining rounds. To the first of these 10 targets it should allocate at most 3 rounds. Suppose that it delivered these 3 rounds with no EK indication (or a kill was evident only by the third round).

Table 4.4 Expected Number of Killed Targets for FBPS Tactic m = 10

n	$p_k = 0.2$			$p_k = 0.5$			$p_k = 0.8$		
	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$
5	1.00	1.00	1.00	2.50	2.50	2.50	4.00	4.00	4.00
10	2.00	2.00	2.00	5.00	5.00	5.00	8.00	8.00	8.00
20	3.62	3.71	3.91	7.55	7.68	8.71	9.62	9.64	9.99
30	4.93	5.15	5.70	8.80	9.25	9.90	9.93	9.98	10.00
40	5.97	6.41	7.26	9.41	9.78	10.00	9.99	10.00	10.00
50	6.81	7.36	8.47	9.71	9.94	10.00	10.00	10.00	10.00

Table 4.5 Optimal Upper Bounds for DBPS Tactic ($p_k = p_{s/k} = 0.5$)

n	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10
10	3 (3.93)	2 (4.28)	2 (4.50)	2 (4.67)	2 (4.83)	1 (5.00)
11	3 (4.12)	2 (4.52)	2 (4.81)	2 (5.00)	2 (5.17)	2 (5.33)
12	3 (4.28)	3 (4.74)	2 (5.09)	2 (5.32)	2 (5.50)	2 (5.67)
13	3 (4.41)	3 (4.93)	2 (5.33)	2 (5.63)	2 (5.83)	2 (6.00)
14	4 (4.52)	3 (5.10)	3 (5.54)	2 (5.90)	2 (6.15)	2 (6.13)
15	4 (4.62)	3 (5.24)	3 (5.74)	2 (6.13)	2 (6.45)	2 (6.66)
16	5 (4.69)	4 (5.37)	3 (5.91)	3 (6.34)	2 (6.71)	2 (6.98)
17	5 (4.75)	4 (5.47)	3 (6.07)	3 (6.54)	2 (6.93)	2 (7.26)
18	5 (4.80)	4 (5.56)	4 (6.20)	3 (6.73)	3 (7.15)	2 (7.51)
19	6 (4.84)	5 (5.64)	4 (6.32)	3 (6.89)	3 (7.35)	2 (7.74)
20	6 (4.88)	5 (5.70)	4 (6.42)	3 (7.03)	3 (7.54)	3 (7.95)

The shooter is now to engage n=9 targets with n=17 rounds. The number of rounds to be delivered to the first of these 9 targets should not exceed 2 (8th row, 5th column). Suppose that this target has become EK by the first round. There are now 16 rounds to engage 8 targets, so the next target should be engaged by at most 3 rounds, and so on...

Table 4.6 shows expected kills for the DBPS tactic.

Note that the improvement of the Fixed Bound PS (FBPS) tactic over the Basic PS (BPS) tactic is much more significant than the improvement of the Dynamic Bound PS (DBPS) tactic over the FBPS tactic. For example, if n=20, m=10 and $p_k = p_{s/k} = 0.5$ then the BPS tactic results in expected kills of 5.49, where FBPS results in 7.68 and DBPS results in 7.95. For values of p_k and $p_{s/k}$ further to the extreme, the difference between FBPS and DBPS is even less notable.

Table 4.6 Expected Number of Killed Targets for DBPS Tactic m = 10

n	$p_k = 0.2$			$p_k = 0.5$			$p_k = 0.8$		
	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$
5	1.00	1.00	1.00	2.50	2.50	2.50	4.00	4.00	4.00
10	2.00	2.00	2.00	5.00	5.00	5.00	8.00	8.00	8.00
20	3.62	3.74	3.93	7.56	7.95	8.80	9.63	9.81	9.99
30	4.94	5.24	5.75	8.84	9.37	9.91	9.94	9.99	10.00
40	6.00	6.49	7.33	9.46	9.85	10.00	9.99	10.00	10.00
50	6.84	7.49	8.53	9.76	9.97	10.00	10.00	10.00	10.00

We can conclude that by imposing upper bounds on the PS tactic, one can considerably improve its effectiveness. However, in many cases it may be impractical to rely on an algorithmic procedure to determine the upper bounds.

Next we present a simple and straightforward heuristic for DBPS which can be used for determining the upper bounds. Using the realistic examples in this section, it is shown that its effectiveness may be almost as good as that of the DBPS tactics.

4.4 Heuristics for Determining Upper Bounds for the PS Tactics (HBPS). A simple rule for determining the upper bound on the number of rounds to be allocated to the next target is to divide the number of rounds available by the number of targets yet to be engaged and take the smallest integer greater than or equal to that number. In other words, when v targets are to be shot upon by n munitions, then the upper bound on the first target to be engaged is $[n/v]^+$, where $[x]^+$ is the smallest integer not smaller than x. This tactic represents an "average" principle in allocating munitions. Hence, after terminating the engagement of a certain target - either by evidently killing it or by expending all the rounds that were allocated to it, the shooter recalculates the updated "average" and sets it as the new upper bound.

The expected number of killed targets under this tactics is given recursively by

$$R_A(v, n)$$

$$= \sum_{i=1}^{[n/v]^+} (1-p_s)^{i-1} p_s [1 + R_A(v-1, n-i)] + (1-p_s)^{[n/v]^+}\left[1 - \left(\frac{1-p_k}{1-p_s}\right)^{[n/v]^+} + R_A(v-1, n-[n/v]^+)\right] \quad (4.15)$$

Table 4.7 shows the expected kills when the HBPS tactic is applied.

Table 4.8 summarizes the analysis so far. It compares the performance of the four aforementioned Persistent Shooting tactics and the shooting tactic with no damage assessment (Urn Model). The comparison is applied to the case of m=10 targets, n=10, 20, 50 rounds and five representative pairs of p_k and $p_{s/k}$.

From Table 4.8 we can draw the following operational conclusions:

(a) If the number of available rounds (n) is not greater than the number of targets (m), then each round should be directed towards a different target. It can be shown [10] that this tactic is optimal. Evidently, the expected number of killed targets is independent of $p_{s/k}$ and it is equal to np_k.

(b) Compared to the no-damage Urn model and the basic PS tactic, each one of the three bounded PS tactics improve the effectiveness of the engagement considerably.

(c) Although DBPS is mathematically superior to FBPS, and HBPS is just a heuristic which is independent of the probability parameters of the engagement, these three bounded PS tactics perform similarly.

(d) Considering the relatively ease by which HBPS can be implemented, we conclude that it is the most efficient bounding tactic if one chooses to adopt a "persistent" strategy in engaging the targets - a strategy that has several operational advantages in terms of target acquisition and fire control.

Clearly, the HBPS tactic is not optimal. Thus, in view of the above, a practical question that may be asked is: how close is HBPS to an optimal strategy?

5. Comparing the HBPS tactic to an Optimal Tactic.

In [10] it is shown that the *Greedy Shooting* (GS) tactic - a tactic in which each round is shot upon the least previously engaged, non-EK target - is optimal with respect to the expected kills criterion. This tactic however is very impractical in actual combat since it requires excessive control measures. Specifically, a shooter must label the targets and then maintain and update a list of these labels, according to the status of the corresponding targets. Moreover, the GS tactic requires frequent redirection of fire which imposes additional constraint on the shooter in terms of acquiring the targets and setting up the weapon system for the new target. Note that the HBPS tactic requires some effort of target labeling too, however it is considerably less demanding than the optimal GS tactic. In

Table 4.7. Expected Number of Killed Targets for HBPS Tactic m = 10

n	$p_k = 0.2$			$p_k = 0.5$			$p_k = 0.8$		
	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$	$p_{s/k} = 0.1$	$p_{s/k} = 0.5$	$p_{s/k} = 0.9$
5	1.00	1.00	1.00	2.50	2.50	2.50	4.00	4.00	4.00
10	2.00	2.00	2.00	5.00	5.00	5.00	8.00	8.00	8.00
20	3.62	3.74	3.89	7.56	7.92	8.39	9.63	9.79	9.89
30	4.94	5.21	5.53	8.84	9.24	9.47	9.94	9.98	9.99
40	6.00	6.41	6.82	9.46	9.72	9.79	9.99	10.00	10.00
50	6.84	7.34	7.75	9.75	9.89	9.91	10.00	10.00	10.00

Table 4.8. A Comparison of Expected Number of Killed Targets m = 10

n	$p_k/p_{s/k}$	BPS	FBPS	DBPS	HBPS	URN
10	0.2/0.1	1.00	2.00	2.00	2.00	1.83
	0.2/0.9	1.89	2.00	2.00	2.00	1.83
	0.5/0.5	3.00	5.00	5.00	5.00	4.01
	0.8/0.1	1.7	8.00	8.00	8.00	5.66
	0.8/0.9	7.3	8.00	8.00	8.00	5.66
20	0.2/0.1	1.29	3.62	3.62	3.62	3.32
	0.2/0.9	3.70	3.91	3.93	3.89	3.32
	0.5/0.5	5.49	7.68	7.95	7.92	6.42
	0.8/0.1	2.50	9.62	9.63	9.63	8.11
	0.8/0.9	9.99	9.99	9.99	9.89	8.11
50	0.2/0.1	1.90	6.81	6.84	6.84	6.36
	0.2/0.9	8.4	8.47	8.53	7.75	6.36
	0.5/0.5	9.77	9.94	9.97	9.89	9.23
	0.8/0.1	4.89	10.00	10.00	10.00	9.85
	0.8/0.9	10.00	10.00	10.00	10.00	9.85

Table 5.1. A HBPS-GS Comparison of Expected Number of Killed Targets m = 10

n	$p_k/p_{s/k}$	HBPS	GS	n	$p_k/p_{s/k}$	HBPS	GS
	0.2/0.1	1.00	1.00		0.2/0.1	4.94	4.95
	0.2/0.9	1.00	1.00		0.2/0.9	5.53	5.80
5	0.5/0.5	2.5	2.5	30	0.5/0.5	9.24	9.49
	0.8/0.1	4.00	4.00		0.8/0.1	9.94	9.94
	0.8/0.9	4.00	4.00		0.8/0.9	9.99	10.00
	0.2/0.1	2.00	2.00		0.2/0.1	6.00	6.01
	0.2/0.9	2.00	2.00		0.2/0.9	6.82	7.43
10	0.5/0.5	5.00	5.00	40	0.5/0.5	9.72	9.91
	0.8/0.1	8.00	8.00		0.8/0.1	9.99	9.99
	0.8/0.9	8.00	8.00		0.8/0.9	10.00	10.00
	0.2/0.1	3.62	3.63		0.2/0.1	6.84	6.86
	0.2/0.9	3.89	3.94		0.2/0.9	7.75	8.64
20	0.5/0.5	7.92	8.04	50	0.5/0.5	9.89	9.99
	0.8/0.1	9.63	9.63		0.8/0.1	10.00	10.00
	0.8/0.9	9.89	10.00		0.8/0.9	10.00	10.00

particular, if $n > m$, then there are only m-1 incidents of redirection of fire in the HBPS tactic. This number is at least m-1 for the GS tactic.

Table 5.1 shows the expected killed targets for both HBPS and the optimal GS tactics.

It is evident from Table 4.8 that for the spectrum of real-world cases analyzed here, the HBPS tactic performs, in terms of expected kills, very close to the optimal GS tactic.

6. Summary and Conclusions

As advanced, precise and expensive weapon systems (e.g. Precision Guided Munitions - PGM) are introduced into the battle field, questions of their operational efficiency and optimal utilization become more prevalent. Moreover, as the range of weapon systems increases, the problem of fire control, and in particular the effect of damage assessment capabilities, becomes crucial.

In this paper we presented several Shoot-Look-Shoot tactics that may apply to PGM systems in situations where damage information is not necessarily complete.

The Basic Persistent Shooting (BPS) tactic simply instructs the shooter to stick to a certain target and engage it as long as the target is not evidently killed. Two variations of this tactic (FBPS and DBPS) were introduced and were shown to perform better than the basic one. Out of these two tactics emerges a simple tactic - the HBPS - that can be easily implemented in combat. This tactic appear to perform close to optimality in a range of realistic scenarios.

Recall that a number of simplifying assumptions were made in the construction of the models. Specifically, we assumed no cumulative damage in the shooting process, ignored "Type II" errors (that is, declaring a live target as killed) and required that the event "kill" (K) is necessary, at each round, for the event "evident kill" (EK). A natural extension of the analysis presented in this paper may be obtained by considering situations where these assumptions are relaxed.

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