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Theory and Methodology

Multiple criteria modelling and ordinal data: Evaluation in terms of subsets of criteria ★

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Abstract

In an earlier article an ordinal multiple criteria model was presented in which each member of a set of alternatives was given an evaluation on each member of a set K of criteria. In this paper we extend this concept to the situation where each alternative i can be assessed in terms of only a subset K_i of K. Various models are presented for dealing with this partial criteria case and the pros and cons of these models are discussed. © 1997 Elsevier Science B.V.

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1. Introduction

Multiple criteria decision modelling has attracted significant attention in the literature over the past several decades. One particular area of MCDM has dealt specifically with those situations where ordinal data is present. In a recent paper by Cook and Kress [2] (CK) a model was presented in which each of a set of N alternatives is given an ordinal rank on each of K criteria. Furthermore, it is assumed that these K criteria can be ordinally ranked in order of importance, and can be further ranked in order of the clearness with which one can discriminate among the alternatives. Using this ordinal information, CK present a model for assigning a rating to each alternative. The underlying principle behind this model is the data envelopment analysis (DEA) approach of Charnes, Cooper and Rhodes [1], wherein each alternative is evaluated relative to all the other competing alternatives.

A key assumption in the CK model is that each alternative to be ranked is evaluated (given an ordinal rank) in terms of each and every criterion. In many situations, however, as will be discussed later, an alternative i may have the opportunity of being evaluated in terms of only a subset of criteria. The problem then arises as to how to rate, in a fair manner, the relative importance of such an alternative vis-a-vis other alternatives whose ratings are to be based on *different* sets of criteria. A number of possible model structures are developed and the pros and cons of these alternative formulations are discussed. Two of these models, the aggressive and average evaluation models, can be viewed as direct extensions to the CK model. The third model is based on ideal or best performance, wherein each alternative is compared to the best possible rating

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achievable within *its* set of criteria. While this model, like the others, has its shortcomings, its structure is well suited to the partial criteria setting.

In Section 2 we review the basic structure of the CK model in which all criteria apply to each and every alternative. Section 3 examines the partial criteria setting, and proposes three alternative model structures. Section 4 discusses some pros and cons of these models.

2. The CK model

The Cook and Kress [2] model is designed to derive a rating for each of a set of alternatives where multiple criteria and ordinal data are present. Specifically, it is assumed that each alternative $i \in \{1,\dots,N\}$ can be evaluated in terms of a set of criteria $K = \{1,\dots,K\}$. This evaluation amounts to assigning to i a rank position or category $l \in \{1,\dots,L\}$ where $L \leqslant N$. Typical values for L might be 5, 7 or 9. Furthermore, it is assumed that, at a minimum, the criteria can be rank ordered (ordinally) in terms of *importance*, and additionally that they can be ranked relative to the *clearness* with which one would be able to distinguish among alternatives.

CK propose two reasonable models for obtaining an overall rating R_i for each alternative i that make use of this ordinal preference information.

Model 1: A DEA model

For purposes of completeness we summarize the basic principles on which the Cook and Kress model is built. The model addresses three important issues pertaining to multiple criteria settings:

(1) The importance or weight associated with each criterion.
(2) The importance of the various rank levels or positions at which an alternative can be placed.
(3) The clearness or exactness with which one can discriminate among alternatives on any given criterion.

To address these three issues, CK first define the decision variables w_{kl}, the weight or importance accorded an alternative that is ranked in lth place on the kth criterion. The idea behind the CK model is to find the most appropriate set of weights w_{kl} so that each alternative i is given the fairest rating R_i in terms of these. Assume, with no loss of generality, that the criteria k are already ranked in order of importance; i.e., criterion 1 is more important than criterion 2, and so on. By definition, the rank positions l are already ranked in order of importance on each criterion. In deriving a set of weights w_{kl}, it is clear that two sets of conditions must be met, namely, $w_{kl} > w_{k+1l}$ and $w_{kl} > w_{kl+1}$ for all k, l. That is, the worth of being ranked in lth place on the criterion k is greater than being ranked at that place on a lower ranked criterion.

To cover a somewhat broader range of situations where the decision maker may wish to specify various patterns that the w_{kl} might follow, Cook and Kress define three *discrimination intensity functions* G_{kl}, H_k, F. The H_k are supplied scaling factors used to represent the *minimum relative* amounts of difference or discrimination between consecutive criteria weights desired by the decision maker. If, for example, one wished to impose the condition that the minimum gaps $w_{kl} - w_{k+1l}$ between consecutive criteria should be a decreasing set of values, one might choose $H_k = 1/k$. To translate these relative minimum gaps into absolute gaps, the H_k are augmented (multiplied) by a decision variable v, and the above simple restrictions are replaced by

$$w_{kl} - w_{k+1l} \geqslant vH_k.$$

This same reasoning is applied to the rank position l. Here, the CK model represents the discrimination intensity function G_{kl} as $G_{kl} = g_l t_k$ where g_l sets the relative positioning of the rank levels l versus $l+1$, and $t_k = \sum_{j=k}^{K} H_j / \sum_{j=1}^{K} H_j$ represents the contraction in rank position gaps relating to the criterion k in question. Thus, t_k decreases as k increases, meaning that the minimum relative gaps get smaller as we go to lesser important criteria. Analogous to the variable v introduced above, define for each criterion k a variable u_k and replace the simple restrictions on the rank position l by

$$w_{kl} - w_{kl+1} \geqslant g_l t_k u_k.$$

The variable u_k can further be used to capture the fuzziness or degree to which we can clearly discriminate between rank positions l when that criterion k is involved. Arguably, if alternatives can be distinguished more clearly on the basis of criterion k_1 then k_2, the minimum gap between l and $l+1$ when k_1 is involved should be greater than when k_2 is involved. This can be accomplished by requiring that the restriction $u_{k_1} - u_{k_2} \geqslant F$ or more correctly

$$u_{\langle j \rangle} - u_{\langle j+1 \rangle} \geqslant F,$$

where $u_{\langle j \rangle} = u_k$ if criterion k is ranked jth in terms of clearness.

The basic problem then is to derive a set of weights or multipliers w_{kl} that measure the *worth* of being ranked lth on the kth criterion. To represent in notational form the rank positions occupied by a given alternative i, define

$$d_{kl}(i) = \begin{cases} 1, & \text{if } i \text{ is ranked in } l\text{th place on criterion } k, \\ 0, & \text{otherwise.} \end{cases}$$

If it is now assumed that the final rating, that is the overall worth accorded any alternative, is the (linear) sum of the credits w_{kl} received from the K criteria, then whatever w_{kl} are chosen, this *composite* rating R_i is given by

$$R_i = \sum_{k=1}^{K} \sum_{l=1}^{L} d_{kl}(i) w_{kl}.$$

CK propose choosing a set of w_{kl} which render the most favourable (largest) value for R_i. This concept, in the spirit of the data envelopment analysis (DEA) methodology of Charnes, Cooper and Rhodes [1], amounts to solving N linear programming problems. Specifically, for each alternative i_o solve the problem:

$$\text{(P)} \qquad R^*_{i_o}(z) = \max R_{i_o}(z) = \sum_{k=1}^{K} \sum_{l=1}^{L} d_{kl}(i_o) w_{kl} \tag{2.1}$$

s.t.

$$\sum_{k=1}^{K} \sum_{l=1}^{L} d_{kl}(i) w_{kl} \leqslant 1, \quad i = 1, \ldots, N; \tag{2.2}$$

$$w_{kl} - w_{kl+1} - g_l t_k u_k \geqslant 0, \quad k = 1,2,\ldots,K, \quad l = 1,2,\ldots,L-1; \tag{2.3a}$$

$$w_{kL} - g_L t_k u_k \geqslant 0, \quad k = 1,2,\ldots,K;$$

$$w_{k1} - w_{kL} - t_k(w_{11} - w_{1L}) \leqslant 0, \quad k = 1,2,\ldots,K; \tag{2.3b}$$

$$w_{kl} - w_{k+1l} - vH_k \geqslant 0, \quad k = 1,2,\ldots,K-1, \quad l = 1,2,\ldots,L; \tag{2.4}$$

$$w_{Kl} - vH_K \geqslant 0, \quad l = 1,2,\ldots,L;$$

$$u_{\langle j \rangle} - u_{\langle j+1 \rangle} - F \geqslant 0, \quad j = 1,2,\ldots,K-1; \tag{2.5}$$

$$v \geqslant z; \tag{2.6}$$

$$F \geqslant z; \tag{2.7}$$

$$w_{kl}, u_k, v, F \geqslant 0, \quad \forall k, l. \tag{2.8}$$

The constraints in this model are essentially of three types. Constraints (2.2) impose an upper limit on the size that R_i can achieve. The limit of 1 is arbitrary, however, the overall R_i in relative terms are invariant to this choice. Constraints (2.3a), (2.4) impose limits on the gaps between the importance weights (w_{kl}, w_{kl+1}) attached to consecutive rank positions and (w_{kl}, w_{k+1l}) to consecutive criteria, as discussed above. Constraints (2.3b) are intended to render the overall maximum range $w_{k1} - w_{kN}$ for criterion k proportionally smaller than that for criterion 1, by utilizing the t_k contraction parameters. Constraints (2.5) provide a mechanism for distinguishing between those criteria that are *clear* and those that are less so as per the earlier discussion. Finally, constraints (2.6) and (2.7) restrict the discrimination factors v and F to be strictly non zero by at least z.

The reader is referred to Cook and Kress [2] for a detailed development and rationale behind this model.

Model 2: A common weight set model

In Model 1 the parameter z is generally taken to be a small positive value. In the DEA setting z is represented by ε and is assumed to be an infinitesimal. Clearly, since the N problems (P) will yield a different set of w_{kl} for each alternative i, the choice of a value for z can affect what those w_{kl} turn out to be. Moreover, the numerical values for the R_i are also affected by the choice of z.

Arguably, this issue of an appropriate value for z and the controversy that often surrounds the use of different weights to evaluate different alternatives point to the desirability of a single or *common* set of weights.

In that regard, CK suggest solving a single optimization problem and thereby arrive at the desired single set of w_{kl}. Specifically, it is suggested to solve the problem:

$$\text{(PC)} \quad z^* = \max z \qquad (2.9)$$
$$\text{s.t.} \quad (2.2)\text{–}(2.8).$$

In determining the maximum value for the *discrimination* parameter z we are, in a sense, distinguishing between consecutive rank positions and between consecutive criteria to the greatest extent possible. At the same time one set of multipliers w_{kl} materializes, and the left hand sides of constraints (2.2) are the corresponding ratings R_i for the alternatives.

In the CK model(s) the principal assumption is that each alternative i receives a rank position l on each criterion k in the set K. In the general, and often prevalent case where only criteria in some proper subset $K_i \subset K$ are pertinent to the evaluation of alternative i, these models may no longer apply. We now examine a number of possible approaches to this more general case. We do not advocate the use of any particular one of these models over the others since the choice depends upon one's definition of *fair evaluation*. We do, however, discuss the pros and cons of the various approaches.

3. Evaluation relative to partial criteria

In the previous sections it was assumed that any given alternative i could be evaluated (assigned a rank position) in terms of each member k of the full set of criteria K. In many decision environments, however, this requirement is not pertinent. Consider, for example, the case where in ranking projects in an electric utility company, one may be considering alternatives such as construction of power lines, additions and modifications to nuclear reactors, upgrades to buildings, maintenance of office facilities, and so on. In such a varied set of alternatives, criteria such as "impact on environment", or "contribution to technological advancement" may apply to some options (e.g., reactor construction), but may be entirely inapplicable to others such as building maintenance. In a completely different setting, consider one of the principal application areas of data envelopment analysis, namely the evaluation of productivity of a set of bank branches. See, for example, Sherman and Gold [4] and Oral and Yolalan [3]. The traditional settings examined to date and cited in the literature, view banks at a given point in time and assume each branch can be evaluated in terms of the same criteria (inputs and outputs). If we want to compare, however, the *new* full service type of banking environment to the traditional branches, problems arise. The new style banks now offer services such as life and property insurance policies, mutual fund investment options, and so on, that are not available in the current (conventional) branches. The comparison of old and new as a single set will then need to consider the partial criteria issue.

The problems associated with comparing a set of alternatives (projects, bank branches, etc.) when some criteria are relevant to certain members of the set but not to others, revolve around the interpretation of missing data and how to account for it. One ad hoc approach to this has been to generate synthetic data by using, for example, an average value for a criterion, where the average is over those alternatives for which that criterion is relevant. In the case of the bank branches, for instance, this would mean looking at the average of insurance sales for the new style branches, and then crediting each of the old style branches with that average value. In assessing projects, one option clearly is to fully penalize an alternative for "failing to perform" on a given dimension. Being fully penalized may mean being credited with the worst possible rank position on the given criterion, or being assigned no rank at all. This latter is the basis for the aggressive model to follow. On the other hand, if one argues that an alternative should not be penalized for not being eligible to be ranked on a given criterion, then a more benevolent action should be taken.

We now consider the general case in which an alternative i can be evaluated in terms of only a subset $K_i \subseteq K$ of the criteria. The manner in which the set of N alternatives is to be evaluated in this *partial criteria* case depends upon the assumptions one makes regarding *fair comparison*. We present three approaches to the evaluation:

3.1. Aggressive evaluation

One point of view regarding evaluation of the N alternatives is to adopt the original full criteria model ((2.1)-(2.8)), and replace the term $\sum_{k=1}^{K}\sum_{l=1}^{L} d_{kl}(i) w_{kl}$ by $\sum_{k\in K_i}\sum_{l=1}^{L} d_{kl}(i) w_{kl}$. In this case when a criterion k_o is not part of the pertinent set K_{i_o} for alternative i_o, a credit of 0 is given. That is $d_{k_o l}(i_o) = 0$ for all l. This approach subscribes to the concept that part of any alternative's worth (e.g., the worth of a project to an organization) is the benefit w_{kl} derived from each criterion. Hence, the fact that the project cannot compete in terms of a particular criterion k only serves to put that project at a disadvantage vis-a-vis other projects which *do* obtain a rank position on k. Thus, projects must compete *aggressively* (or at least are evaluated aggressively) with no compensation for failure to achieve a standing relative to certain criteria.

Clearly, this approach rewards those alternatives for which the cardinality $|K_i|$ of K_i is large, and penalizes those for which the cardinality is small.

While the approach has the advantage of treating all alternatives on an equal footing, it could be judged as being unfairly harsh in situations where criteria are simply inapplicable. In a situation, for example, where environmental impact is one of the factors used for evaluation, the 1 $\rightarrow$ L scale may, in some circumstances, be interpreted as "good" to "bad". Thus, a rating of $l = 1$ means that an alternative has a very positive effect vis-a-vis environmental benefits, while $l = L$ may imply a very negative impact. An alternative (e.g., building maintenance) which is *neutral* should, if given a rank at all, be rated somewhere in the middle of the scale. Hence, the manner in which scales are defined can influence the applicability of the standard model in the partial criteria case.

3.2. Average performance evaluation

To avoid the potential problems created by cardinality differences among the sets K_i, as cited in the previous model, an approach which utilizes an *average performance* per pertinent criterion can be adopted. Specifically, we replace $\sum_{k=1}^{K}\sum_{l=1}^{L} d_{kl}(i) w_{kl}$ in (2.1) and (2.2) by $\sum_{k\in K_i}\sum_{l=1}^{L} d_{kl}(i) w_{kl}/|K_i|$. In a certain sense, this model is a natural extension of (P). That is, if in the full criteria case (i.e., $|K_i| = N$ for all i) we replace $\sum_{k=1}^{K}\sum_{l=1}^{L} d_{kl}(i) w_{kl}$ by $\sum_{k=1}^{K}\sum_{l=1}^{L} d_{kl}(i) w_{kl}/N$, we get a formulation equivalent to (P). This formulation avoids the size differences in the K_i, but does penalize the alternative i whose criteria set K_i contains low ranked criteria versus an alternative that may be evaluated in terms of a similar number, but of higher ranked criteria. As with the previous model, there may be circumstances where this is a desirable property, and others where it is not.

3.3. Benevolent evaluation: performance relative to the ideal

In the case where we want to evaluate alternatives in the fairest possible (i.e., most *benevolent*) way, it can be argued that such an evaluation should not penalize an alternative for failing to be considered in terms of a large portion of the criteria, nor for failing to be evaluated relative to the most important criteria. This approach would then advocate evaluating an alternative in terms of only those criteria k on which it receives a ranking l. Only the importance of these "pertinent" criteria *relative to one another* would then come into play, and the standing of these criteria vis-a-vis the complementary set (the set on which i is not evaluated) would not enter the picture.

One means of accomplishing the aforementioned benevolent approach is to compare each alternative i to the best possible or ideal performance for that alternative. In the notation of CK in Section 2, the *ideal alternative* would receive a rating of

$$R_{\text{ideal}} = \sum_{k\in K} w_{k1}.$$

Clearly, any alternative i which ranks lower than first place ($l > 1$) on any criterion k will score worse than this ideal, hence $R_i \leqslant R_{\text{ideal}}$. Thus, the measure

$$\hat{R}_i = R_i / R_{\text{ideal}},$$

is a reasonable and convenient way of expressing the performance level of i. $\hat{R}_i$ is similar in some respects to an industrial productivity measure where we compare actual to *standard* performance, although it could be argued that R_{standard} is probably something less than R_{ideal}. For our purposes, R_{ideal} represents the only tangible (and, in principle, achievable) measure that can be used as a backdrop against which to evaluate alternatives.

With this concept as a basis, and proceeding in a manner analogous to problem (P), consider the following N problems:

$$\text{(PI)} \quad \hat{R}^*_{i_o} = \max \hat{R}_{i_o} = \frac{\sum_{k\in K_{i_o}} \sum_{l=1}^{L} d_{kl}(i_o) w_{kl}}{\sum_{k\in K_{i_o}} w_{k1}} \tag{3.1}$$

$$\text{s.t.} \quad \frac{\sum_{k\in K_i} \sum_{l=1}^{L} d_{kl}(i) w_{kl}}{\sum_{k\in K_i} w_{k1}} \leqslant 1, \quad i = 1,\dots,N; \tag{3.2}$$

(2.3a)-(2.8).

In this ratio formulation, the numerator in (3.2) represents the actual performance of alternative i, with the denominator being the theoretical or best possible performance. It is noted that in this formulation constraints (3.2) are redundant, and can, therefore, be removed from the problem. Unlike the linear problem (P), (PI), having a fractional objective function, is nonlinear, and in general can be difficult to solve. By way of a transformation, however, (PI) can be converted to a linear format. Specifically, let

$$\tau_o = 1 / \sum_{k\in K_{i_o}} w_{k1}$$

and define the variables $\tilde{w}_{kl} = \tau_o w_{kl}, \tilde{u}_k = \tau_o u_k, \tilde{\nu} = \tau_o \nu$ and $\tilde{F} = \tau_o F$. Problem (PI) (in the absence of constraints (3.2)) can then be written in the form:

$$\text{(PIL)} \quad \hat{R}^*_{i_o} = \max \hat{R}_{i_o} = \sum_{k\in K_{i_o}} \sum_{l=1}^{L} d_{kl}(i_o) \tilde{w}_{kl} \tag{3.3}$$

s.t.

$$\sum_{k\in K_{i_o}} \tilde{w}_{kl} = 1; \tag{3.4}$$

$$\tilde{w}_{kl} - \tilde{w}_{kl+1} - g_l t_k \tilde{u}_k \geqslant 0, \quad k = 1,\dots,K; \quad l = 1,\dots,L-1;$$

$$\tilde{w}_{kL} - g_L t_k \tilde{u}_k \geqslant 0, \quad k = 1,\dots,K; \tag{3.5a}$$

$$\tilde{w}_{kl} - \tilde{w}_{kL} - t_k(\tilde{w}_{11} - \tilde{w}_{1L}) \leqslant 0, \quad k = 1,\dots,K; \tag{3.5b}$$

$$\tilde{w}_{kl} - \tilde{w}_{k+1l} - \tilde{\nu} H_k \geqslant 0, \quad k = 1,\dots,K-1; \; l = 1,\dots,L;$$

$$\tilde{w}_{Kl} - \tilde{\nu} H_K \geqslant 0, \quad l = 1,\dots,L; \tag{3.6}$$

$$\tilde{u}_{(j)} - \tilde{u}_{(j+1)} - \tilde{F} \geqslant 0, \quad j = 1,\dots,K-1; \tag{3.7}$$

$$\tilde{\nu} \geqslant \tau_o z; \tag{3.8}$$

$$\tilde{F} \geqslant \tau_o z; \tag{3.9}$$

$$\tau_o, \tilde{w}_{kl}, \tilde{u}_k, \tilde{\nu}, \tilde{F} \geqslant 0, \quad \forall k, l. \tag{3.10}$$

Lemma 1. *There exists an optimal solution to* (PI) *in which* $\sum_{k\in K_{i_o}} w_{k1} \leqslant 1$.

Proof. For any feasible solution $W = (w_{kl})$ to (PI), cW is also a feasible solution for any $c \geqslant 1$. Hence, we may impose a bounding constraint $\sum_{k\in K_{i_o}} w_{k1} \leqslant \ominus$ in (PI) for some $\ominus$ and still have a problem equivalent to (PI). Furthermore, for z small enough we may, with no loss of generality, arbitrarily choose $\ominus = 1$. Hence, the result. □

Lemma 2. *There exists an optimal solution* $\tilde{w}^*_{kl}$, $\tilde{u}^*_k$, $\tilde{\nu}^*$, $\tilde{F}^*$, τ^*_o *to* (PIL) *in which* $\tau^*_o = 1$.

Proof. Due to Lemma 1 and the definition of τ_o, we have

$$\tau^*_o = \frac{1}{\sum_{k\in K_{i_o}} w^*_{k1}} \geqslant 1.$$

To yield maximum flexibility in the problem, it is optimal to force τ_o to its lower limit (the problem is the least restricted in this case). Hence $\tau^*_o = 1$. □

Theorem 3. *In the special case where all* $K_i = K$ *and* $|K| = K$, *problem* (PIL) *is equivalent to problem* (P) *if an* $(N+1)$*st alternative, the ideal alternative, is added to the latter.*

Proof. From Lemma 2 $\tau^*_o = 1$, hence $w^*_{kl} = \tilde{w}^*_{kl}$, $u^*_k = \tilde{u}^*_k$, $\nu = \tilde{\nu}^*$, $F^* = \tilde{F}^*$. Furthermore, constraint (3.4) may be replaced by $\sum_{k=1}^{K} w_{k1} \leqslant 1$, the upper limit on the rating for the ideal alternative. Since constraints (2.2) are redundant in the presence of this inequality, the result follows. □

By virtue of Theorem 3, problem (PI) can be written in the form:

$$(\text{PIL}') \quad \hat{R}_{i_o} = \max R_{i_o} = \sum_{k \in K_{i_o}} \sum_{l=1}^{L} d_{kl}(i_o) w_{kl} \tag{3.11}$$

$$\text{s.t.} \quad \sum_{k \in K_{i_o}} w_{k1} \leqslant 1; \tag{3.12}$$

$$(2.3\text{a})–(2.8).$$

Common set of weights

As with problem (P), (PIL′) will generally yield a different set of weights w_{kl} for each alternative i_o being evaluated. Along the lines of the previous section, a common set of weights can be derived by solving the problem:

$$(\text{PC}') \quad z^* = \max z \tag{3.13}$$

$$\text{s.t.} \quad \sum_{k \in K_i} w_{kl} \leqslant 1, \quad i = 1, \ldots, N; \tag{3.14}$$

$$(2.3\text{a})–(2.8).$$

This problem is clearly bounded since every criterion k can be assumed to lie in at least one subset K_i, hence $w_{kl} \leqslant 1$ for all k. Thus, z will achieve an optimum. The final ratings to be assigned to any alternative i is given by

$$\tilde{R}_i = \frac{\sum_{k \in K_i} \sum_{l=1}^{L} d_{kl}(i) w_{kl}^*}{\sum_{k \in K_i} w_{k1}^*},$$

where the w_{kl}^* are the optimal variables from problem (PC′).

Model (PI) (hence model (PC′)) has the advantage that it provides a fair evaluation to an alternative i, regardless of the status of those criteria K_i that pertain to that alternative. Specifically, an alternative is not penalized for or given an unfair advantage because of the nature of its particular criteria. This very property may in certain circumstances, however, be seen as a weakness of the approach. If in a project rating situation, for example, the contribution of projects to a specific management goal is a key element in deciding on the set of choices to be funded, then the model of this section may not be appropriate. On the other hand, if projects from different departments are to be fairly assessed so that all contenders have an opportunity to compete, then it may be desirable not to have criteria not pertinent to an alternative, affect how that alternative is rated in a relative sense.

4. Discussion

In this paper we extend the full criteria model of Cook and Kress [2] to the general case where only a subset K_i of the full set K of criteria are applicable to a given alternative i. Depending upon the situation, different models for this partial criteria environment can be formulated. Three possible models are given herein. The first two models, the *aggressive* and *average* performance evaluation formulations, can be viewed as direct extensions of the CK model. They apply to those environments where criteria importance must be maintained and rewarded (or penalized) whether applicable to an alternative or not. The aggressive model can be criticized for harshly penalizing those alternatives that only relate to a small number of the criteria. Thus, an alternative that can be given a rating, even a very low one, on a large proportion of the criteria will be rated higher than an alternative that is able to compete (and perhaps have top performance) on only a few criteria. The average performance model corrects for this shortcoming by utilizing the mean over those criteria that are applicable.

In the case where it is desirable to view an alternative strictly in terms of those criteria that are applicable, and where the importance of inapplicable criteria is irrelevant, the ideal performance model is proposed. Here, we take as a project's rating the ratio of the sum of weights for the alternative to the sum of the weights for an ideal alternative (evaluated in the same criteria). Although not discussed here, other options to the ideal performance model are possible. Clearly, the denominator of model (PI) could, for example, be replaced by the worst (i.e. $\sum_{k \in K_i} w_{kL}$) rather than the best performance. With a suitable adjustment to the right hand side of constraints (3.2), that is by replacing 1 by a fixed constant Θ (chosen large enough), another definition of relative performance arises. Here, the ratio would be interpreted as the gain in performance over the lowest achievable performance.

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