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On popular response to violence during insurgencies

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ABSTRACT

Population behavior is a key factor in the evolution and outcome of insurgencies. This behavior is affected by the violence exerted by the insurgents and the regime. In this paper we model the effect of targeted (i.e., coercion) and misguided (i.e., collateral casualties) violence on the behavior of the population. It is shown that excess violence and poor targeting accuracy may lead to situations where a population's support for a certain side will vanish.

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1. Introduction

Popular behavior is a key factor in the evolution and outcome of an insurgency [1,5,6,9]. An insurgency is an asymmetrical combat situation where a relatively small, ill equipped and poorly trained grass-root force, typically motivated by ideology, religion and social grievances, challenges the regime, which is backed by a sizable, trained and well equipped military force. The civilian population, caught in between these two adversaries, is an important third player; each one of the two sides – the insurgents and the regime – needs the population's support. For the insurgency, the population is a source of recruitment, physical support, and information. For the regime, the population is a source of valuable information, which is utilized for better targeting the low-signature insurgents, who are diffused in the population. Each side carries out violent actions to attain its objectives. The regime uses violence to fight the insurgents and put pressure on their identified collaborators, while the insurgents attack the regime forces and coerce people to deter them from supporting the regime. Civilians get hurt, directly or indirectly (e.g., injury or death of a family member or a friend), by these violent actions. The exposure to violence affects the people's sense of security and is a major factor in shaping their allegiance to one side or another [7,12]. Two recent studies in the political science literature examine the impact of violence on popular behavior in Iraq [3] and Afghanistan [2]. Both studies find that civilian causalities caused by coalition forces increase future insurgent violence and civilian causalities caused by insurgents decrease future insurgent violence. These results illustrate the importance of popular support, and how that support changes based on civilian casualties. Notwithstanding the cultural and ideological background of individuals, and their response to social and economic incentives offered by each side, people will support the side that is perceived as providing better security for the population at large. Thus, violence is a double edge sword for both the insurgents and the regime. On the one hand violence is needed to fight the other side and perhaps deter individuals in the population from supporting the other side, but on the other hand it can turn the population against the source of that violence. The problem is amplified when a certain side has poor targeting accuracy that results in many unintended casualties among the civilian population. Before proceeding, we note that a recent study using data from Chechnya [11] found that indiscriminate violence from government forces can *reduce* insurgent violence. The author admits however that this finding is opposed to much of the research in the area (e.g., [3,2]) and believes such a relationship is the exception, rather than the rule. The paper focuses on indiscriminate violence against civilians, which may be appropriate for describing the current situation in Syria (Spring 2012), but not the situation studied in this paper. Therefore, we will assume the relationship found in [3,2] where civilian causalities have a negative impact for the side responsible for the violence.

We develop in this note a dynamic, differential equations, model that captures key aspects of the relations among violence, targeting accuracy and population behavior. Specifically, we consider four key parameters: the violence intensity ratio between the insurgents and the regime, the effectiveness of coercion, the targeting accuracy of the violence and the sensitivity of the population to that violence, as manifested in the way they "remember" violent events that affected them directly or indirectly. The main conclusions of the analysis are: (1) Excess violence and poor targeting accuracy may lead to situations where the population's support for

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a certain side will vanish; (2) The regime should not be discouraged by an initial small level of popular support because there are situations where this would actually play to its advantage; (3) The effect of the initial distribution of opinions in the population on the outcome of the insurgency depends on the population's response pattern to violence. For some response patterns the outcome is insensitive to this initial distribution.

2. Setting and assumptions

In insurgency situations violence is generated by the regime and the insurgents. The regime carries out targeted violent actions against the insurgents, but absent perfect situational awareness regarding the location and identity of the insurgents, and employing imprecise weapons, the regime's forces may inadvertently hit civilians. We assume that the regime does not intentionally coerce the population, and, as stated before, it does not use indiscriminate violence against the civilian population (e.g., as happened in Chechnya [11]). Civilian casualties from the regime's violence only occur as a result of collateral damage. The insurgents attack the regime's forces but also exert coercive actions against individuals suspected to be collaborating with the regime, attempting to deter these suspected collaborators from doing so. While the insurgents can clearly identify the military targets when fighting the regime, they cannot do that while coercing potential supporters of the regime. The insurgents' knowledge regarding the identities of these supporters is limited, which may result in erroneously coercing the wrong people—some of whom may be their own supporters.

We assume a very simple binary response pattern to violence; the result of experiencing violence generated by one side (insurgents or regime) is becoming an opponent to that side. This is consistent with the findings in [3,2]. The question is what is the endurance of that effect – how long this violent experience will shape an individual's behavior – and what would happen if that individual is suddenly hit by the other side? What happens if the individual is hit multiple times by both sides? How will this individual change his allegiance? These type of questions, which involve emotion and memory, have been studied in the behavioral psychology literature [16]. Much work has been done examining how people react to multiple events or items. Examples include advertising [14] (i.e., which commercials people remember most strongly) and political campaigns [13] (i.e., what timing and content of campaign messages resonate most with potential voters). We could not find any studies that analyze how individuals react, process, and remember *multiple* traumatic or violent events. Because memories of traumatic and non-traumatic events are processed differently [15], we did not want to extrapolate the results from the study of non-traumatic memory to our civilian casualty context. Instead, our model can handle multiple types of memory paradigms. We assume that without an experience of violence, an individual is *sensitive* and thus will turn against the side that first hits him. Because of the traumatic effect, an individual becomes *insensitive* immediately after experiencing violence, where new violent events do not change his behavior for a while. Eventually the effect fades away and an individual may transition back to the sensitive state where a new experience of violence by the other side may change his allegiance. This behavioral pattern is consistent with the results in [3] that found that a side was "punished" immediately after causing collateral casualties, but then, after a few weeks, the disadvantage faced by the responsible side disappeared. While we model the general case where individuals transit from an insensitive stage back to a sensitive stage, we only analyze the two extreme cases, sometimes called in the literature of behavioral psychology *Primacy* and *Recency* (see e.g., [10,13]). Primacy means that the first experience to some stimulus shapes the behavior of an individual and therefore the transition is always one way: sensitive (to the stimulus) to insensitive. On the other extreme, Recency postulates that the last experience is the dominant one, that is, an individual always remains sensitive to the stimulus (violence).

The population is divided into three parts: (1) active supporters of the regime, called henceforth *Actives* and denoted as A, (2) neutrals or latent supporters of the regime, called henceforth *Latents* and denoted as L, and (3) opposition to the regime (supporters of the insurgents), called henceforth *Contrarians* and denoted as C. We do not explicitly model neutrals who support (or hate) both the regime and insurgents. Actives provide intelligence and aid to the regime to help them defeat the insurgents. While Latents prefer the regime to the insurgents, they do not help the regime out of fear of reprisals from the insurgents. From an operational standpoint (e.g., intelligence gathering), the regime only benefits from the Actives in the population. Latents can switch to Actives, and vice-versa, depending upon the violence generated by the regime and insurgents. Since we assume that the regime does not intentionally coerce civilians with violence, we do not distinguish between active and latent Contrarians and assume that all Contrarians are active. The misguided violence is detrimental to the side exerting it; collateral casualties by the regime's actions may generate Contrarians, and collateral casualties by insurgents' actions may become active supporters of the regime. Active supporters of the regime who are successfully coerced by the insurgents become latent. At any given time, the population is divided into individuals who are sensitive to violent actions – they change their behavior according to the source of violence they experience (directly or indirectly) – and individuals who are insensitive. Insensitive individuals do not react to violence. A sensitive individual, who experiences violence and changes his behavior, becomes insensitive for a while. Insensitive individuals become sensitive again at a certain constant rate. As is discussed in the next section, this insensitive–sensitive transition rate determines the effect of the violent actions. If this rate is zero, then the population only comprises *first-event* individuals who determine their behavior according to the first event of violence they experience and do not change their behavior thereafter. This scenario represents the Primacy effect discussed above. If this transition rate is very high (infinity), then the population only comprises *last-event* (memoryless) individuals who change their behavior based on the last act of violence they experience. This scenario represents the Recency effect. In reality, it is most likely that the population behavior lies in-between these two extremes.

The accuracy of the targeting of actions carried out by both sides – the regime and the insurgents – is represented by the fraction of violent actions that are correctly targeted (see [9]). The rest of the violence is "shooting in the dark"—violence that is spread over the population in the region contested by the two forces in a spatially uniform manner. The targeting accuracy depends on the information available to each side and we focus on two extreme cases: perfect targeting accuracy and none ("shooting in the dark"). Finally, we assume that the rate of change in population behavior as a result of insurgents and regime actions is higher than the rate at which the violence intensities of the two sides change due to mutual attrition and other operational and environmental factors. Thus, the violence rate, on both sides, is assumed to remain constant throughout.

Based on these assumptions, we develop in the next section a model that captures the dynamics of popular behavior as a result of insurgents' and regime's violence. Before, we proceed to the model, we note the asymmetry between the regime and insurgents in our framework. The violence generated by the regime is only directed at the insurgents. While some of that violence may fall mistakenly on the population and produce collateral damage, the regime does not target civilians directly with violence (e.g., coercion). The violence generated by the insurgents is only the coercion directed at civilians to deter them from supporting the regime (i.e., Actives). From the insurgents' perspective, increasing violence may be an effective tool to manipulate the population by decreasing the fraction of Actives. However, the regime never benefits in terms of population behavior by increasing its violence level because this only serves to potentially alienate their supporters and drive them to become Contrarians. The conclusion is not that the regime should avoid generating any violence. The objective of the regime is *not* to maximize popular support; it is to defeat the insurgents and they need to generate violence in order to accomplish this. Thus the level of violence generated by the regime is a constant exogenous to the model that reflects the strategic situation and the tactics of the regime's forces. We discuss this issue further in the Conclusion.

3. Notation and model

Let $A^H \geq 0, L^H \geq 0$ and $C^H \geq 0$, denote the fraction of Actives, Latents and Contrarians, respectively, who are insensitive to violent actions. We use the superscripts S and H to denote sensitive and insensitive ("hardened") individuals, respectively. We define $A^S \geq 0, L^S \geq 0$ and $C^S \geq 0$, as the fraction of Actives, Latents and Contrarians, respectively, who are sensitive to violent action and therefore will change their behavior when experiencing violence. We have, $A^H + L^H + C^H + A^S + L^S + C^S = 1$. Insensitive individuals are those who recently have been affected by violence and thus have a firm attitude towards the regime and the insurgents, according to the source of the violence that affected them. Over time, the memory of the violence may fade out, the attitude may become less firm, and people will become sensitive again to violence, that is, they may change their attitude according to the new source of the violence that affects them. We assume that insensitive individuals become sensitive again at a rate ρ. Let λ_R and λ_I denote the rate of violent actions by the regime and the insurgents, respectively. Recall that according to our assumption, the values of λ_R and λ_I remain constant throughout. The parameters representing targeting accuracy are denoted by υ_R and υ_I for the regime and the insurgents, respectively; $\upsilon = 1$ means perfect targeting accuracy while $\upsilon = 0$ implies none. Finally, let ϵ denote the fraction of sensitive Actives who are successfully coerced by the insurgents and therefore become Latents. This parameter reflects the "compliance" rate—the effect of the insurgents' coercive actions on the intended target population of Actives. The set of equations that represents this situation is:

$$\dot{A}^H = (1-\epsilon)\lambda_I \left(\upsilon_I \frac{A^S}{A} + (1-\upsilon_I)A^S\right) + \lambda_I(1-\upsilon_I)(L^S + C^S) - \rho A^H \tag{1}$$

$$\dot{A}^S = -\lambda_R(1-\upsilon_R)A^S - \lambda_I \left(\upsilon_I \frac{A^S}{A} + (1-\upsilon_I)A^S\right) + \rho A^H \tag{2}$$

$$\dot{L}^H = \epsilon\lambda_I \left(\upsilon_I \frac{A^S}{A} + (1-\upsilon_I)A^S\right) - \rho L^H \tag{3}$$

$$\dot{L}^S = -\left(\lambda_I(1-\upsilon_I) + \lambda_R(1-\upsilon_R)\right) L^S + \rho L^H \tag{4}$$

$$\dot{C}^H = \lambda_R(1-\upsilon_R)(L^S + A^S + C^S) - \rho C^H \tag{5}$$

$$\dot{C}^S = -[\lambda_I(1-\upsilon_I) + \lambda_R(1-\upsilon_R)]C^S + \rho C^H \tag{6}$$

where $A = A^S + A^H$.

The last term on the right hand side of each one of the Eqs. (1)–(6) represents transitions from insensitive individuals to sensitive ones. The right hand side of (1) has three terms. The first term represents sensitive Actives who are not deterred by the insurgents' coercion and remain Actives. Note that this term comprises direct targeted coercion, $\lambda_I \upsilon_I$, and indirect dispersed coercion $\lambda_I(1-\upsilon_I)A^S$ (see [9]). The direct targeted coercion falls correctly on the Actives, as intended, due to accurate information (i.e., intelligence) the insurgents may have on the identities of Actives in the population. The direct targeted coercion is scaled by the fraction of sensitive individuals among the Actives ($\frac{A^S}{A}$), reflecting the fact that the insurgents cannot distinguish between sensitive and insensitive Actives. The indirect dispersed coercion represents "coercing in the dark" that happens to fall on the Actives by chance. The second term represents sensitive Latents and Contrarians, who experience (unintended) violence by the insurgents and become Actives. The first term on the right hand side of (2) represents sensitive Actives who are affected by the regime violence and therefore become Contrarians. The second term represents sensitive Actives that are coerced by the insurgents and either become insensitive Actives (with probability $1-\epsilon$) or Latents (with probability ϵ). The interpretations of the right hand side of the rest of the equations are similar. Note in (6) that sensitive Contrarians are affected by two types of unintended violence: mistargeted coercion by the insurgents and collateral effects of regime counterinsurgency activities. The former results in insensitive Actives while the latter results in insensitive Contrarians. As mentioned above, these individuals will become sensitive again at a rate ρ.

Next we consider two extreme cases: $\rho = 0$ and $\rho = \infty$. The first case represents Primacy behavior; an individual's attitude is determined by the first act of violence he experiences and therefore once he becomes insensitive to violence, he remains like this forever. On the other extreme ($\rho = \infty$) we have the Recency behavior; an individual changes his attitude according to the last violence event he experiences. We assume for simplicity that initially the entire population is sensitive. This assumption is not necessary, but the general case, where initially a fraction of the population is sensitive while the other fraction is not, is notationally cumbersome and does not provide any additional insights. Let A_0, L_0 and C_0 denote the initial distribution of the population to Actives, Latents and Contrarians, respectively. We denote by A^*, L^* and C^* the end-state fractions of Actives, Latents and Contrarians, respectively. Finally, let $\alpha = \frac{\lambda_I}{\lambda_R}$ denote the *violence intensity ratio* between the insurgents and the regime.

4. Primacy behavior

In this case the first experience of violence is dominant. Therefore $\rho = 0$, and the last term on the right hand side of each one of Eqs. (1)–(6) vanishes. We consider four extreme cases regarding the accuracy functions, corresponding to perfect and no accuracy. Proofs of the results are given in the Appendix.

Case P1: $\upsilon_R = \upsilon_I = 1$.

In this case both the regime and the insurgents have perfect targeting accuracy. The violence is targeted correctly and therefore there are no collateral casualties. It follows that $C^* = C_0$, $L^* = L_0 + \epsilon A_0$, $A^* = (1-\epsilon)A_0$. Note that in this case the end states are independent of the violence intensity ratio (α).

Case P2: $\upsilon_R = \upsilon_I = 0$.

In this case both sides are "shooting in the dark", generating collateral casualties as they fight and, in the case of the insurgents, coerce. The end-states in this case are: $C^* = \frac{1}{1+\alpha}$, $L^* = \frac{\alpha\epsilon}{1+\alpha}A_0$, $A^* = \frac{\alpha}{1+\alpha}(1-\epsilon A_0)$, which are independent of the initial ratio between Contrarians and Latents. The regime should minimize its violence as much as operationally possible and encourage violence by the insurgents, so that it is the insurgents who cause the collateral casualties.

Case P3: $\upsilon_R = 1$; $\upsilon_I = 0$.

This is the best case for the regime where it has perfect targeting accuracy, while its opponent – the insurgency – has none. The violence is targeted correctly by the regime and therefore there are no collateral casualties generated by the regime, only by the insurgents. Here obviously $C^* = 0$ and $L^* = \epsilon A_0$, $A^* = 1 - \epsilon A_0$. As in case P1, the end-states are independent of the violence intensity ratio, and as in case P2, they are independent of the initial ratio between Contrarians and Latents. It is interesting to note that the regime is better off with a small initial body of support (small A_0) than a large one. This also occurs in case P2. The reason for this surprising observation is a combination of the lack of targeting accuracy by the insurgents and the Primacy effect; initial Actives are sensitive and therefore susceptible to coercion, while Actives generated from the insurgents' collateral casualties are insensitive and therefore will never be successfully coerced.

Case P4: $v_R = 0$; $v_I = 1$.

This is the worst-case scenario for the regime. In this case the insurgents have perfect targeting accuracy while the regime has none. Mathematically, this is also the most complicated case. The end-states in this case are $A^* = W\left(\frac{A_0+\alpha\epsilon}{\alpha(1-\epsilon)} e^{\frac{\epsilon}{1-\epsilon}}\right)\alpha(1-\epsilon) - \alpha\epsilon$, $L^* = \frac{\epsilon}{1-\epsilon}A^*$, $C^* = 1 - A^* - L^*$, where $W(\cdot)$ is the Lambert W function (see e.g., [4]). As in cases P2 and P3, the end-states are independent of the initial ratio between Contrarians and Latents. The regime is better off with α large (small λ_R) because their violent acts only serve to alienate the population and push them to become Contrarians. However the regime may find it difficult to reduce λ_R because it needs a large level of violence to effectively combat the insurgents. As in case P1, the number of Actives in the end-state increases with the number of individuals who are initially Active. If possible, the regime should also take actions to decrease the effectiveness of coercion (i.e., decrease ϵ).

5. Recency behavior

In this case the last violent event is dominant and therefore the states X^H, $X = A, L, C$ in Eqs. (1)–(6) vanish. The representation of the dynamics reduces to the following two differential equations. To simplify notation we drop the superscript S, which is redundant.

$$\dot{A} = -\lambda_R(1-v_R)A - \lambda_I\epsilon\,(v_I + (1-v_I)A) + \lambda_I(1-v_I)(1-A) \tag{7}$$

$$\dot{C} = -\lambda_I(1-v_I)C + \lambda_R(1-v_R)(1-C) \tag{8}$$

with $L = 1 - A - C$. We derive Eq. (7) by combining Eqs. (1)–(2) and dropping the terms that transition between A^H and A^S. The first term in Eq. (7) represents the Actives who become Contrarians because the regime has mistakenly hit them with violence. The second term is the individuals moving from Active to Latent after being effectively coerced by the insurgents. As with Eqs. (1)–(2) this second term contains both direct targeted coercion, $\lambda_I v_I$, and indirect dispersed coercion $\lambda_I(1 - v_I)A$. The final term of (7) represents non-Active individuals who become Actives after the insurgents mistakenly coerce them. The two terms in Eq. (8) can be derived in a similar fashion: the first term is the Contrarians mistakenly coerced by the insurgents and the second term is the individuals who become Contrarians after being hit by the regime.

The system of equations defined by (7)–(8) has an analytic solution. The equilibrium point of this system (assuming that v_R and v_I are not both 1) is

$$A^* = \frac{\alpha(1-v_I(1+\epsilon))}{1-v_R+\alpha(1-v_I)(1+\epsilon)} \tag{9}$$

$$xC^* = \frac{1}{\alpha\frac{1-v_I}{1-v_R}+1}. \tag{10}$$

The regime benefits when coercion has limited effect (i.e., ϵ is small) and the regime has the targeting advantage (i.e., large v_R and small v_I). By inspection of (9), the regime will receive no active support if $v_I > \frac{1}{1+\epsilon}$. If the regime does receive active support, then it prefers α to be as large as operationally possible because the fraction of Actives will increase and the fractions of Contrarians will decrease. In this case the insurgents have imperfect targeting accuracy, and thus it is in the regime's best interest to encourage the insurgents to instigate violence so they, and not the regime, are the source of collateral damage. For comparison to the Primacy situation in Section 4, we next consider four extreme cases.

Case R1: $v_R = v_I = 1$.

If both the regime and the insurgents have perfect targeting accuracy there are no collateral casualties and therefore we are in a situation similar to Case P1 above. However, since the Actives, as the rest of the population, remain sensitive, and each coercion attempt is successful with probability $\epsilon > 0$, eventually all actives will become latents. Thus, $A^* = 0$, $C^* = C_0$, $L^* = 1 - C_0$. The end-states are independent of everything except the initial fraction of Contrarians. In fact, these end-states are obtained for more general behavior then pure Recency. That is, for any non-zero ρ the aforementioned end-states prevail. The reason is that every insensitive Active becomes, at one point or another sensitive, and thus susceptible to coercion, while Latents and Contrarians remain unharmed.

Case R2: $v_R = v_I = 0$.

Both sides are essentially "shooting in the dark". The fractions of Contrarians, Latents and Actives converge to end-states that depend on the violence intensity ratio α and the coercion effectiveness ϵ. The end-states do not depend on the initial fractions of Contrarians, Latents and Actives. Specifically, $A^* = \frac{\alpha}{1+(1+\epsilon)\alpha}$, $C^* = \frac{1}{1+\alpha}$, $L^* = 1 - (A^* + C^*)$. Clearly, the insurgents and the regime must exercise violence with care; each side would like the other side to be more of the "bad guy" thus enhancing its own support. For example, if the intensities of violent actions on both sides are equal ($\alpha = 1$), then at equilibrium $A^* = \frac{1}{2+\epsilon}$, $C^* = \frac{1}{2}$, $L^* = \frac{\epsilon}{2(2+\epsilon)}$, that is, half of the population are Contrarians, and out of the 50% supporters the fraction of Actives range between $\frac{2}{3}$, if the coercion is perfectly effective, and 1, if it is not effective at all. When $\alpha \to 0$ (which means the regime is much more violent than the insurgents: $\lambda_R \gg \lambda_I$) then $A^* = L^* = 0$, $C^* = 1$; the regime, who exerts much more violence than the insurgents, alienates the people and therefore the entire population becomes Contrarians. Conversely, when $\alpha \to \infty(\lambda_R \ll \lambda_I)$ then $A^* = \frac{1}{1+\epsilon}$; $L^* = \frac{\epsilon}{1+\epsilon}$, $C^* = 0$; the insurgents exert such a high level of violence compared to the regime that, if their violence is accurately targeted ($\epsilon = 1$) the best they can get is an even split between the Latents and Actives; no Contrarians are left to support them. If the accuracy is poor ($\epsilon = 0$) everyone becomes Active.

Case R3: $v_R = 1$; $v_I = 0$.

Here the regime has perfect targeting accuracy while the insurgents have none. This is the best-case scenario for the regime. The violence is targeted correctly by the regime and therefore no collateral casualties are generated by the regime, only by the insurgents. Here $C^* = 0$ (obvious), $A^* = \frac{1}{1+\epsilon}$, $L^* = \frac{\epsilon}{1+\epsilon}$. Note that these are the end-states in Case R2 when $\alpha \to \infty$.

Case R4: $v_R = 0$; $v_I = 1$.

In this case the insurgents have perfect targeting accuracy while the regime has none. This is the worst-case scenario for the regime. Here $A^* = L^* = 0$, $C^* = 1$. Clearly, in this situation in particular the regime should minimize its violence as much as operationally possible. Indeed, if $\lambda_R = 0$ then $C^* = C_0$, $L^* = 1 - C_0$, which is the best the regime can hope for in this case. Note that if, in addition, the regime can take actions that render the insurgents' coercion ineffective ($\epsilon = 0$) then $A^* = A_0$, $L^* = L_0$, $C^* = C_0$.

6. Discussion and conclusions

In this note we modeled and studied the impact of three key factors on popular behavior in insurgency situations: the relative levels of violence intensities of the regime and the insurgents, the targeting accuracy of each side, and most importantly, the response pattern to these acts of violence by the civilian population. We studied eight cases corresponding to extreme scenarios regarding the population response to violence and targeting accuracy, and showed that the popular behavior is crucially dependent on the assumptions regarding those three key factors. First, if both sides have perfect targeting accuracy then the initial fraction of Contrarians remains unaffected. The difference between the Primacy and Recency cases is that in the former there may remain some Actives, whose fraction will depend on the effectiveness of the insurgents coercive actions, while in the latter all the supporters of the regime will become Latents—there will be no Actives. Second, if both sides have poor targeting accuracy then in both cases – Primacy and Recency – the fraction of Contrarians at the end-state are identical. This common end-state value is independent of the initial fraction of Contrarians and only depends on the violence intensity ratio. At violence parity 50% of the population will end up as Contrarians. The end-state fraction of Actives depends on its initial fraction in the Primacy case, but is independent of it in the Recency case. If this initial fraction of Actives is small enough, the Primacy case is better for the regime than the Recency one—at the end it will result in a higher fraction of Actives. Third, in the best-case scenario for the regime, when it has perfect targeting there will be no Contrarians at the end. The end-state fraction of Actives is independent of the violence intensity ratio and it depends on its initial fraction only in the Primacy case. If the initial fraction of Actives is not greater than 50%, the end-state fraction of actives will be at least 50%, regardless of the response behavior of the population. Fourth, in the worst case for the regime, when it is shooting in dark, the regime wants to reduce its violence (i.e., decrease λ_R) as much as is operationally feasible to avoid alienating all of the Actives. If possible the regime should also try reducing the impact of coercion on Actives. To summarize the main insights: if the regime has poor targeting accuracy (i.e., limited intelligence about the identity and location of the insurgents and/or imprecise weapons) it must be very careful when using violence against the insurgents. The regime should attempt to reach its best-case scenario (perfect accuracy for the regime and poor for the insurgents) by investing in intelligence and accurate (vs. lethal) weapons on the one hand, and applying information operations on the insurgents and enticing them to act, on the other hand.

While in reality an individual's reaction to a series of violent events will be a function of all the events, a reasonable question is under what situations will the Primacy model be more realistic than the Recency model and vice-versa? While a full study of this is more appropriate for the behavioral and cognitive psychology experts, we can examine the conflicts in Iraq and Afghanistan to determine if one effect dominates. Condra and coauthors examine the impact of collateral casualties in Iraq [3] and Afghanistan [2]. Both studies show that a side that generates civilian casualties is "penalized" by the population. For example, if coalition forces cause collateral damage, then in the future there will be more insurgent attacks. However, the manifestation of this penalty is different in Iraq and Afghanistan. In Iraq there is an immediate impact, but after several weeks the disadvantage disappears [3]. This suggests Recency may be the dominant factor in Iraq as the impact of civilian violence is only temporary; presumably superseded by later violent events. However, in Afghanistan the impact is not immediate, but results in a long-run disadvantage for the offending side [2]. This suggests that Primacy may be the dominant factor in Afghanistan because individuals may exact retribution for civilian causalities, even if it takes a long time. This is consistent with the cultural norms of the Pashtuns (the dominant ethnic group of Taliban) where revenge is a key component of society, potentially festering for generations [2,8]. If further studies had confirmed that the Primacy paradigm is a reasonable representation for the population in Afghanistan, then our analysis could have provided an insight into counterinsurgency operations for that conflict. The initial low level of popular support for coalition forces would not have been a great concern. Because of the Primacy effect, the impact of collateral damage is severe, so the coalition forces would benefit from keeping violence levels as low as possible and baiting the Taliban to lash out against the population by using information operations.

The most obvious extension of our analysis would be to examine cases with an intermediate ρ. It appears that the problem would lose all analytic tractability, but numerical techniques could examine specific scenarios. Other modifications would have the regime targeting some of their violent actions on the population and modeling a neutral population more explicitly who supports or hates both the regime and insurgents. Perhaps more interesting than these descriptive models would be a prescriptive model that analyzes how much violence the regime (or insurgents) should generate. The regime's objective is to defeat the insurgents and it must generate violence and have popular support to accomplish this. Our model of popular support could be combined with an insurgency attrition model (e.g., [9]) to analyze this problem. Optimization techniques such as optimal control theory could then be utilized to calculate the regime violence profile.

Appendix. Proofs of properties

Lemmas 1–4 apply to the Primacy case.

Lemma 1. *If $\upsilon_R = \upsilon_I = 1$, then $C^* = C_0$ and $A^* = (1-\epsilon)A_0$.*

Proof. The right hand side of Eqs. (4)–(6) are 0. Eqs. (1)–(3) become

$$\dot{A}^H = (1-\epsilon)\lambda_I \upsilon_I \frac{A^S}{A} \tag{11}$$

$$\dot{A}^S = -\lambda_I \upsilon_I \frac{A^S}{A} \tag{12}$$

$$\dot{L}^H = \epsilon\lambda_I \upsilon_I \frac{A^S}{A}. \tag{13}$$

Thus a fraction $(1-\epsilon)$ of the initial Actives (A^S) will remain Active (A^H), implying $A^* = (1-\epsilon)A_0$. □

Lemma 2. *If $\upsilon_R = \upsilon_I = 0$, then $C^* = \frac{1}{1+\alpha}$ and $A^* = \frac{\alpha}{1+\alpha}(1-\epsilon A_0)$.*

Proof. Eqs. (1)–(6) become

$$\dot{A}^H = (1-\epsilon)\lambda_I A^S + \lambda_I(L^S + C^S) \tag{14}$$

$$\dot{A}^S = -(\lambda_I + \lambda_R)A^S \tag{15}$$

$$\dot{L}^H = \epsilon\lambda_I A^S \tag{16}$$

$$\dot{L}^S = -(\lambda_I + \lambda_R)L^S \tag{17}$$

$$\dot{C}^H = \lambda_R(L^S + A^S + C^S) \tag{18}$$

$$\dot{C}^S = -(\lambda_I + \lambda_R)C^S. \tag{19}$$

A fraction $\frac{\lambda_R}{\lambda_R+\lambda_I}$ of the initial Latents and Contrarians ($L^S + C^S$) will transition to insensitive Contrarians (C^H), and the rest will become Actives (A^H). A fraction $\frac{\lambda_R}{\lambda_R+\lambda_I}$ of the initial Actives (A^S) also will become Contrarians (C^H). A fraction $(1-\epsilon)$ of the remaining initial Actives will remain Active (A^H). Aggregating the components together produces the result. □

Lemma 3. *If $\upsilon_R = 1$, $\upsilon_I = 0$, then $C^* = 0$ and $A^* = 1 - \epsilon A_0$.*

Proof. Eqs. (1)–(6) become

$$\dot{A}^H = (1-\epsilon)\lambda_I A^S + \lambda_I(L^S + C^S) \tag{20}$$

$$\dot{A}^S = -\lambda_I A^S \tag{21}$$

$$\dot{L}^H = \epsilon\lambda_I A^S \tag{22}$$

$$\dot{L}^S = -\lambda_I L^S \tag{23}$$

$$\dot{C}^H = 0 \tag{24}$$

$$\dot{C}^S = -\lambda_I C^S. \tag{25}$$

All of the individuals who are initially Latents and Contrarians ($L^S + C^S$) will become Active (A^H). A fraction $(1-\epsilon)$ of the initial Actives (A^S) will remain Active (A^H), with the rest switching to Latent (L^H). Thus only ϵA_0 will become Latent, and everyone else will be Active. □

Lemma 4. *If $v_R = 0$, $v_I = 1$, then $A^* = W\left(\frac{A_0+\epsilon\alpha}{(1-\epsilon)\alpha}e^{\frac{\epsilon}{1-\epsilon}}\right)(1-\epsilon)\alpha - \epsilon\alpha$ and $L^* = \frac{\epsilon}{1-\epsilon}A^*$.*

Proof. Eqs. (1)–(6) become

$$\dot{A}^H = (1-\epsilon)\lambda_I \frac{A^S}{A} \tag{26}$$

$$\dot{A}^S = -\lambda_R A^S - \lambda_I \frac{A^S}{A} \tag{27}$$

$$\dot{L}^H = \epsilon\lambda_I \frac{A^S}{A} \tag{28}$$

$$\dot{L}^S = -\lambda_R L^S \tag{29}$$

$$\dot{C}^H = \lambda_R(L^S + A^S + C^S) \tag{30}$$

$$\dot{C}^S = -\lambda_R C^S. \tag{31}$$

All of the individuals who are initially Latents and Contrarians ($L^S + C^S$) will transition to insensitive Contrarians (C^H). Thus we only need to focus on how the initial Actives (A^S) will transition to the three insensitive populations. Dividing Eq. (28) by Eq. (26) yields $\frac{dL^H}{dA^H} = \frac{\epsilon}{1-\epsilon}$. This produces the desired relationship for L^*. Next dividing Eq. (27) by Eq. (26) produces the following differential equation:

$$\frac{dA^S}{dA^H} = -\frac{\lambda_R}{(1-\epsilon)\lambda_I}(A^S + A^H) - \frac{1}{(1-\epsilon)}. \tag{32}$$

We can solve Eq. (32) using standard integrating factor techniques to determine the relationship between A^S and A^H at any point in time:

$$A^S = -A^H - \epsilon\alpha + (A_0 + \epsilon\alpha)e^{-\frac{1}{(1-\epsilon)\alpha}A^H}. \tag{33}$$

Substituting $A^S = 0$ into Eq. (33) produces an implicit function for the end-state fraction of Actives in the population

$$A_0 + \epsilon\alpha = (A^* + \epsilon\alpha)e^{\frac{1}{(1-\epsilon)\alpha}A^*}. \tag{34}$$

The solution to Eq. (34) is the Lambert W function expression specified in the lemma. □

Lemmas 5–7 apply to the Recency case.

Lemma 5. *The equilibrium point of the Recency equations described by (7)–(8) is the pair (A^*, C^*) defined by Eqs. (9)–(10).*

Proof. Eqs. (7)–(8) form a decoupled system. Rewriting those two equations yields a pair of equations easily solved using standard integrating factor techniques:

$$\begin{aligned}\dot{A} = &-(\lambda_R(1-v_R) + \lambda_I(1-v_I)(1+\epsilon)) \\ &\times A + \lambda_I(1 - v_I(1+\epsilon))\end{aligned} \tag{35}$$

$$\dot{C} = -(\lambda_I(1-v_I) + \lambda_R(1-v_R))\,C + \lambda_R(1-v_R). \tag{36}$$

The solutions to these two equations are

$$\begin{aligned}A(t) = &\frac{\alpha(1-v_I(1+\epsilon))}{1-v_R+\alpha(1-v_I)(1+\epsilon)} \\ &+ \left(A_0 - \frac{\alpha(1-v_I(1+\epsilon))}{1-v_R+\alpha(1-v_I)(1+\epsilon)}\right) \\ &\times e^{-(\lambda_R(1-v_R)+\lambda_I(1-v_I)(1+\epsilon))t}\end{aligned} \tag{37}$$

$$\begin{aligned}C(t) = &\frac{(1-v_R)}{\alpha(1-v_I)+(1-v_R)} \\ &+ \left(C_0 - \frac{(1-v_R)}{\alpha(1-v_I)+(1-v_R)}\right) \\ &\times e^{-(\lambda_I(1-v_I)+\lambda_R(1-v_R))t}.\end{aligned} \tag{38}$$

As $t \to \infty$ Eqs. (37) and (38) converge to the fixed point values defined in (9) and (10), respectively. □

Lemma 6. *If $v_R = v_I = 1$ and $\varepsilon > 0$, then $C^* = C_0$ and $A^* = 0$.*

Proof. Eqs. (7)–(8) become

$$\dot{A} = -\lambda_I\epsilon v_I \tag{39}$$

$$\dot{C} = 0 \tag{40}$$

and since $\varepsilon > 0$, the results follow. □

Lemma 7. *The following relationships hold*

- *If $v_R = v_I = 0$, then $C^* = \frac{1}{1+\alpha}$ and $A^* = \frac{\alpha}{1+(1+\epsilon)\alpha}$.*
- *If $v_R = 1$, $v_I = 0$, then $C^* = 0$ and $A^* = \frac{1}{1+\epsilon}$.*
- *If $v_R = 0$, $v_I = 1$, then $C^* = 1$ and $A^* = 0$.*

Proof. These results follow immediately by substituting the specific values of v_R and v_I into Eqs. (9) and (10), proved in Lemma 5. □

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